1. (a) (**2 marks**) If $a, b \in \mathbb{R}^*$, then $a \neq 0$ and $b \neq 0$, so $a/b \neq 0$ and $a/b \in \mathbb{R}^*$. Thus / is a binary operation.

(b) (**4 marks**) Suppose $e \in \mathbb{R}^*$ is the identity. Then $a/e = a$ for each $a \in \mathbb{R}^*$, so that $e = 1$. But $e/a = 1/a \neq a$ in general. For example, when $a = 2$, $1/2 \neq 2$. It follows that $(\mathbb{R}^*, /)$ has no identity.

(c) (**4 marks**) For $a, b, c \in \mathbb{R}^*$, $(a/b)/c = a/(bc)$ and $a/(b/c) = (ac)/b$. But $a/(bc) \neq (ac)/b$ in general, for example, when $a = 1, b = c = 2$, $a/(bc) = 1/4$ and $(ac)/b = 2/2 = 1$. Thus / is not associative.

2. (i) (**3 marks**) Let $a, b \in \mathbb{R}^+$. Then $a > 0$ and $b > 0$, so that $ab > 0$ and hence $ab \in \mathbb{R}^+$. Thus the multiplication is a binary operation.

(ii) (**2 marks**) 1 is the identity, since $1a = a1 = a$ foe any $a \in \mathbb{R}^+$.

(iii) (**3 marks**) For $a \in \mathbb{R}^+$, $a > 0$ and so $1/a > 0$. Thus $1/a \in \mathbb{R}^+$ and $1/a$ is the inverse of a, since $a(1/a) = (1/a)a = 1$.

(iv) (**2 marks**) The associativity $a(bc) = (ab)c$ holds for all real numbers a, b, c, so it also holds for $a, b, c \in \mathbb{R}^+$.

It follows that $R^+ = (0, \infty)$ is a group under multiplication.

3. (a) (**8 marks**) Suppose

R_0 is the rotation of $0°$,

R_{90} is the rotation of $180°$,

H is the rotation of $180°$ about horizontal axis and

V is the rotation of $180°$ about vertical axis.

Then $S(X) = \{R_0, R_{180}, H, V\}$.

(b) (**7 marks**)

$\circ$	R_0	R_{180}	H	V
R_0	R_0	R_{180}	H	V
R_{180}	R_{180}	R_0	V	H
H	H	V	R_0	R_{180}
V	V	H	R_{180}	R_0

(c) (**3 marks**) $R_0^{-1} = R_0$, $R_{180}^{-1} = R_{180}$, $H^{-1} = H$ and $V^{-1} = V$.

4. (a) (**9 marks**)

$*$	R_0	R_{120}	R_{240}	V	V'	V''
R_0	R_0	R_{120}	R_{240}	V	V'	V''
R_{120}	R_{120}	R_{240}	R_0	V'	V''	V
R_{240}	R_{240}	R_0	R_{120}	V''	V	V'
V	V	V''	V'	R_0	R_{240}	R_{120}
V'	V'	V	V''	R_{120}	R_0	R_{240}
V''	V''	V'	V	R_{240}	R_{120}	R_0

(b) (**3 marks**) $R_{120} * V = V' \neq V'' = V * R_{120}$.