1. (a) (**5 marks**) If $n \not| a$ and $n \not| b$, then $n \not| ab$.

(b) (**10 marks**) True, for example, $A(1)$ is true.

(c) (**10 marks**) False. For example, if $a = 2$ and $b = 2$, then $A(4)$ is false.

2. (**25 marks**) After relabeling, we may suppose $m_1 \leq m_2 \leq \ldots \leq m_n$.

For $n \in \mathbb{N} \cup \{0\}$, let P_n be the statement that $W(n)$ is well-ordered.

Base: P_0 is the statement that $W(0) = \mathbb{N}$ is well-ordered, which is true as shown in the class.

Inductive step: Suppose $k \in \mathbb{N} \cup \{0\}$ and P_k is true, that is, $W(k)$ is well-ordered. Then $W(k+1) = \{-m_{k+1}\} \cup W(k)$. Let K be a non-empty subset of $W(k+1)$.

If $-m_{k+1} \in K$, then $-m_{k+1}$ is its least element.

If $-m_{k+1} \notin K$, then $K \subseteq W(k)$ and by induction assumption, K also has a least element. It follows that P_{k+1} is true.

Hence, by induction, P_n is true for all $n \in \mathbb{N} \cup \{0\}$.

3. (**25 marks**) **Reflexive**: $a \sim a$ because $|a| = |a|$.

Symmetric: $a \sim b \iff |a| = |b| \iff |b| = |a| \iff b \sim a$.

Transitive: $(a \sim b) \wedge (b \sim c) \iff (|a| = |b|) \wedge (|b| = |c|)$, so $|a| = |c| \iff a \sim c$.

It follows that $\sim$ is an equivlence relation on $\mathbb{Z}$.

Let $x \in \mathbb{Z}$. Then $x \in [2]$, if and only if $|x| = |2| = 2$, so that $x = 2$ or -2. Thus

$$[2] = \{2, -2\}.$$

4. (**25 marks**) Suppose h is onto and $A, B \in \mathcal{P}(Y)$. If $A \subseteq B$ and $x \in h^{-1}(A)$, then $y = h(x) \in A$ and $y \in B$ as $A \subseteq B$. Thus $x \in h^{-1}(B)$ and $h^{-1}(A) \subseteq h^{-1}(B)$. We have shown that if $A \subseteq B$, then $h^{-1}(A) \subseteq h^{-1}(B)$.

Conversely, suppose $h^{-1}(A) \subseteq h^{-1}(B)$ and $y \in A$. Since h is onto, $y = h(x)$ for some $x \in X$. In particular, $x \in h^{-1}(A)$ and so $x \in h^{-1}(B)$. Thus $h(x) \in B$ and hence $y = h(x) \in B$. We have shown that if $h^{-1}(A) \subseteq h^{-1}(B)$, then $A \subseteq B$.

Therefore $A \subseteq B \iff h^{-1}(A) \subseteq h^{-1}(B)$ and $h^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is order preserving.