1. (a) (**5 marks**) A Cayleyb table of $(U(5), \cdot_5)$ is

$\cdot_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

(b) (**4 marks**) Define $\varphi : U(5) \to \mathbb{Z}_4$ by $\varphi(1) = 0$, $\varphi(2) = 1$, $\varphi(3) = 3$ and $\varphi(4) = 2$.
(**4 marks**) Then the image of the Cayley table of $U(5)$ is

	0	1	3	2
0	0	1	3	2
1	1	2	0	3
3	3	0	2	1
2	2	3	1	0

(**3 marks**) which is a Cayley table of $(\mathbb{Z}_4, +_4)$. It follows that $\varphi(x \cdot_5 y) = \varphi(x) +_4 \varphi(y)$ for $x, y \in U(5)$. (**2 marks**) Since φ is also a bijection, φ is an isomorphism and so $(U(5), \cdot_5) \simeq (\mathbb{Z}_4, +_4)$.

2. (**3 marks**) Since $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in H$, it follows that $H \neq \emptyset$.

(**7 marks**) If $A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$ are elements of H, then

$$A - B = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix} - \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix} = \begin{pmatrix} 0 & (a-b) \\ -(a-b) & 0 \end{pmatrix}.$$

Since $a, b \in \mathbb{R}$, it follows that $a - b \in \mathbb{R}$ and so $A - B \in H$. By One-Step-Test, H is a subgroup of $\text{Mat}_2(\mathbb{R})$.

3. (a) (**6 marks**) Since $1^{-1} = 1 \in K$ and $6^{-1} = 6 \in K$, it follows that $K \neq \emptyset$ and $x^{-1} \in K$ for each $x \in K$. Similarly, $1 \cdot_7 1 = 6 \cdot_7 6 = 1 \in K$ and $1 \cdot_7 6 = 6 \in K$, which implies that $x \cdot_7 y \in K$ for any $x, y \in K$. By Two-Step-Test, K is a subgroup of $U(7)$.

(b) (**8 marks**) For a group $(G, *)$, let $X_G = \{x \in G : x * x = e_G\}$. Then by Proposition 1.27, if $H \simeq G$ as groups, then the number of elements of X_G and X_H are the same.
Since $X_{U(7)} = \{1, 6\}$ and since $X_{D_3} = \{R_0, V, V', V''\}$, it follows that $U(7)$ is not isomorphic to D_3.

(c) (**8 marks**) The left cosets of K in $U(7)$ are
$K = \{1, 6\}$
$2K = \{2, 5\}$ and
$3K = \{3, 4\}$.