NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

1. (**15 marks**)

(a) Let A, B C be sets such that $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7, 8, 9\}$ and $C = \{a, b, c, d\}$ (distinct elements). Let $f : A \to B$ and $g : B \to C$ be functions such that $f = \{(1, 4), (2, 6), (3, 8)\}$ and $g = \{(4, a), (5, b), (6, d), (7, c), (8, b), (9, d)\}$.

Show that $g \circ f$ is one-to-one. Verify that f is one-to-one and g is not one-to-one.

(b) Let X and Y be sets, and let $h : X \to Y$ and $w : Y \to Z$ be functions such that the composition $w \circ h$ is one-to-one. Show that h is one-to-one.

2. (**15 marks**) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2$ for all $x \in \mathbb{R}$, and let A_1 be the interval $(0, 4] \subseteq \mathrm{Dom}(f)$, $A_2 = \{-2, -1, 1, 3, 10\} \subseteq \mathrm{Dom}(f)$, $B_1 = \{-1, 1, 4, 100\} \subseteq \mathrm{Codom}(f)$ and B_2 the interval $(0, 16) \subseteq \mathrm{Codom}(f)$.

(a) Find $f(A_1), f(A_2)$ and $f(A_1 \cap A_2)$.

(b) Verify that $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$.

(c) Find $f^{-1}(B_1), f^{-1}(B_2)$ and $f^{-1}(B_1 \cap B_2)$.

(d) Verify that $f^{-1}(B_1) \cap f^{-1}(B_2) = f^{-1}(B_1 \cap B_2)$.

3. (**20 marks**)

(a) Let $h : X \to Y$ be a function.

(i) If $Y_1 \subseteq Y$ and $Y_2 \subseteq Y$, show $h^{-1}(Y_1 \cap Y_2) = h^{-1}(Y_1) \cap h^{-1}(Y_2)$.

(ii) If moreover, h is one-to-one, show that the function $h : \mathcal{P}(X) \to \mathcal{P}(Y)$ by $A \mapsto h(A)$ is an order preserving function, where $\mathcal{P}(X)$ and $\mathcal{P}(Y)$ are regarded as posets under inclusion $\subseteq$.

(b) Let $A = \{\frac{1}{n} : n \in \mathbb{N}\}$ and $B = \{-n : n \in \mathbb{N}\}$. View A and B as totally ordered set under the usual ordering on $\mathbb{R}$. Define $f : A \to B$ by $f(\frac{1}{n}) = -n$. Show that f is an order isomorphism between A and B.