STRIP TEMPERATURE IN A METAL COATING LINE ANNEALING FURNACE

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We discuss the work done at MISG 2004 on the mathematical modelling of a long, electric radiant furnace used to anneal strips of steel. The annealing process involves heating the steel, which is passed continuously through the furnace, to certain temperatures and then cooling it, resulting in a change in the crystalline structure of the steel. The furnace settings are often changed to cater for products with different metallurgical properties and varying dimensions. The mathematical model is desired to optimise the running of the furnace, especially during periods of change in furnace operation.

1. Introduction

New Zealand Steel (NZS) use a unique process to convert New Zealand ironsand into steel sheet products at its Glenbrook mill near Auckland. Traditional galvanised steel (Galvsteel^{TM}) and the new product Zincalume® are produced in a range of dimensions, grades and coating weights.

The steel strip is annealed prior to being coated, by heating to a predetermined temperature for a definite time. Annealing produces desirable changes in the crystalline structure of the steel, allowing NZS to tailor its strength and ductility.

Strips of steel sheet are passed through a 150m long, 4.6 MW electric radiant furnace at speeds of up to 130 metres per minute in order to achieve the strip temperatures required for annealing, and subsequent coating. The temperature along the furnace is controlled by varying the power supplied to the heating elements and by use of cooling tubes. The cooling tubes are located in the last half of the furnace and consist of steel tubes through which ambient air is pumped. It is important that steel exit the furnace with the correct temperature for the coating that is applied at the exit point.

The line speed through the furnace is reduced for strips of large thickness and width in order to achieve the required temperatures. At the beginning of

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the annealing–coating line there is an automatic welding process which welds the beginning of a new coil of steel sheet to the end of its predecessor, allowing the line to run continuously.

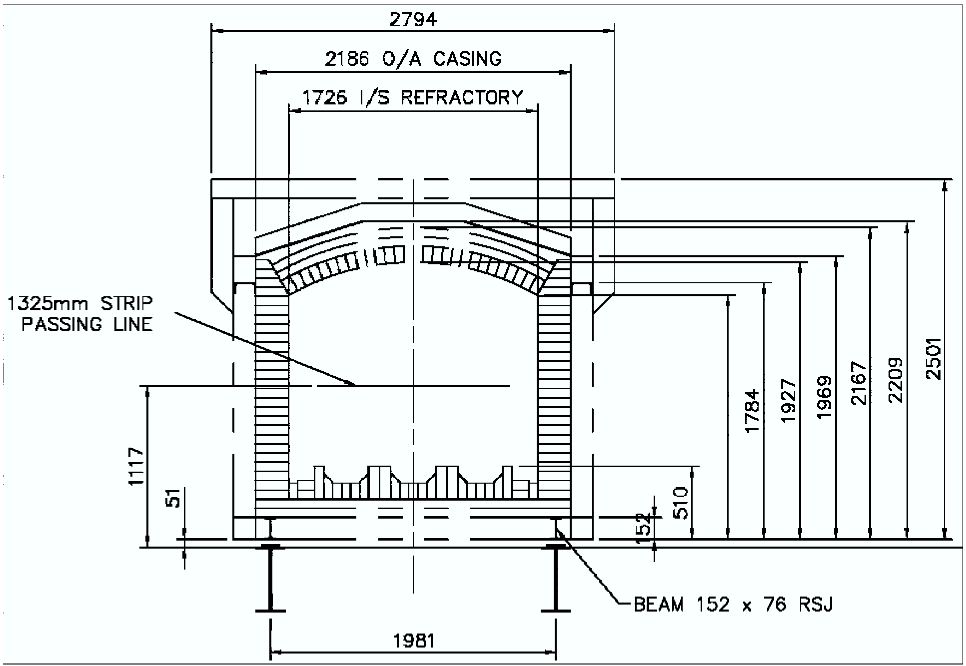

Figure 1: A Cross-section of the furnace.

In each of the twenty zones of the furnace, there are thermocouples in steel tubes, which are used to measure furnace temperature. The thermocouple temperatures are compared with desired temperature set-points, and the heating elements are controlled accordingly. Steel strip temperature is also measured, using non-contact pyrometers at three positions in the furnace.

If there is no variation in strip dimensions and annealing settings then the line is able to run in a *steady state*, with the furnace temperatures remaining steady at the desired thermocouple settings. NZS have already developed a mathematical model of furnace and strip temperatures for this steady state operation. Challenges occur when there is variation in strip dimensions or annealing settings because the furnace–strip system has a large amount of thermal inertia. Consequently the line is in a *transient* state for up to 50% of its operation, with varying effects on quality control of the product.

Two improvements are planned for the line in the very near future; a 3 MW induction heater and a gas jet cooler. The induction heater is capable of heating the strip rapidly. The steel strip will pass directly from the induction heater into the radiant furnace. The extra heating power should allow the system to achieve greater line speeds for strips of large thickness and width. Further, with its more rapid response, the induction heater has the potential to reduce the time spent in transient modes of operation. In the gas jet cooler, which will replace part of the existing cooling zone, cooled furnace gas is blown directly onto the steel strip. The new cooler section is expected to respond more rapidly than the existing cooling tubes, giving more precise control of dipping temperatures.

