1. Show from first principles that $g(x) = \frac{1}{\sqrt{x+2}}$ is continuous at $x = 2$.

2. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0, \\ 1 - 2x & \text{if } 0 \leq x \leq 1, \\ x^3 & \text{if } x > 1. \end{cases}$$

(a) Prove from first principles that $f(x)$ is continuous at 0.

(b) Prove from first principles that $f(x)$ is *not* continuous at 1.

3. Show that if $\lim_{x\to 0} f(x) = L$, then $\lim_{x\to 0} f(ax) = L$ for all $a \neq 0$.

4. Suppose f and g are continuous functions on the interval $[a, b]$ and suppose that $f(a) < g(a)$ and $g(b) < f(b)$ show that there exists a number $c \in (a, b)$ such that $f(c) = g(c)$.