1. (a) (**5 marks**) Suppose $x \in C \setminus (A \cap B)$. Then $x \in C$ and $x \notin A \cap B$. Thus either $x \notin A$ or $x \notin B$. If $x \notin A$ then $x \in C \setminus A$, so $x \in (C \setminus A) \cup (C \setminus B)$. If $x \notin B$ then $x \in C \setminus B$, so $x \in (C \setminus A) \cup (C \setminus B)$. Since x is arbitrary, it follows that

$$C \setminus (A \cap B) \subseteq (C \setminus A) \cup (C \setminus B).$$

(**5 marks**) Conversely, suppose $x \in (C \setminus A) \cup (C \setminus B)$. Then $x \in C \setminus A$ or $x \in C \setminus B$. If $x \in C \setminus A$ then $x \notin A$. If $x \in C \setminus B$ then $x \notin B$. In both cases $x \notin A \cap B$, so $x \in C \setminus (A \cap B)$ and since x is arbitrary, it follows that

$$(C \setminus A) \cup (C \setminus B) \subseteq C \setminus (A \cap B).$$

Hence $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$.

(b) (**3 marks**) Since $A \setminus B \subseteq A$, $B \setminus A \subseteq B$ and $A \cap B \subseteq A$, it follows that

$$(A \setminus B) \cup (B \setminus A) \cup (A \cap B) \subseteq A \cup B.$$

(**3 marks**) Let $x \in A \cup B$, so that $x \in A$ or $x \in B$. Suppose moreover, $x \in A$. If $x \in B$, then $x \in A \cap B$. If $x \notin B$, then $x \in A \setminus B$, so

$$x \in (A \setminus B) \cup (A \cap B) \subseteq (A \setminus B) \cup (B \setminus A) \cup (A \cap B).$$

(**3 marks**) Suppose $x \in B$. If $x \in A$, then $x \in A \cap B$. If $x \notin A$, then $x \in B \setminus A$, so

$$x \in (B \setminus A) \cup (A \cap B) \subseteq (A \setminus B) \cup (B \setminus A) \cup (A \cap B).$$

(**1 mark**) Hence $x \in (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$. Since x is arbitrary, it follows that

$$A \cup B \subseteq (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$$

and so $A \cup B = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$.

2. (a) (**3 marks**) $\mathcal{P}(A) = \{\varnothing, \{1\}, \{3\}, \{5\}, \{1,3\}, \{1,5\}, \{3,5\}, \{1,3,5\}\}$.

(b) (**3 marks**) $\mathcal{P}(B) = \{\varnothing, \{3\}, \{7\}, \{3,7\}\}$.

(c) (**4 marks**) $\mathcal{P}(A \cap B) = \mathcal{P}(\{3\}) = \{\varnothing, \{3\}\}$.

(d) (**5 marks**) $\mathcal{P}(A \cup B) = \mathcal{P}(\{1,3,5,7\})$, so

$$\begin{aligned}\mathcal{P}(A \cup B) = \{&\varnothing, \{1\}, \{3\}, \{5\}, \{7\},\\ &\{1,3\}, \{1,5\}, \{1,7\}, \{3,5\}, \{3,7\}, \{5,7\},\\ &\{1,3,5\}, \{1,3,7\}, \{1,5,7\}, \{3,5,7\}, \{1,3,5,7\}\}.\end{aligned}$$

3. (a) (**5 marks**) Let $A = \{3\}$ and $B = \{7\}$. Then $\mathcal{P}(A) \cup \mathcal{P}(B) = \{\varnothing, \{3\}, \{7\}\}$ and $\mathcal{P}(A \cup B) = \mathcal{P}(\{3, 7\}) = \{\varnothing, \{3\}, \{7\}, \{3, 7\}\}$, so that

$$\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$$

(b) (**5 marks**) Suppose $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$ and let $x = A \cup B \in \mathcal{P}(A \cup B)$. Then $x \in \mathcal{P}(A)$ or $x \in \mathcal{P}(B)$.

If $x \in \mathcal{P}(A)$, then $x = A \cup B \subseteq A$. But $B \subseteq A \cup B$, so $B \subseteq A$.

If $x \in \mathcal{P}(B)$, then $x = A \cup B \subseteq B$. But $A \subseteq A \cup B$, so $A \subseteq B$.

Thus either $B \subseteq A$ or $A \subseteq B$.

(**5 marks**) Suppose $B \subseteq A$ or $A \subseteq B$.

If $B \subseteq A$ and $x \in \mathcal{P}(B)$, then $x \subseteq B \subseteq A$ and so $x \in \mathcal{P}(A)$. Thus $\mathcal{P}(B) \subseteq \mathcal{P}(A)$ and $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A)$.

On the other hand if $B \subseteq A$ then $B \cup A = \{x : x \in B \text{ or } x \in A\} = \{x : x \in A\} = A$, $\mathcal{P}(A \cup B) = \mathcal{P}(A) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

Similarly, if $A \subseteq B$ then we replace A by B and B by A in the proof above, so $\mathcal{P}(A \cup B) = \mathcal{P}(B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.