NZS set the following tasks for the Study Group:

- Develop a mathematical model for transient furnace conditions.
- Investigate the accuracy of the existing steady state model.
- Predict transient strip temperatures for actual production schedules with changes in product dimension, steel grade and furnace temperature settings.
- Couple the temperature model to a metallurgical model.

The paper is set out as follows:

We begin in Section 2. with an introduction to radiative heat transfer, which is the primary mode of heat transfer within the furnace.

In Section 3. we model the temperature of the strip itself as it receives radiant heat energy from the furnace. We see that the strip's temperature can be accurately modelled as a function of time and just one spatial coordinate, the distance from the entry point of the furnace. Temperatures rapidly equilibrate across the thickness of the steel and thermal diffusion along the strip is found to be negligible for the length of time that any part of the strip was in the furnace.

Next, in Section 4., we model the radiation by studying the heat transfer between surfaces within the furnace. Completing a task of this scale was beyond the scope of MISG. However, the group at MISG was able to develop a simple model for the furnace, capturing the main features of the system and identifying the principles from which a more complex model may be developed.

In Section 4.2 we investigate the time and length scales of the model and find that while it takes hundreds of hours for the furnace to come to equilibrium, the inner surface of the furnace responds much more rapidly to changes in furnace settings.

Our dynamical model for the strip–furnace system leads, of course, to a steady state model, and this is discussed in Section 4.3. The model differs from NZS's model in that MISG's model allows for continuous changes in temperature along the length of the furnace while NZS's model is discrete, involving one value of strip temperature and one value of the furnace (wall) temperature for each of the furnace's twenty zones.

We consider the temperatures that are measured by the thermocouples in each section of the furnace in Section 5., and do a premlinary analysis of the effect of cold steel on the thermocouples. Finally, in Section 6., we discuss our conclusions and ideas for on–going work.

2. Radiative Heat Transfer

Radiative heat transfer is the primary mode of heat transfer within the furnace, so here we give a brief summary of the theory that we need. More details may be found in some of the excellent texts on the subject, including Sigel and Howell [5], Sparrow and Cess [6], and Modest [2]. We follow Modest in this description.

Real opaque surfaces *emit*, *absorb* and *reflect* electromagnetic radiation and these three properties depend on the temperatures of the surfaces. The medium containing the surfaces may also participate in thermal radiation heat transfer, but in the case of the NZS furnace the medium, a mixture of nitrogen and hydrogen, is *non-participating.*

Surfaces emit a spectrum of thermal radiation when they are heated. The distribution of wavelengths in the spectrum depends on what material the surface is made from and its temperature. Likewise, the absorption and reflection of thermal radiation is temperature dependent and it also has a dependence on a material's response to different wavelengths. Moreover, there may be a directional dependence; surfaces may emit, absorb or reflect radiation more in one direction than another.

It is reassuring to know that most engineering problems involving radiative heat transfer may be solved with sufficient accuracy under the assumption that the surfaces have ideal properties. The most common of these assumptions is that the surfaces are grey, diffuse emitters, absorbers and reflectors. This means that the net absorption, reflection and emission properties of a surface have no directional dependence. Such surfaces which do not reflect thermal radiation at all are said to be *black* or *black bodies.* The total emissive power of a black body at absolute temperature T is given by

$$E_b(T) = \sigma T^4, \quad \sigma = 5.670 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}, \tag{1}$$

where σ is called the *Stefan–Boltzmann constant.*

Properties of a surface are given by three non–dimensional parameters, defined in terms of the energy of the radiation. These are

Reflectance, $\rho = \dfrac{\text{reflected part of incoming radiation}}{\text{total incoming radiation}}$,

Absorbtance, $\alpha = \dfrac{\text{absorbed part of incoming radiation}}{\text{total incoming radiation}}$,

Emittance, $\epsilon = \dfrac{\text{energy emitted by a surface}}{\text{energy emitted by a black surface at the same temperature}}$.

Transmittance is another important parameter, but we do not need to consider this because all of the surfaces within the furnace are opaque. These parameters may vary in value between 0 and 1 and it can be shown that

$$\alpha = \epsilon = 1 - \rho$$

for diffuse, grey surfaces.

Let x be any point on a surface within an enclosure. Let ϕ_x be the heat flux supplied from inside the surface body to the surface at x, E_x the power emitted by the surface at x per unit surface area and H_x the irradiation at x, i.e. the radiant heat power per unit area arriving at x from all other surface points within the enclosure. The power supplied to the surface is due to the flux from inside the surface body and the absorbed irradiation and this power must equal the power emitted from the surface; i.e. $E_x = \phi_x + \alpha H_x$. This equation must hold for all surface points within the enclosure, so we simply write

$$E = \phi + \alpha H. \tag{2}$$

In order to calculate H we need the notion of *view factors*, which are sometimes called *shape factors* or *configuration factors.* The view factor between two infinitesimal surface elements dA_i and dA_j, located at points x_i and x_j respectively, is

$$dF_{dA_i - dA_j} = \frac{\text{diffuse energy leaving } dA_i \text{ directly toward and intercepted by } dA_j}{\text{total diffuse energy leaving } dA_i}.$$

