COMPSCI 225

THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2001
Campus: City

MATHEMATICS

Discrete Mathematics

(Time allowed: TWO hours)

NOTE: Answer **All** questions. The questions in Section A (multiple choice questions) are worth 1 mark each: the marks for each question in Section B are as shown. Section A is worth a total of 20 marks and Section B is worth a total of 40 marks: you are advised to spend about one hour on Section A and two hours on Section B.
Answer Section A (multiple choice questions) on the coloured answer sheet provided.

SECTION A

MULTIPLE CHOICE QUESTIONS

Circle the preferred choice **on the coloured answer sheet provided.**

1. Let A and B and C be subsets of a universal set U. Which of the following sets is equal to the shaded region in the following Venn diagram?

(a) $(A \cup (B \cap C)) \setminus (A \cap (B \cup C))$.

(b) $A \cup (B \cap C)$.

(c) $A \cap (B \cup C)$.

(d) $(A \setminus B) \cap (A \setminus C)$.

(e) $(A \setminus (B \cup C)) \cup (B \cap C)$.

2. Let $f : A \to B$ and $g : B \to C$ be functions. Which of the following statements is NOT true?

(a) if $g \circ f$ is 1–1 then f and g are both 1–1.

(b) if $g \circ f$ is 1–1 then f is 1–1.

(c) if $g \circ f$ is onto then g is onto.

(d) if $g \circ f$ is a bijection then f is 1–1 and g is onto.

(e) if f and g are both 1–1 then $g \circ f$ is 1–1.

3. A multiple-choice exam has 20 question, and each question has 5 possible answers. The same 5 answers for each question will appear on each exam paper, but they can appear in any order. The questions will appear in the same order on every exam paper. How many different exam papers can be printed?

(a) $(5!)^{20}$.

(b) $\binom{20}{5}$.

(c) $\binom{20+5-1}{5}$.

(d) $P(20, 5)$.

(e) $\frac{20!}{5!}$.

SECTION B

4. (a) For each of the following propositions, determine whether it is a tautology, a contradiction or a contingent proposition. You may find it helpful to refer to the list of standard taultologies in the ATTACHMENT on page 4.

(i) $p \to (q \wedge p)$.

(ii) $p \to (q \vee p)$.

(iii) $p \to \neg(q \wedge p)$.

(iv) $\neg(q \vee (q \to p))$.

(b) Let $A_1, A_2, \ldots, A_n, B$ be propositions. What does it mean to say that the argument $A_1, A_2, \ldots, A_n, \therefore B$ is *valid*? Identify the simple propositions in the following argument and determine whether or not it is valid:

> "If interest rates are not high, then house prices will rise. If interest rates are high, then the exchange rate will be low. Therefore if the exchange rate is not low then house prices will rise."

(5 marks)

5. (a) Let $p(x, y)$ denote the predicate "$x < y$", with domain of definition $\mathbb{Z} \times \mathbb{Z}$. Translate the following statements into predicate logic.

(i) "For every integer x there is an integer y with $x < y$."

(ii) "For all integers x, y and z, if $x < y < z$ then $x < z$."

(iii) "For each integer x there is an integer y such that $x < y$ but there is no integer z with $x < z < y$."

(b) (i) What does it mean to say that a proposition in predicate logic is a *tautology*?

(ii) Show that $\forall x\, p(x) \rightarrow \exists x\, p(x)$ is a tautology.

(iii) Show that $\exists x\, p(x) \rightarrow \forall x\, p(x)$ is not a tautology. (7 marks)

6. (a) Let A and B be sets, and let $f : A \rightarrow B$ be a function. Define a relation $\approx$ on A by

$$x \approx y \qquad \Leftrightarrow \qquad f(x) = f(y).$$

Show that $\approx$ is an equivalence relation.

(b) Define the relation $\approx$ on $\mathbb{R} \times \mathbb{R}$ by

$$(x, y) \approx (u, v) \qquad \Leftrightarrow \qquad x^2 + y^2 = u^2 + v^2.$$

By part (a) we know that this is an equivalence relation. For $x, y \in \mathbb{R}$, we denote the equivalence class of (x, y) by $[x, y]$. Suppose we define the sum of two equivalence classes by

$$[x, y] \oplus [u, v] = [\sqrt{x^2 + u^2}, \sqrt{y^2 + v^2}\,].$$

(i) What does it mean to say that $\oplus$ is *well-defined*?

(ii) Show that $\oplus$ is well-defined. (10 marks)

$p \Leftrightarrow \neg\neg p$	Double Negation
$p \wedge q \Leftrightarrow q \wedge p$	Commutative
$\vee q \Leftrightarrow q \vee p$	Laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	De Morgan's
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	Laws

Standard tautologies

ANSWER SHEET FOLLOWS

Candidate's Name: ______________________ ID No: ____________

SECTION A

Circle the preferred answer.

If you make a mistake, mark a cross through your wrong choice and circle your next alternative.

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e