Sensing God's Will is Fixed Parameter Tractable

Michael Fellows, Frances Rosamond and Arkadii Slinko

Abstract

One of the goals of Artificial Intelligence is to create autonomous software agents which exhibit social intelligence, i.e are capable to act as a group member in various groups and, in particular, to participate in group decision making. For such agents to be programmed we need protocols which are formalizable. In this paper we are suggesting such a protocol based on one of the most famous and controversial social choice rules - Dodgson's rule. We prove that the problem of implementation of this protocol is fixed parameter tractable.

Keywords: consensus, protocol, Dodgson rule, complexity

1 Introduction

Wooldridge and Jennings [19] start their landmark paper with the following descriptions of a hypothetical event that occur sometime in the future:

> "The key air-traffic control systems in the country of Ruritania suddenly fail, due to freak weather conditions. Fortunately, computerised air-traffic control systems in neighbouring countries negotiate between themselves to track and deal with all affected flights, and the potentially disastrous situation passes without major incident."

To make this a reality we must teach autonomous software agents to achieve a *consensus*. Do we know what it is and what are the algorithms of achieving it? Let us look at historical examples of making decisions by consensus.

Quakers or The Religious Society of Friends, which is a Christian religious denomination founded in England in the 17th century [1], believed that on every issue there is a God's will and that the society as a whole is better positioned to sense it than a single individual. When faced by the necessity of taking a business decision, they would gather at a "Meeting for Worship with a Concern for Business", or simply "Business Meeting".Each member of the meeting was expected to listen to the voice of God within themselves and, if called, to contribute to the group their opinion for reflection and consideration. A decision is reached when the Meeting, as a whole, feels that the "way forward" has been discerned (also called "coming to unity") or there is a consensus. This is known as Quakers' method of making a decision-by-consensus.

In the modern society some decisions can be made by consensus only. Complex problems of this kind are numerous: global warming, toxic chemicals in the environment, fighting terrorism, deforestation, overfishing and so on. Each involves a certain public good and externalities (effects that one agent's actions have on an otherwise unrelated agent). There are currently no good mechanisms for efficient and direct aggregation of preferences in such domains; rather we have to rely on political parties and a few non-governmental organizations to act on our behalf. Due to the largely atheistic nature of modern society, the Quakers' method is not applicable. Yet still, some organisations like the Internet Engineering Task Force (IETF) [4] or Wikipedia [2] claim to make decisions by consensus or rough consensus. To date, no protocols of consensus decision making have been published: perhaps from fear of having it manipulated (IETF) or due a non-formalisable nature of such protocols. All we know is that in each case, building consensus is a process in the course of which agents who initially disagreed with the final decision are persuaded to accept it, perhaps with some compromise.

We believe that some kind of compromise (or compensation) is always involved in consensus building, and any consensus comes at a price. In some cases, like church meetings or elections of the Pope, preserved unity may be a sufficient compensation. In other cases, like countries negotiating fishing quotes, monetary compensations could be inevitable.

Modern social choice theory, following Kenneth Arrow, treats voting as a method for aggregating diverse preferences and values. An earlier view, initiated by Marquis de Condorcet, is that voting is a method for aggregating information. Voters' opinions differ because they make errors of judgment; absent these errors they would all agree on the best choice. The goal is to design a voting rule that identifies the best choice with highest probability. This approach is called maximum likelihood estimation and it has been pursued by Peyton Young in papers [21, 20, 22]. The main idea of this line of research is that there is a 'correct' ranking and that voters express opinions that aggregately represent the correct opinion with some 'noise'. Conitzer and Sandholm [9] showed that several rules can be viewed as maximum likelihood estimators but others cannot.

A slightly different approach was suggested by Dodgson[1] (1832–1898). His approach implicitly suggests that not individual voters but the society as a whole knows the "correct answer" but the profile that results in the election is contaminated with some random mistakes (e.g. mistakes in casting a ballot) and some voters deliberately acting irrationally obstructing the electoral process.

Dodgson's considered that, given the votes, the administrator is allowed to make a minimal number of swaps of neighboring alternatives to obtain a profile for which a Condorcet Winner exists, that is then declared the winner of the election. This approach is based on two assumptions:

- If a Condorcet Winner exists, then the society has spoken unambiguously. In

[1] better known by the pen name Lewis Carroll

consensus-decision-making terminology we may say that in this case there is a "rough consensus" in the society [3, 4].

- A mininal change in a voter's opinion is for the voter to swap in their linear order two neighboring alternatives, ceteris paribus, and that, in general, the magnitude of a change in opinion can be measured in the number of these minimal changes.