Since energy leaves the surfaces diffusely, view factors depend only on the geometry of the enclosure and it is not difficult to derive the formula

$$dF_{dA_i - dA_j} = \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_j,$$

where S is the distance between x_i and x_j, θ_i and θ_j are the angles between the line from x_i to x_j and the outer normal vectors at x_i and x_j respectively. An important observation from this equation is the *law of reciprocity*

$$dA_i dF_{dA_i - dA_j} = dA_j dF_{dA_j - dA_i}.$$

Surfaces within enclosures are often approximated by a finite number of isothermal surfaces and one needs the view factor between two such surfaces of areas A_i and A_j. This is given by

$$F_{A_i - A_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi S^2} \, dA_j dA_i, \tag{3}$$

and there is a *law of reciprocity*

$$A_i F_{A_i - A_j} = A_j F_{A_j - A_i}.$$

Consider an enclosure consisting of N isothermal surfaces of areas A_i, $i = 1, 2, \ldots N$ with emittances ϵ_i, reflectances $\rho_i = 1 - \epsilon_i$, emissive powers E_i, temperatures T_i, outward surface fluxes ϕ_i and irradiations H_i. The contribution to H_i from A_j is due to the radiation emitted and the irradiation reflected from A_j, so it is given by

$$F_{A_j - A_i} \frac{A_j}{A_i} (E_j + \rho_j H_j) = F_{A_i - A_j} (E_j + \rho_j H_j),$$

by reciprocity of the view factors. Hence

$$H_i = \sum_{j=1}^{N} F_{A_i - A_j} (E_j + \rho_j H_j).$$

But $E_i = \epsilon_i E_b(T_i)$ and, by Equation (2), $H_i = (E_i - \phi_i)/\alpha_i = (E_i - \phi_i)/\epsilon_i$. Hence

$$E_b(T_i) - \frac{1}{\epsilon_i}\phi_i = \sum_{j=1}^{N} F_{A_i - A_j} \left(E_b(T_j) - \left(\frac{1}{\epsilon_j} - 1 \right) \phi_j \right). \tag{4}$$

This important equation relates the heat fluxes from within the surface bodies to the surface temperatures. For the case of view factors between infinitesimal surfaces the relationship between the heat fluxes and surface temperatures is an integral equation and Equation (4) may be regarded as a discretisation of this integral equation.

3. Modelling the strip

If we assume that the sheet is perfectly straight with a rectangular cross section then the portion of the strip within the furnace occupies a region of space

$$\mathcal{S} = \{(x, y, z) : 0 \le x \le L, -w(x,t)/2 \le y \le w(x,t)/2, 0 \le z \le h(x,t)\},$$

where

- L is the length of the furnace,
- x measures distance from the point of entry of the strip into the furnace,
- $w(x,t)$ and $h(x,t)$ are respectively the width and thickness of the strip,
- z is a distance coordinate in the vertical direction and y is a distance coordinate across the strip.

The strip thickness h and width w are piecewise constant functions of x and t because the strip is formed by welding together straight sheets that may have different dimensions.

The temperature u within the strip may be modelled by the heat equation with an advection term corresponding to the strip's speed v through the furnace:

$$\rho_S C_S \left(\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial x} \right) = k_S \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \quad t > 0,\ (x, y, z) \in \mathcal{S}.$$

In this equation ρ_S and C_S are the strip's density and specific heat capacity respectively.

We can determine the relative importance of the different terms in the equation by using dimensionless coordinates $\tilde{x} = x/L$, $\tilde{y} = y/w$, $\tilde{z} = x/h$, $\tilde{t} = tv/L$, where h and w are typical values of the thickness and width of the strip. In terms of the dimensionless variables, the equation takes the form

$$\frac{\partial u}{\partial \tilde{t}} + \frac{\partial u}{\partial \tilde{x}} = \frac{k_S L}{v \rho_S C_S} \left(\frac{1}{L^2} \frac{\partial^2 u}{\partial \tilde{x}^2} + \frac{1}{w^2} \frac{\partial^2 u}{\partial \tilde{y}^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \tilde{z}^2} \right).$$

Taking $L = 150$ m, $v = 2$ m s^{-1}, $w = 0.5$ m, $h = 0.5$ mm, $k_S = 50$ W m^{-1} K^{-1}, $C_S = 500$ J Kg^{-1} K^{-1} and $\rho_S = 7854$ Kg gives the equation

$$\frac{\partial u}{\partial \tilde{t}} + \frac{\partial u}{\partial \tilde{x}} = 4.2 \times 10^{-8} \frac{\partial^2 u}{\partial \tilde{x}^2} + 3.8 \times 10^{-3} \frac{\partial^2 u}{\partial \tilde{y}^2} + 3.8 \times 10^{3} \frac{\partial^2 u}{\partial \tilde{z}^2},$$

which shows that the heat conduction terms in the x and y directions may be ignored. Further, the large coefficient of the heat conduction term in the z direction indicates that the strip responds rapidly to changes in temperature in this direction.

