NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

1. (**10 marks**) Which of the following sentences are statements, which are predicates, and which are neither?

(a) 0 is a positive number.
(b) m is an even number.
(c) If n is an even number then $n-1$ is odd.
(d) Is 2 a prime number?
(e) Every odd number is the sum of three odd numbers.

2. (**16 marks**) Let A, B and C be statements. Construct truth tables for the following statements. For each statement, state whether it is a tautology, a contradiction or neither.

(a) $(A \implies \sim B) \wedge (\sim A \implies B)$.
(b) $(\sim A \implies B) \wedge (A \implies B)$.
(c) $(\sim A \implies \sim B) \vee (A \implies B)$.
(d) $B \wedge \sim (A \implies B)$.

3. (**14 marks**) For any natural number n, let $A(n)$ be the statement

"If n is a natural number then $n+1$ is an odd number."

(a) Write down the contrapositive of $A(n)$.
(b) Write down the converse of $A(n)$.
(c) Write down the negation of $A(n)$.
(d) Is $A(n)$ true for some natural number n? If so, give an example, if not give a proof.
(e) Is $A(n)$ true for every natural number n? If so, give a proof, if not give a counterexample.
(f) Is the contrapositive of $A(n)$ true for some natural number n? Is it true for every natural number n? Give brief reasons for your answer.
(g) Is the converse of $A(n)$ true for some natural number n? Is it true for every natural number n? Give brief reasons for your answer.

4. (**10 marks**) Let $f : \mathbb{N} \to \mathbb{N}$ be given by $f(x) = x^2 + 2$.

(a) Use a **direct proof** to show that if n is odd then $f(n)$ is odd.
(b) Use a **proof by contraposition** to show that if $f(n)$ is odd then n is odd.
(c) Use a **proof by contradiction** to show that if $f(n+k)$ is even then n and k are both odd or both even.