The implementation of Dodgson's rule as it was envisaged by Dodgson himself is that the administrator or an electoral committee should be allowed to make the smallest possible number of such swaps so that the resulting profile has a Condorcet Winner.

It is perfectly reasonable for the administrator to be reluctant to swap someone's first and second preferences but more willing to swap someone's fifth and sixth preferences since the voter feels more strongly about the position of his best alternatives than he does about the position of his fifth best alternative. So it is reasonable to assume that there is a certain "cost" c_{ij} involved in swapping the ith and $(i+1)$th preferences of the jth voter. At the first glance it may seem reasonable to assume that this cost decreases as i grows, i.e., $c_{1j} \geq c_{2j} \geq \ldots \geq c_{mj} \geq \ldots$. However, this may not always be justified. Imagine the situation when a voter classifies candidates into two large groups: acceptable and not acceptable ones. She has ranking within each group too. For her, the movement of candidates within each group will incur only a small cost while moving candidates from one group to another will be costly.

Our first goal is to introduce the concept of a generalised Dodgson score relative to an arbitrary cost function which is individual to the particular voter. We require, however, that the cost function is anonymous, that is, voters with the same preference orders incur identical costs. We claim that this protocol is an ideal tool for making decisions by consensus. This is the first such formalised protocol that we know.

Our second goal is to dispel the stigma of exorbitant complexity associated with Dodgson's rule. From the classical complexity point of view things look really bad (we discuss this in the Section 3). However from the parameterised complexity point of view things are not gloomy at all. Recently Fellows and Rosamond[2] and Betzler, Guo and Niedermeier [5] showed that the problem Dodgson Winner is in FPT, if it is parameterised by the score. The result of this paper is more general as we extend this result to the Generalised Dodgson's score.

Our third goal is to attract attention to Generalised Dodgson's rule as a protocol for achieving a consensus. As McCabe-Dansted showed [16, 17] for classical Dodgson's rule; with less that 25 alternatives and less than 100 voters, his algorithm finds the winner in less than one tenth of a second in the worst case. We expect that similar results can be obtained in the generalised case.

2 Preliminaries from Social Choice Theory

Let A be a finite set of alternatives which are under consideration by a group of agents who need to choose the "best" alternative. We assume that the ith voter ranks all

[2]See the remark added in proof in [8]

alternatives without ties and submits a ballot which contains a linear order R_i on A. The sequence $\mathcal{R} = (R_1, R_2, \dots, R_n)$ of linear orders is called a *profile*. It contains all available information, i.e. which voter submitted which ballot. This information is not always available. Voters are often anonymous and in this case we only know how many votes of each type were submitted. This information is contained in the *voting situation* which corresponds to the profile $R = (R_1, \dots, R_n)$. This is a multiset $\mathcal{V} = \mathcal{V}(\mathcal{R})$ on the set of all linear orders $\mathcal{L}(A)$ on A, i.e. a mapping $\mu\colon \mathcal{L}(A) \to \mathbb{N}$ such that, for each linear order $Q \in \mathcal{L}(A)$, the value $\mu(Q)$ is the number of indices i for which $Q = R_i$. Of course, the cardinality of this multiset $\sum_{Q\in\mathcal{L}(A)} \mu(Q) = n$, the number of voters.

We define n_{xy} to be the number of linear orders in $\mathcal{R}$ that rank x above y, i.e. $n_{xy} \equiv \#\{i \mid xR_iy\}$. This information is stored in the matrix $N_{\mathcal{R}}$, where $(N_{\mathcal{R}})_{ab} = n_{ab}$. A function $W_{\mathcal{R}}\colon A \times A \to \mathbb{N}$ given by $W_{\mathcal{R}}(a, b) = n_{ab} - n_{ba}$ for all $a, b \in A$, will be called the *weighted majority relation* on $\mathcal{R}$. It is obviously skew symmetric, i.e. $W_{\mathcal{R}}(a, b) = -W_{\mathcal{R}}(b, a)$ for all $a, b \in A$. We note that the information which is contained in the voting situation $\mathcal{V}(\mathcal{R})$ is sufficient to calculate $W_{\mathcal{R}}$. An arbitrary skew symmetric function $W\colon A \times A \to \mathbb{N}$ is called a *weighted tournament* on A. So, given a profile $\mathcal{R}$, the weighted majority relation is a weighted tournament.

Profiles, voting situations and weighted tournaments are three levels of aggregation of the original information contained in voting ballots.

The following lemma is obvious but useful.