Returning to the original variables, we find

$$\rho_S C_S \left(\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial x} \right) = k_S \frac{\partial^2 u}{\partial z^2}. \tag{5}$$

Let $T(x, y, t)$ denote the temperature of the strip *averaged over the z direction.* Thus

$$T(x, y, t) = \frac{1}{h} \int_0^h u(x, y, z, t)\, dz.$$

The advantage of dealing with T rather than u is that T depends on fewer variables than u and so it is easier to compute. Moreover, T should be an excellent approximation for u because heat conduction in the z direction is so rapid. Integrating each side of (5) with respect to z gives

$$\begin{aligned} \rho_S C_S \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right) &= \frac{k_S}{h} \left(\left.\frac{\partial u}{\partial z}\right|_{z=h} - \left.\frac{\partial u}{\partial z}\right|_{z=0} \right) \\ &= \frac{1}{h} (\text{flux in at top surface} + \text{flux in at bottom surface}). \end{aligned}$$

Assuming that the strip receives only radiation evenly across it in the y direction and that the total radiation it receives is $q(x, t)$ per unit length in the x direction, we obtain

$$\rho_S C_S \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right) = \frac{q}{wh}. \tag{6}$$

We note that there are significant variations in C_S, the specific heat capacity of steel, over the range of temperatures to which the steel is subjected. Tables 1 and 2, taken from Incropera and DeWitt [1], show the temperature dependence of the thermal properties of steel.

Table 1: Properties of steel at 300 K.

ρ (Kg/m^3)	C_p (J/Kg.K)	k (W/m.K)
7854	434	60.5

Polynomial interpolation of the C_p data for steel yields the interpolation function

$$C_S(T) = 345 - 0.504333\, T + 0.004895\, T^2 - 9.06667 \times 10^{-6}\, T^3 + 5.5 \times 10^{-9}\, T^4, \tag{7}$$

Table 2: Properties of steel at various temperatures.

T (K)	300	400	600	800	1000
k (W/m.K)	60.5	56.7	48.0	39.2	30.0
C_p (J/Kg.K)	434	487	559	685	1169

and we use this expression for C_S in Equation (6). The C_p values of steel and this interpolating function are graphed in Figure 2.

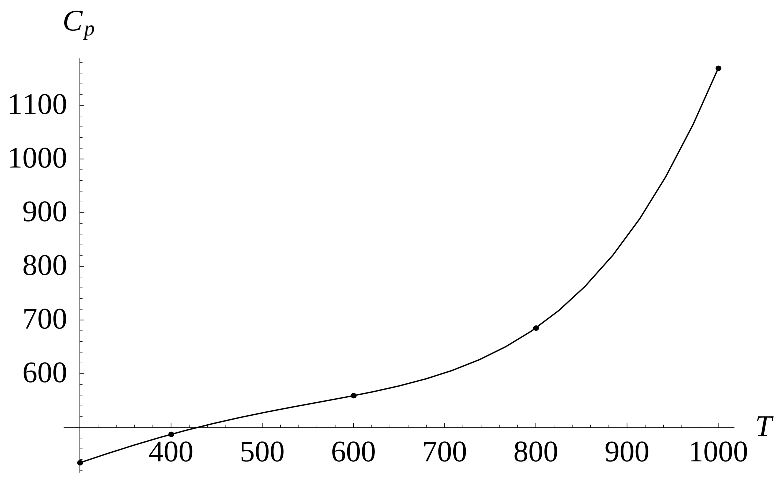

Figure 2: Variation of C_p (J/Kg.K) for steel with absolute temperature T in Kelvin.

4. Simple furnace model

Our aim in this section is to develop a simple model of the furnace–strip system that is detailed enough to exhibit the main dynamical properties of the system. NZS's steady state model is essentially a two surface model in that in each zone it is assumed that there is surface at temperature T_{zone} interacting with the strip's surface which is at temperature T. Thus there is an assumption that little net radiation travels from one zone into another. This assumption seems to work because of the fact that, with one exception, temperatures change very gradually along the length of the furnace. The exception is at the interface between the cooling section and the heating section of the furnace. But at this interface there is a wall of refractory bricks with a narrow opening in it, through which the strip passes. This wall acts as a radiation shield, reducing thermal radiation transfer from the heating section to cooling section of the furnace. Hence, even at this interface, the assumption of no net radiation transfer from one zone to another seems to be a reasonable first approximation and we adopt this approximation for our simple dynamical model. Thus, our assumptions are:

Assumptions

- The inner surface temperature of the furnace walls depends on time t and distance x along the furnace measured from the entry point of the strip.
- The temperature of a heating element is the same as the temperature of the inner surface of the wall adjacent to the element.
- Temperature changes within the furnace are so gradual that we can ignore radiative or convective heat transfer along the length of the furnace.
- At this point, we ignore the cooling tubes. This model applies to the heating zones of the furnace.

4.1 Model equations

Consider a length Δx of the furnace. Within this length there are only two surfaces that interact: walls and strip. Let $2p\Delta x$ denote the total area of the inner surfaces of the walls in this length of furnace and let $w\Delta x$ denote the total surface area of one side of the strip in this length of furnace. Thus, w represents the width of the strip and p is approximately height+width of the inside of the furnace.

The assumptions of approximate isothermality of these surfaces simplify the calculation of radiative heat transfer between the surfaces. The relevant view factor for radiation from the strip to the walls is $F_{SW} = 1$. By reciprocity, $F_{WS} = w/p$ and, because the rows of the view factor matrix sum to one, $F_{SS} = 0$ and $F_{WW} = 1 - w/p$.

