NB: Please deposit your solutions in the appropriate box by 4 p.m. on the due date. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

- 1. (17 marks) Suppose that S is a nonempty, bounded set of real numbers and that T is a nonempty subset of S.
 - (a) Show that T is bounded.
 - (b) Prove that glb $S \leq$ glb $T \leq$ lub $T \leq$ lub S.
- **2.** (10 marks) Let a and b be real numbers with a < b. Show that the interval (a, b) has no least element.
- **3.** (23 marks) Let $\{a_n\}_{n=1}^{\infty}$ be the sequence, where $a_n = \frac{3}{1+5^{-n}}$.
 - (a) Show $\{a_n\}_{n=1}^{\infty}$ is monotonic and bounded.
 - (b) Find the greatest lower bound and least upper bound of the set $\{a_n\}_{n=1}^{\infty}$, and determine whether either is an element of $\{a_n\}_{n=1}^{\infty}$.
 - (c) Show from first principles (i.e. directly from the definition) that $\{a_n\}_{n=1}^{\infty}$ converges to its least upper bound.