Lemma 1. *Let $\mathcal{V}_1$ and $\mathcal{V}_2$ be two voting situations and $\mathcal{V}_1 \cup \mathcal{V}_2$ be their multiset union, i.e. the multiset for which the multiplicity function is the sum of the multiplicity functions of $\mathcal{V}_1$ and $\mathcal{V}_2$. Then $W_{\mathcal{V}_1\cup\mathcal{V}_2} = W_{\mathcal{V}_1} + W_{\mathcal{V}_2}$.*

Many of the rules to determine the winner use the numbers

$$\mathrm{adv}_{\mathcal{R}}(a, b) = \max(0, n_{ab} - n_{ba}) = (n_{ab} - n_{ba})^+,$$

which will be called *advantages*. If a wins a pairwise simple majority contest against b, then $\mathrm{adv}_{\mathcal{R}}(a, b)$ shows by how many votes this contest is won. Note that $\mathrm{adv}_{\mathcal{R}}(a, b) = \max(0, W_{\mathcal{R}}(a, b)) = W_{\mathcal{R}}(a, b)^+$, where $W_{\mathcal{R}}$ is the weighted majority relation of $\mathcal{R}$.

We will not write the subscript if it is clear which profile is under consideration. A Condorcet Winner is an alternative a for which $\mathrm{adv}_{\mathcal{R}}(b, a) = 0$ for all b. If $\mathrm{adv}_{\mathcal{R}}(b, a) > 0$, then, for a to win the pairwise contest against b we have to persuade at least

$$\left\lceil \frac{\mathrm{adv}_{\mathcal{R}}(b, a)}{2} \right\rceil$$

people to swap a and b in their linear orders. It might involve even more swaps of neighboring alternatives since a and b may be far apart in some orders.

We will need the following lemma, the proof of which uses ideas from McGarvey [18].

Lemma 2. *Let $A = \{a_0, a_1, \dots, a_m\}$ and $w_1, \dots, w_m$ be arbitrary integers which are either all even or all odd. Then there exists a profile $\mathcal{R}$ on A of cardinality not greater than $w_1 + \ldots + w_m$ such that for the weighted tournament $W_{\mathcal{R}}$ associated with $\mathcal{R}$ we have $W_{\mathcal{R}}(a_0, a_i) = w_i$ for all $i = 1, \dots, m$ and $W_{\mathcal{R}}(a_s, a_t) = 0$, when $(s, t) \neq (0, i)$.*

Proof. Suppose all w_i's are even. Then, for each $i = 1, \ldots, m$, we add $|w_i/2|$ pairs of linear orders. If $w_i > 0$, then the linear orders in those pairs will be

$$a_0 > a_i > b_1 > b_2 > \ldots > b_{m-1}, \qquad b_{m-1} > b_{m-2} > \ldots > b_1 > a_0 > a_i,$$

where $\{b_1, \ldots, b_{m-1}\} = A \setminus \{a_0, a_i\}$. If $w_i < 0$, then a_0 and a_i in these orders should be swapped. If all w_i's are odd, we firstly add a linear order

$$b_1 > \ldots > b_p > a_0 > c_1 > \ldots > c_q,$$

where $\{b_1, \ldots, b_p\} = \{a_i \in A \mid w_i < 0\}$ and $\{c_1, \ldots, c_q\} = \{a_j \in A \mid w_j > 0\}$. In light of Lemma 1 this reduces the problem to the first case. $\square$

3 Dodgson's Rule and Classical Complexity Theory

As Dodgson's rule is anonymous we may have either a profile or a voting situation as input. The protocol of Dodgson's rule stipulates that initially the majority relation has to be calculated and it has to be checked whether or not a Condorcet Winner exists. If so, it is elected. If not, the protocol, for every alternative $a \in A$, calculates the score $\mathrm{Sc}(a)$ defined to be the smallest possible number of swaps of neighboring alternatives in the linear orders of the profile or the voting situation that makes a a Condorcet Winner. The alternative with the smallest score is elected.

Classical complexity provides us with plenty of negative results about Dodgson's rule. Bartholdi, Tovey, and Trick [6], in their seminal paper "Voting Schemes for which It Can Be Difficult to Tell Who Won the Election," raised the issue of the computational complexity of determining a winner. In sharp contrast to most other known voting rules, Bartholdi et al. proved that Dodgson's election protocol has the disturbing property that it is NP-hard to determine whether a given candidate has won a given election (a problem they called DODGSON WINNER), and that it is NP-hard even to determine whether a given candidate has tied-or-defeated another given candidate (a problem they called DODGSON RANKING). E. Hemaspaandra, L. Hemaspaandra, and J. Rothe [15] proved that DODGSON WINNER and DODGSON RANKING are complete for parallel access to NP establishing its complexity exactly. One of the most interesting problems is given below.