We need to calculate q, the radiation per unit length of the furnace from the walls to the strip. Instead of using i and $j = 1, 2$ in Equation (4) we use subscripts W and S to denote quantities associated with the walls and strip, respectively. Solving Equation (4) and using $q = 2p\phi_W$, or equivalently $q = -2w\phi_S$, leads to the required expression for q:

$$q = \frac{2w\epsilon_S\sigma(T_W^4 - T^4)}{1 + \dfrac{\epsilon_S(1-\epsilon_W)}{\epsilon_W}\dfrac{w}{p}}. \tag{8}$$

Note that

$$q \approx 2w\epsilon_S\sigma(T_W^4 - T^4) \tag{9}$$

because $p > w$, $\epsilon_W \approx 1$ and ϵ_S is small. Note also that an alternative approach is to model the strip and walls as parallel planes. If this approach is adopted then one obtains the formula

$$\frac{2w\sigma(T_W^4 - T^4)}{\dfrac{1}{\epsilon_S} + \dfrac{1}{\epsilon_W} - 1}$$

for q. This formula is similar to one used by NZS in their steady state model and it also differs only slightly from the approximation (9).

Next consider the energy balance for the wall surface and heating elements in the length Δx of furnace. Let C_E denote the specific heat of the element material and let $m(x)$ denote the mass of heating elements per length of furnace ($m(x)$ will be a step function). Since the heating elements and the inner wall surface are treated as being a lumped isothermal object,

$$mC_E\frac{\partial T_W}{\partial t}\Delta x = P\Delta x - \Phi 2p\Delta x - q\Delta x$$

where Φ is the heat flux into the walls and P is the power supplied to the heating elements per unit length of the furnace. Assuming that the heating elements have little thermal inertia, this simply gives

$$\Phi = \frac{P - q}{2p}. \tag{10}$$

We simplify the modelling of heat flow through the walls by treating each wall as a separate slab. Thus we obtain a simple one dimensional heat conduction problem

$$\rho_W C_W \frac{\partial T_B}{\partial t} = k_W \frac{\partial^2 T_B}{\partial r^2}, \qquad 0 < r < d, \tag{11}$$

$$T_B(x, 0, t) = T_W(x, t), \tag{12}$$

$$k_W \frac{\partial T_B}{\partial r}\bigg|_{r=0} = -\Phi, \tag{13}$$

$$k_W \frac{\partial T_B}{\partial r}\bigg|_{r=d} = H(T_\infty - T_B(x, d, t)), \tag{14}$$

where d is the thickness of the furnace wall, T_∞ is the external ambient temperature, H is a convection coefficient and $T_B(x, r, t)$ is the internal wall (brick) temperature at a distance x along the furnace and a depth r into the wall.

The thermal properties of refractory brick, of which the furnace walls are made, are summarised in Table 3.

Table 3: Properties of refractory brick (provided by NZS).

T (K)	478	1145
k (W/m.K)	0.25	0.30
C_p (J/Kg.K)	≈ 900	≈ 900
ρ (Kg/m^3)	≈ 2000	≈ 2000

4.2 Characteristic time and length scales for furnace wall heating

Dimensional analysis has already played a roll in our analysis; in Section 3. we used it to simplify the equation modelling the heating of the strip. Here we use it to gain insight into the furnace's response to changes in heating.

For *bulk changes* in the furnace's temperature, the dimensional parameters that are relevant are those that appear in the heat equation (11) and the wall thickness, d. These combine to give a time constant

$$t_1 = \frac{\rho_W C_W d^2}{k_W} \approx 320 \text{ hours},$$

using $d = 0.4$m (see Fig. 1) and values from Table 3. This gives a measure of the time it would take for the furnace bricks to effectively come to equilibrium if exposed to a constant source of heat.

However, heat sources within the furnace change much more rapidly than this and one would expect that the furnace walls will respond quite rapidly in the locality of their inner surfaces. To get a measure of such *local changes* to furnace temperature, two approaches are presented here. The first approach is the simpler, and is the method used during the Study Group. In this approach, the geometry of the oven and the presence of steel strip is ignored. Diffusion in the oven wall is given by equation (11), but the boundary conditions are simplified. Radiant heating of the wall by nearby electric heaters is modelled by the boundary condition

$$k_W \frac{\partial T_B}{\partial r} = f\sigma(T_W^4 - T_h^4), \quad r = 0 \tag{15}$$

and the wall is taken to be infinitely thick. Taking the heaters to be parallel to the walls and of the same width gives $f = \epsilon_S \approx 0.2$.

We consider the effect of changing from a constant initial state $T_W = T_0$ which is in equilibrium with the heaters ($T_h = T_0$), by changing the temperature of the heaters to a new value $T_0 + \Delta T_0$. We linearise the response of the wall temperature about T_0 by using $T_W = T_0 + \theta\Delta T_0$ and nondimensionalise to obtain

$$\begin{aligned} \theta_t &= \theta_{rr} \,, \quad r > 0 && (16) \\ \theta_r &= \theta - 1 \,, \quad r = 0 && (17) \\ \theta &= 0 \,, \quad t = 0 \,, && (18) \end{aligned}$$

with a lengthscale

$$L = \frac{k}{4\epsilon_S \sigma T_0^3} \approx 5\text{mm}$$

and timescale

$$\tau = \frac{L^2 \rho_W C_W}{k} \approx 2\text{mins}$$

That is, the characteristic time for the wall to respond to a change in heater temperatures is about 2 minutes, and only the first 5 mm of depth needs to respond. This is shorter than on-site experience suggests.

