Maths 255SC Class Test: Overview of questions. Detach for reference

1. (**25 marks**) Let a, b be integers and let $A(n)$ be the predicate:

If n divides ab, then n divides a or n divides b.

(a) Write down the contrapositive of $A(n)$.

(b) Determine with reason whether it is true that $\exists n \in \mathbb{N}\ A(n)$.

(c) Determine with reason whether it is true that $\forall n \in \mathbb{N}\ A(n)$.

2. (**25 marks**) Let n be a non-negtive integer and $m_1, m_2, \ldots, m_n$ natural numbers and let $W(n)$ be the set

$$W(n) = \{-m_1, -m_2, \ldots, -m_n\} \cup \mathbb{N}.$$

Prove by mathematical induction on n that $W(n)$ is a well-ordered set.

3. (**25 marks**) Let $\sim$ be a relation on $\mathbb{Z}$ defined by $x \sim y$ if and only if $|x| = |y|$. Show that $\sim$ is an equivalence relation, and find the equivalence class $[2]$ containing 2.

4. (**25 marks**) Let $h : X \to Y$ be a function. If $h : X \to Y$ is onto, show that the function $h^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ by $B \mapsto h^{-1}(B)$ is an order preserving function, where $\mathcal{P}(Y)$ and $\mathcal{P}(X)$ are regarded as posets under inclusion $\subseteq$.