The problem: DODGSON'S SCORE

> *Instance:* A set of alternatives A, an alternative $a \in A$, a multiset of linear orders V and a nonnegative integer k.
> *Question:* Does a Dodgson's score of a satisfy $\mathrm{Sc}(a) \leq k$?

We need to consider multisets here since several voters might have identical opinions. Bartoldi et al. [6] proved that DODGSON SCORE is NP-complete by reducing it to EXACT COVER BY 3-SETS (X3C), known to be NP-complete [13]. McCabe-Dansted proved that DODGSON SCORE does not admit a Polynomial Time Approximation Scheme (PTAS), unless $P = NP$ [16, 17].

Further investigation by methods of parameterised complexity [10] has revealed that things are not as bad as they seemed. Two important parameters may be small and are of interest to parameterised complexity, namely, the number of alternatives m, and the threshold k, hence we have two possible parameterised variants of DODGSON SCORE. Both parametrised problems have been shown to be in FPT ([16, 5, 8]. McCabe-Dansted showed [16] that for $m \leq 25$ and $n \leq 100$ the time required to calculate Dodgson's winner is less than than 1/10 of a second. This makes Dodgson's rule feasible as a protocol computing consensus since consensus.

4 Generalised Dodgson Rule and Its Parameterised Complexity

Now we will deal with monetary compensations. As in the standard Dodgson's rule, the administrator calculates the cost of making an alternative a a Condorcet Winner. The voters reveal their costs and the problem for the administrator is to decide which alternative is cheaper to make a Condorcet Winner. We will assume that voters with identical preferences bear the same costs. Since voters remain anonymous even after revealing their costs, we call this an *anonymous cost structure*.

Mathematically, this leads us to introducing a cost function $c\colon \mathcal{L}(A) \times A \to \mathbb{N} \cup \{\infty\}$ such that $c(Q, x)$ is the cost for the voter with preferences Q of swapping the alternative x with the one just above it (and $c(Q, x) = \infty$ if x is the top alternative already).

Given a set of cost functions c, we can introduce the concept of a generalised score for each of the alternatives denoting the minimal cost of making an alternative a a Condorcet Winner as $Sc(\mathcal{R}, c, a)$ or $Sc(\mathcal{V}, c, a)$.

The problem: GENERALISED DODGSON'S SCORE

> *Instance:* A set of alternatives A, a set of voters V a profile $\mathcal{R}$ of V on A, a cost function c and an alternative $a \in A$.
>
> *Parameter:* k (representing the threshold cost)
>
> *Question:* Is it true that $Sc(\mathcal{R}, c, a) \leq k$?

We would like to show that this problem is fixed parameter tractable. However we would like to do more than that and give information on the degree of FPT-ness of the problem. We need the following definition.

Definition 1. *A parameterised language L is kernelizable if there is a parametric transformation of L to itself that satisfies:*

1. *the running time of the transformation of (x, k) into (x', k') (the kernel) is bounded by a polynomial $q(n, k)$, where $n = |x|$.*
2. *$k' \leq k$, and*

3. $|x'| \leq h(k)$*, where* h *is an arbitrary function.*

So that in fact this is a polynomial time transformation of L to itself, considered classically, although with the additional structure of a parametric reduction. It is known that a parameterised language L is fixed-parameter tractable if and only if it is kernelisable [12]. The size $h(k)$ of the kernel (x', k') is a very important parameter of a kernelisable parameterised language. If $h(k)$ is a polynomial, we say that the problem has a polynomial kernel, and if $h(k) = e^{O(k)}$, we say that it has a simply exponential kernel.

Theorem 1. Generalised Dodgson Score *has a kernel of size* $e^{O(k^2)}$*.*

Proof. Let $(A, V, \mathcal{R}, c, a)$ be an instance of the problem and let $\mathcal{V}$ be the voting situation corresponding to $\mathcal{R}$. We then can calculate the majority relation and identify the set of candidates X from A who win over a in pairwise contests. This can be done in polynomial time. Since $c(Q, a) \geq 1$ for all i, we may assume that $|X| \leq k$, otherwise $(A, V, \mathcal{R}, c, a)$ is a 'no'-instance. For each problem candidate $x \in X$ we calculate

$$\mathrm{gap}(x) = \left\lceil \frac{\mathrm{adv}_{\mathcal{R}}(x, a)}{2} \right\rceil .$$

This is how many voters have to be persuaded to vote for a so that a wins (or draws) a simple majority contest against x.