Numerical solutions of equations (16)–(18), conducted at the Study Group and graphed in Fig. (3) confirm that, as expected, the rescaled temperature changes are of order one when time changes are of order one, at the surface of the oven wall.

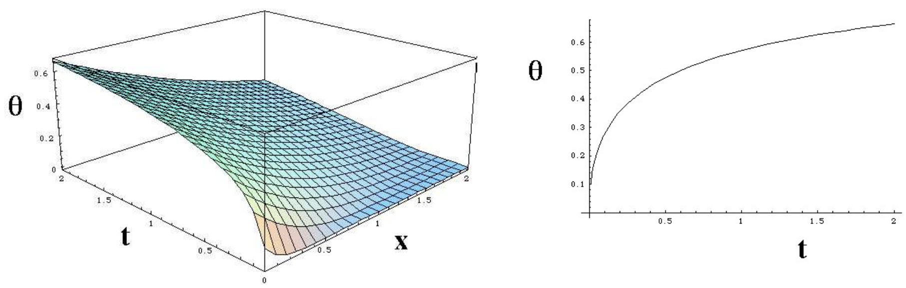

Figure 3: Numerical solutions of the simpler wall heating model, after nondimensionalisation.

A second, more sophisticated model (developed subsequently to the Study Group in the course of writing this report) takes more careful account of the actual oven geometry and the presence of the steel strip, by using boundary condition (13). In this model, the power supplied per unit length of heaters changes from P to $P + \Delta P$. Let $\theta(r, t)$ denote the resulting deviation in brick temperature from its steady value T_0, so that we expand $T_W = T_0 + \theta$. For simplicity, we only consider the first few heating zones where the strip temperature T is relatively small. In this region we ignore the fourth power of the strip temperature because it is much less than the fourth power of the wall temperature.

Hence, linearising Equations (9)–(13) for small θ gives

$$\rho_W C_W \frac{\partial \theta}{\partial t} = k_W \frac{\partial^2 \theta}{\partial r^2}, \qquad 0 < r < d,$$
$$\left(k_W \frac{\partial \theta}{\partial r} - 4T_0^3 \frac{w}{p} \epsilon_S \sigma \theta \right)\bigg|_{r=0} = -\frac{\Delta P}{2p},$$

with initial condition $\theta = 0$ and boundary condition $\theta \to 0$ as r becomes large (but is much smaller than d). These equations may be cast into dimensionless form by setting

$$r = \frac{pk_W}{4T_0^3 w \epsilon_S \sigma} \tilde{r} \equiv R\,\tilde{r}\ ,$$
$$t = \frac{\rho_W C_W}{k_W} R^2 \tilde{t}\ ,$$
$$\theta = \frac{\Delta P R \tilde{\theta}}{2pk}\ .$$

The resulting dimensionless equations are the same as for the simpler model:

$$\frac{\partial \tilde{\theta}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{\theta}}{\partial \tilde{r}^2}, \qquad \tilde{r} > 0\ , \tag{19}$$
$$\frac{\partial \tilde{\theta}}{\partial \tilde{r}} = \tilde{\theta} - 1, \qquad \tilde{r} = 0\ . \tag{20}$$

The scaling parameters are different to those for the simpler model. With a strip width $w = 0.938$m, oven perimeter $p = 3.4$m, and a temperature $T_0 = 1000$K, the scaling factors give a length scale $R = 20$mm and a time scale of 40 minutes.

The response of the furnace-strip system depends on its settings, but a figure of 40 minutes is comparable to the actual period of time taken by the furnace to respond to changes, especially in the front where the steel strip is relatively cool. The simpler model result of 2 minutes is too short to be realistic.

Since the simpler model calculation at MISG led to time and length scales that were too short, attention was then shifted to the steel hearth rolls (the rollers which carry the strip along the furnace), to see if they could be the main source of thermal inertia within the furnace. Preliminary calculations indicated that the hearth rolls do indeed respond to temperature changes on the correct time scale, so that it would be useful to include the hearth rolls in any transient thermal model of the oven.

4.3 Steady state solutions

The steady state equations are:

$$\frac{dT}{dx} = \frac{q}{wh\rho_S C_S v}, \tag{21}$$

$$\frac{\partial^2 T_B}{\partial r^2} = 0, \tag{22}$$

$$T_B(x,0) = T_W(x), \tag{23}$$

$$k_W \frac{\partial T_B}{\partial r}\bigg|_{r=d} = H(T_\infty - T_B(x,d)), \tag{24}$$

which must be solved together with (7) and (8). The last equation (24) gives

$$T_B = T_W - \frac{T_W - T_\infty}{d + k_W/H} r. \tag{25}$$

This equation allows us to estimate k_W/H from temperature measurements. NZS estimate $d = 0.4$m and the external furnace temperature, $T_B(d) = 60$°C. Taking the internal wall temperature $T_B(0) = T_W = 900$°C and $T_\infty = 20$°C gives

$$\frac{k_W}{H} \approx d/21 = 0.019\text{m}.$$

Equation (25) also gives

$$\Phi = -k_W \frac{\partial T_B}{\partial r} = \frac{k_W}{d + k_W/H}(T_W - T_\infty).$$

