1. (**2 marks each**)

(a) "0 is a positive number" is a statement.

(b) "m is an even number" is a predicate (with m as a free variable).

(c) "If n is an even number then $n-1$ is odd" is a predicate (with n as a free variable) and a statement.

(d) "Is 2 a prime number?" is neither a statement nor a predicate.

(e) "Every odd number is the sum of three odd numbers" is a statement.

2. (**4 marks each**)

(a) We have the following truth table:

A	B	$\sim A$	$\sim B$	$A \implies \sim B$	$\sim A \implies B$	$(A \implies \sim B) \wedge (\sim A \implies B)$
T	T	F	F	F	T	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	F	F

Since the last column contains both "T" and "F", $(A \implies \sim B) \wedge (\sim A \implies B)$ is neither a tautology nor a contradiction.

(b) We have the following truth table:

A	B	$\sim A$	$\sim A \implies B$	$A \implies B$	$(\sim A \implies B) \wedge (A \implies B)$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	T	F

Since the last column contains both "T" and "F", $(\sim A \implies B) \wedge (A \implies B)$ is neither a tautology nor a contradiction.

(c) We have the following truth table:

A	B	$\sim A$	$\sim B$	$\sim A \implies \sim B$	$A \implies B$	$(\sim A \implies \sim B) \vee (A \implies B)$
T	T	F	F	T	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Since the last column contains only "T"s, $(\sim A \implies \sim B) \vee (A \implies B)$ is a tautology.

(d) We have the following truth table:

A	B	$A \implies B$	$\sim(A \implies B)$	$B \wedge \sim(A \implies B)$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

Since the last column contains only "F"s, $B \wedge \sim(A \implies B)$ is a contradiction.

3. (**2 marks each**)

(a) The contrapositive of $A(n)$ is "If $n+1$ is not an odd number then n is not a natural number".

(b) The converse of $A(n)$ is "If $n+1$ is an odd number then n is a natural number".

(c) The negation of $A(n)$ is "n is a natural number but $n+1$ is not an odd number".

(d) Yes, $A(n)$ is true for some n: for example, $A(2)$ is true.

(e) No, $A(n)$ is not true for every natural number n: as a counterexample, $A(3)$ is false.

(f) Since the contrapositive is equivalent to $A(n)$ itself, from (d) and (e) we see that the contrapositive is true for some $n \in \mathbb{N}$ but not for all $n \in \mathbb{N}$.

(g) The converse is true for all n, since by assumption $n \in \mathbb{N}$ is a natural number.

4. (a) (**3 marks**) Suppose n is odd. Then $n = 2k+1$ for some natural number k, so

$$\begin{aligned} f(n) &= (2k+1)^2 + 2 \\ &= 4k^2 + 4k + 1 + 2 \\ &= 2(2k^2 + 2k + 1) + 1 \end{aligned}$$

and $2k^2 + 2k + 1 \in \mathbb{N}$, so $f(n)$ is odd.

(b) (**3 marks**) Suppose n is not odd. Then n is even, so $n = 2k$ for some $k \in \mathbb{N}$. But then

$$\begin{aligned} f(n) &= (2k)^2 + 2 \\ &= 4k^2 + 2 \\ &= 2(2k^2 + 1), \end{aligned}$$

and $(2k^2 + 1) \in \mathbb{N}$, so $f(n)$ is even. Thus $f(n)$ is not odd. Hence, by contraposition, if $f(n)$ is odd then n is odd.

(c) (**4 marks**) Suppose, for a contradiction, that $f(n+k)$ is even and (n and k are both even or odd) is false, in other words one of n and k is even and another is odd. But then $n+k$ is odd, so by part (a) we have $f(n+k)$ is odd, a contradiction. Hence, by contradiction, if $f(n+k)$ is even then n and k are either both odd or both even.