Let $\sigma(Q)$ be the linear order which is obtained as a result of a swap of a one position upwards (which is not defined when a is already the top alternative). Given $Q \in \mathcal{L}(A)$ and a cost function c, let $\ell_r(Q, c, a) = \sum_{i=0}^{r-1} c(\sigma^i(Q), a)$. This is the cost of moving the alternative a up r times in a linear order of type Q with the cost function c. Let us define a couple of useful things. Firstly, let $L(a, Q)$ be the lower contour set of a in Q, i.e. $L(a, Q) = \{x \in A \mid aQx\}$. Also let $\mathrm{pos}(a, Q) = |A| - |L(a, Q)|$ be the position of a in Q. Let us also set $X_i = X \cap L(a, \sigma^i(Q))$. Then the *footprint* of a in Q will be the sequence of pairs $(\ell_0(Q, c, a), X_0), \ldots, (\ell_r(Q, c, a), X_r)$, where r is the maximum value such that $\ell_r(Q, c, a) \leq k$.

Since $X \supseteq X_0 \supseteq X_1 \supseteq \ldots \supseteq X_r$, we have $|X_0| \leq k$ and $\ell_r(Q, c, a) \leq k$. Therefore there are at most 2^{2k^2} footprints. Denote the set of all footrpints as F. We can now calculate the following multiset corresponding to the profile $\mathcal{R} = (R_1, \ldots, R_n)$. This is a multiset $\mathcal{F} = (F, \mu)$ on the set of all footprints F, with the multiplicity function $\mu\colon F \to \mathbb{N}$ such that, for each footprint $f \in F$ the value $\mu(f)$ is the number of indices i for which a has footprint f in R_i. Of course, the cardinality of $\mathcal{F}$ is $n = |V|$. This multiset can be computed in polynomial time. Now we construct another multiset $\mathcal{G} = (F, \nu)$, where $\nu(f) = \min(\mu(f), k)$. This multiset has cardinality not greater than $k2^{2k^2}$. This multiset determines a voting situation which we will denote $\mathcal{V}_1$. To construct it, for any footprint f, we take any $\nu(f)$ linear orders with that footprint.

Define $\bar{A} = X \cup \{a\}$ and let $\bar{Q} \in \mathcal{L}(\bar{A})$ be the restriction of Q on $\bar{A}$. Switching from A to $\bar{A}$ we make some alternatives invisible but their existence will be encoded in the new cost function $\bar{c}$. Let $\mathrm{pos}(\bar{Q}, a) = j$ and $\mathrm{pos}(\bar{Q}, b) = j - 1$, i.e. they are neighbours in $\bar{Q}$ and b is ranked directly above a, but $\mathrm{pos}(Q,a) = i$ and $\mathrm{pos}(Q,b) = i - h$ for some positive integer h. Then we set $\bar{c}(\bar{Q}, a) = \ell_h(Q, c, a)$. Restricting all linear orders of $\mathcal{V}_1$ to $\bar{A}$ we obtain a voting situation $\bar{\mathcal{V}_\infty}$. The size of this voting situation depends now only on k but in the process of obtaining this voting situation we discarded several linear orders and the advantages $\mathrm{adv}_{\bar{\mathcal{V}}_1}(x,a)$ for $x \in X$ are now different from $\mathrm{adv}_{\mathcal{V}}(x,a)$. This has to be corrected.

As the cardinality of $\bar{\mathcal{V}}_1$ is less that $k2^{2k^2}$ we have $\mathrm{adv}_{\bar{\mathcal{V}}}(x,a) \le k2^{2k^2}$. By Lemma 2 we need to add to $\bar{\mathcal{V}}_1$ at most another $k^2 2^{2k^2}$ linear orders so that the resulting voting situation $\mathcal{V}_2$ gives us

$$\mathrm{adv}_{\mathcal{V}_2}(x,a) = \mathrm{adv}_{\mathcal{V}}(x,a)$$

for all $x \in X$. For every new linear order $\bar{Q}$ added we set $\bar{c}(\bar{Q}, a) = k + 1$. This concludes the construction of the kernel. Indeed, the size of the constructed voting situation is bounded by a function of k only and it is is exactly the same to decide if $Sc(\mathcal{R}, c, a) \le k$ or $Sc(\mathcal{V}_2, \bar{c}, a) \le k$. $\square$

5 Conclusion

The heart of consensus is a cooperative intent, where the members are willing to work together to find a solution that meets the need of the group. The cooperative nature of search for consensus creates a different mindset from the competitive mindset of voting. In particular, the society might wish to compensate those members of the society whose preferences are far from the collective decision that results from using the agreed protocol. We suggested such protocol and the method of compensation associated with it. We showed that the protocol is feasible for groups of relatively small size. We believe that this is the first such formalised protocol.

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