Inserting this expression for Φ into (10) and using (8) gives an equation of the form used by NZ Steel to model the steady state:

$$P = k_1(T_W^4 - T^4) + k_2(T_W - T_\infty),$$

where

$$k_1 = \frac{2w\epsilon_S\sigma}{1 + \dfrac{\epsilon_S(1-\epsilon_W)}{\epsilon_W}\dfrac{w}{p}},$$

$$k_2 = \frac{2pk_W}{d + k_W/H}.$$

An approximate solution to (21) is easily obtained by replacing T on the right-hand-side of (21) by τ_i, the strip temperature at the start of zone i. Thus the strip temperature in zone i is given by

$$T = \tau_i + \frac{2\epsilon_S\sigma}{hv\rho_S C_S(\tau_i)}\left(T_{Wi}^4 - \tau_i^4\right)(x - x_i), \tag{26}$$

where x_i is the location of the start of zone i, T_{Wi} is the wall temperature of zone i, and the expression $C_S(\tau_i)$ is given by (7). Solving the differential equation in this manner is essentially implementing NZS's discrete model.

Figures 4 and 5 were generated by solving the differential equation (21) using Matlab. There is a small discrepancy between these figures and similar figures that NZS presented, based on their discrete model, at MISG. NZS's model predicts higher strip temperatures in the full anneal. The difference is due to the fact that here we take into account the increase in heat capacity of the steel with increasing temperature, which results in a smaller temperature gain per unit heat energy absorbed by the strip at higher temperatures.

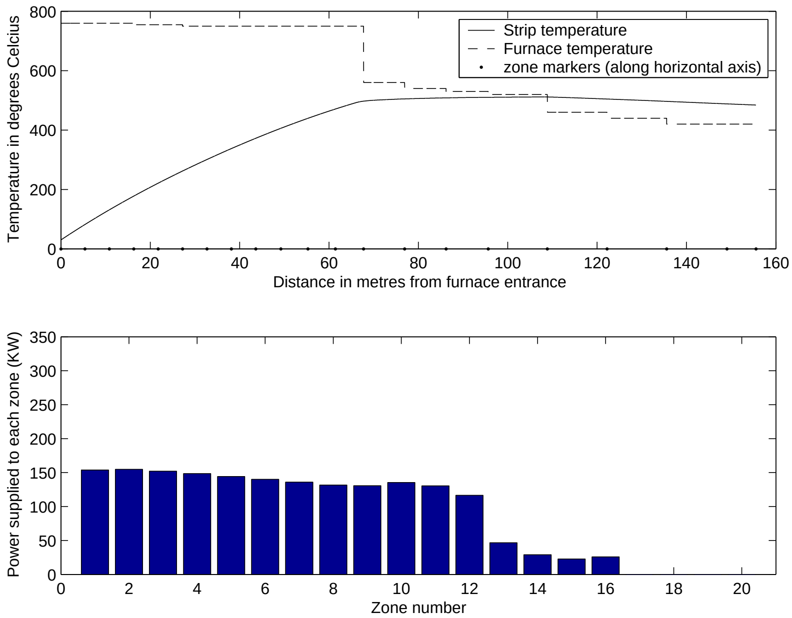

Figure 4: Steady state solution for recovery anneal (soft iron). Here v = 116.4m/minute, $w = 0.940$m, $h = 0.42$mm.

5. Measuring Furnace Temperature

An important measurement used by NZ Steel as an approximation to the transient furnace temperature is the temperature measured by thermocouples, one in each section of the furnace. A furnace section is ~5–6 m long, and each thermocouple is set into a steel tube projecting into the furnace from the ceiling. These tubes are 0.3m long and have an outer diameter of ~0.15m.

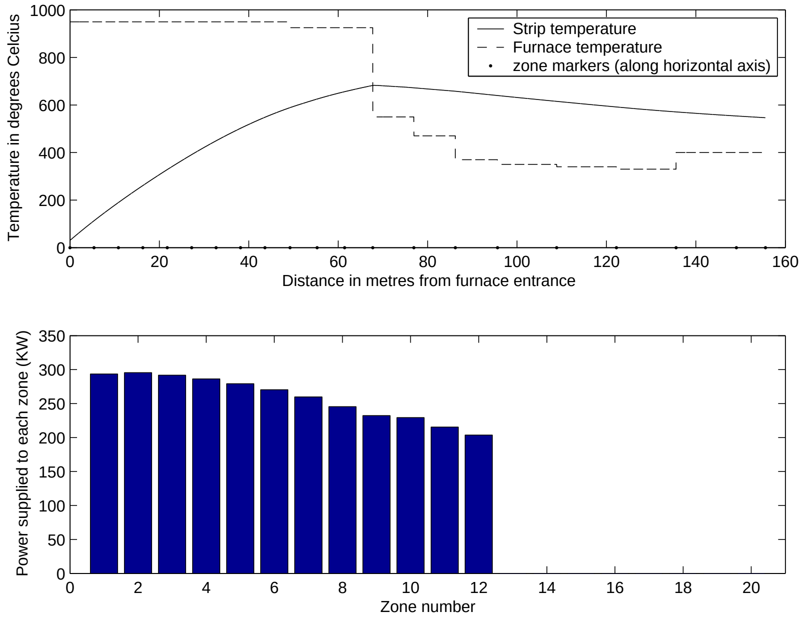

Figure 5: Steady state solution for full anneal (hard iron). Here v = 108.2m/minute, $w = 0.938$m, $h = 0.55$mm.

