**NB:** Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

1. (**20 marks**) Indicate whether each of the following relations on the given set is reflexive, symmetric, antisymmetric, or transitive. Explain your answers.

   (a) $A = \{x \in \mathbb{R} : x > 0\}$ and $x\rho y$ iff $xy = 0$.

   (b) $B = \{x \in \mathbb{Z} : x \geq 0\}$ and $x\rho y$ iff $x + y = 4$.

   (c) $C = \{1, 2, 4\}$ and $\rho = \{(2, 1), (2, 4), (1, 1), (4, 2), (2, 2)\}$.

   (d) $D = \mathbb{Q}$ and $x\rho y$ iff $x - y \in \mathbb{Z}$.

2. (**15 marks**) Let $S$ be the set $\{1, 2, 3, 4, 5, 6\}$.

   (a) Let $\rho$ be the relation

   $$\rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (4, 5), (5, 4)\}$$

   Verify that $\rho$ is an equivalence relation. Find all equivalence classes and check the collection of distinct classes is a partition of $S$.

   (b) Let $S_1 = \{1\}$, $S_2 = \{2, 4, 6\}$ and $S_3 = \{3, 5\}$. Verify that $S_1, S_2, S_3$ is a partition of $S$. Define an equivalence relation $\rho$ such that each $S_i$ is an $\rho$-equivalence class.

3. (**15 marks**) Let $\sim$ be a relation on $\mathbb{Z}$ such that $x \sim y$ iff $x - y = 4b$ for some integer $b \in \mathbb{Z}$.

   (a) Show $\sim$ is an equivalence relation.

   (b) Find all distinct equivalence classes.