1. (**5 marks each**)

(a) Note that $\rho = \{(x, y) \in A \times A : xy = 0\} = \emptyset$ since both x and y are positive. So ρ is **symmetric, antisymmetric** and **transitive**. But ρ is **not reflexive** because $(1, 1) \notin \rho$.

(b) Note $\rho = \{(0, 4), (4, 0), (1, 3), (3, 1), (2, 2)\}$. So ρ is **not reflexive** because $(0, 0) \notin \rho$.
Symmetric: $(x, y) \in \rho \iff x + y = 4 \iff y + x = 4 \iff (y, x) \in \rho$;
Not antisymmetric: $0\rho 4 \wedge 4\rho 0$ but $4 \neq 0$.
Not transitive: $0\rho 4 \wedge 4\rho 0$ but $(0, 0) \notin \rho$.

(c) **Not reflexive**: $(4, 4) \notin \rho$.
Not symmetric: $(2, 1) \in \rho$ but $(1, 2) \notin \rho$.
Not antisymmetric: $2\rho 4 \wedge 4\rho 2$ but $4 \neq 2$.
Not Transitive: $4\rho 2 \wedge 2\rho 1$ but $(4, 1) \notin \rho$.

(d) **Reflexive**: for all $x \in D$, $x - x = 0 \in \mathbb{Z}$.
Symmetric: $(x, y) \in \rho \iff x - y \in \mathbb{Z} \iff y - x = -(x - y) \in \mathbb{Z} \iff (y, x) \in \rho$.
Not antisymmetric: $1\rho 2 \wedge 2\rho 1$ but $1 \neq 2$.
Transitive: $x\rho y \wedge y\rho z \iff x - y \in \mathbb{Z}$ and $y - z \in \mathbb{Z}$, so $x - z = (x - y) + (y - z) \in \mathbb{Z}$ and $x\rho y$.

2. (a) (**8 marks**)
Reflexive: $x \in S \implies (x, x) \in \rho$ and so $x\rho x$.
Symmetric: $(x, y) \in \rho \implies (y, x) \in \rho$.
Transitive: Suppose $(x, y) \in \rho \wedge (y, z) \in \rho$. We may suppose $x \neq y$, $x \neq z$ and $y \neq z$. Check the transitivity when $x = 1, 2, 3, 4, 5$, respectively.
Equivalence classes: $[1] = \{1, 2, 3, \}$, $[4] = \{4, 5\}$ and $[6] = \{6\}$. Let $S_1 = [1]$, $S_2 = [4]$ and $S_3 = [6]$, Then $S = S_1 \cup S_2 \cup S_3$ and $S_i \cap S_j = \emptyset$ whenever $i \neq j$.

(b) (**7 marks**) Since $S = S_1 \cup S_2 \cup S_3$ and $S_i \cap S_j = \emptyset$ whenever $i \neq j$, it follows that $\{S_1, S_2, S_3\}$ is a partition of S. Let

$$\rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (2, 4), (4, 2), (2, 6), (6, 2), (4, 6), (6, 4), (3, 5), (5, 3)\}.$$

Then ρ is an equivalence relation and $S_1 = [1], S_2 = [2]$ and $S_3 = [3]$.

3. (a) (**6 marks**)
Reflexive: Since $x - x = 4 * 0$ and $0 \in \mathbb{Z}$, it follows that $x \sim x$.
Symmetric: If $x \sim y$ then $x - y = 4b$ for some $b \in \mathbb{Z}$, so $y - x = 4(-b)$ and $-b \in \mathbb{Z}$. Thus $y \sim x$.
Transitive: Suppose $x \sim y$ and $y \sim z$. Then $x - y = 4b$ and $y - z = 4t$ for some $b, t \in \mathbb{Z}$, so $x - z = (x - y) + (y - z) = 4(b + t)$ and $b + t \in Z$. Thus $x \sim z$.

(b) (**9 marks**) For $x \in \mathbb{Z}$, $[x] = \{y \in \mathbb{Z} : x - y = 4b \text{ for } b \in \mathbb{Z}\}$, so

$$[x] = \{y \in \mathbb{Z} : y = 4b + x \text{ for } b \in \mathbb{Z}\}.$$

If $x = 4q + r$ with $t \in \mathbb{Z}$ and $0 \leq r \leq 3$, then $y = 4b + x = 4(q + b) + r$ and since b is arbitrary, $q + b$ is also arbitrary and

$$[x] = \{y \in \mathbb{Z} : y - r = 4t \text{ for } t \in \mathbb{Z}\} = [r].$$

It follows that $[0], [1], [2]$ and $[3]$ are all equivalent classes. Since $i \notin [r]$ for i with $0 \leq i \leq 3$ and $i \neq r$, it follows that $[0], [1], [2]$ and $[3]$ are all distinct equivalent classes.