1. First if $|x-2| \leq 1$, then $1 \leq x \leq 3$ and so $\sqrt{3} \leq \sqrt{x+2} \leq \sqrt{5}$.

Let $\epsilon > 0$. Choose $\delta = \min\{1, \epsilon(6+4\sqrt{3})\}$. If $|x-2| < \delta$, then

$$\begin{aligned}\left|\frac{1}{\sqrt{x+2}} - \frac{1}{2}\right| &= \left|\frac{2-\sqrt{x+2}}{2\sqrt{x+2}}\right| \\ &= \left|\frac{4-(x+2)}{2\sqrt{x+2}(2+\sqrt{x+2})}\right| = \frac{|x-2|}{4\sqrt{x+2}+2(x+2)} \\ &\leq \frac{|x-2|}{4\sqrt{3}+6} < \epsilon.\end{aligned}$$

2. (a) Let $\epsilon > 0$. Since x^2+1 is continuous at $x = 0$, there is $\delta_1 = \delta_1(\epsilon) > 0$, such that

$$|x^2+1-1| = |x^2| < \epsilon \quad \text{whenever} \quad |x| < \delta_1.$$

Similarly, since x^2+1 is continuous at $x = 0$, there is $\delta_2 = \delta_2(\epsilon) > 0$, such that

$$|1-2x-1| = |2x| < \epsilon \quad \text{whenever} \quad |x| < \delta_2.$$

Let $\delta = \{\delta_1, \delta_2, 1\}$ and suppose $|x| < \delta$.
If $x > 0$, then

$$|f(x) - f(0)| = |x^2+1-1| = |x^2| < \epsilon.$$

If $x < 0$, then

$$|f(x) - f(0)| = |1-2x-1| = |2x| < \epsilon.$$

It follows that $f(x)$ is continuous at 0.

(b) Take $\epsilon = 1$. Since x^3 is continuous at 1, there exists $\delta > 0$ such that

$$|x^3 - 1| < \epsilon \quad \text{whenever} \quad |x-1| < \delta.$$

Suppose $|x-1| < \delta$ and $x > 1$. Then

$$|f(x) - f(1)| = |x^3 - (-1)| = |x^3 - 1 + 2| \geq |2 - |x^3-1|| > 2 - 1 = 1$$

and f is not continuous at $x = 1$.

3. Let $\epsilon > 0$. Since $\lim_{x\to 0} f(x) = L$, there exists a $\delta_1 > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x| < \delta_1.$$

Let $\delta = \delta_1/|a|$. If $0 < |x| < \delta$, then $0 < |ax| = |a||x| < \delta_1$ and

$$|f(ax) - L| < \epsilon.$$

It follows that $\lim_{x\to 0} f(ax) = L$.

4. Let $h(x) = f(x) - g(x)$. Then $h(x)$ is continuous on $[a, b]$, and $g(a) = f(a) - g(a) < 0$, $g(b) = f(b) - g(b) > 0$. It follows by Bolzano theorem that $f(c) = 0$ for some $c \in (a, b)$.