

**NB:** Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

1. (15 marks) Prove by mathematical induction that, for every  $n \in \mathbb{N}$ ,

$$1 \cdot 2 + 2 \cdot 3 + \dots n(n+1) = \frac{1}{3}n(n+1)(n+2).$$

2. (10 marks) Let  $n \in \mathbb{N}$  and  $W(n) = \{-(n-1), -(n-2), \dots, -1, 0\} \cup \mathbb{N}$ . Prove by mathematical induction on  $n$  that  $W(n)$  is a well-ordered set.
3. (15 marks) Prove by mathematical induction that, for every  $n \in \mathbb{N}$ ,  $3^{4n+1} + 4^{n+1}$  is divisible by 7.
4. (10 marks) Use the Euclidean algorithm to find the greatest common divisor of 3598 and 1603, and find integers  $x$  and  $y$  such that

$$\gcd(3598, 1603) = 3598x + 1603y.$$