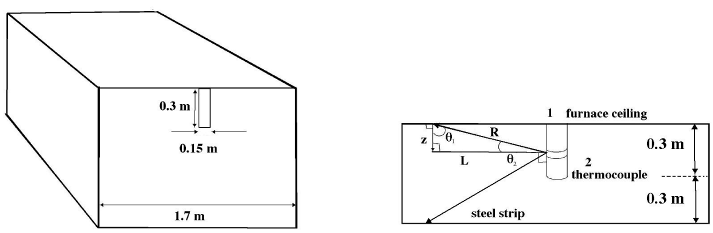

Figure 6: A sketch of the geometry of the thermocouple in each section of the furnace.

Assuming the thermocouple touches the inside of the steel tube, it measures the temperature of the steel tube wall. This temperature will be approximately at equilibrium with its radiation environment, so that the rate at which heat is radiated from the steel tube will approximately match the rate at which heat is being absorbed from its surroundings.

A detailed model would account for the different temperatures of the radiating electric heater elements, the hot inner brick walls, the cooler steel, and reflections from the relatively shiny surface of the steel. As a first approximation, we consider the cylinder (surface number 2) to be projecting from a uniformly hot surface (the ceiling of the furnace, surface number 1), and ignore other radiators in the furnace.

The net heat received from the hot wall by a surface element dA_2 on the cylinder is

$$q_{21} = dA_2\ F_{21}\sigma(T_2^4 - T_1^4)$$

where the shape factor is

$$dA_2\ F_{21} = \int \frac{\cos(\theta_1)\cos(\theta_2)}{\pi R^2} dA_1$$

where R is the variable distance from dA_2 to the wall, θ_1 and θ_2 are the angles between R and the normals to the surface elements as illustrated in Fig. (6), and the integral is taken over the surface of the hot ceiling.

If the temperature of the ceiling varies with R, it too would need to be inside the integral. An examination of the integrand in the shape factor reveals which parts of the hot ceiling have most effect on the temperature of the thermocouple:

$$\frac{\cos(\theta_1)\cos(\theta_2)}{\pi R^2} = \frac{\sin(2\theta_1)}{2\pi R^2}$$

where the geometry gives $\theta_1 + \theta_2 = \pi/2$, so that the area integral looks like

$$\int_0^{\mathrm{Lmax}} dL \int_{\phi_{\min}}^{\phi_{\max}} \frac{L\ d\phi}{(L^2 + z^2)^{3/2}}$$

where $R^2 = L^2 + z^2$ and z is the vertical distance from the ceiling to the area element dA_2 (see Fig. (6)). Lmax is the maximum distance we are integrating away from the cylinder. For Lmax < 0.85m, $\phi_{\text{min}} = 0$ and $\phi_{\text{max}} = 2\pi$. For larger L, the range of ϕ is restricted and depends on L, but this effect on the shape factor is small compared with the factor $\sim L^{-2}$ which reduces the effect of distant parts of the ceiling.

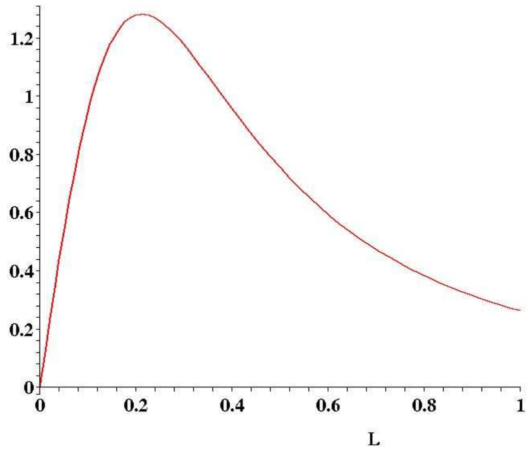

Figure 7: A plot of the kernel of the shape factor integral, for $z = 0.3$.

The integrand depends mostly on L, and a simple analysis (illustrated in Fig. (7) for $z = 0.3$) reveals that it reaches a maximum near $L \sim z$, and decays like L^{-2} as L increases. Hence the region of ceiling most affecting the temperature of the thermocouple is that region in a disk of radius approximately 0.8m around the steel tube, with the dominant effect being from an annulus about 0.3m away.

The effect of cooler steel strip on thermocouple readings can be estimated roughly, by noting that the thermocouple tube extends down to a distance 0.3m from the ceiling, which is 0.3m above the steel. Hence the steel, if it is cooler than the furnace ceiling, will have a small effect on the thermocouple, reducing its temperature. The effect is more pronounced near the lower end of the tube, where it is closer to the cooler steel — at the very tip of the thermocouple tube, the effect of the steel strip is comparable to that of the hot ceiling, and the recorded thermocouple temperature will be roughly an average of the two (steel strip and ceiling temperatures). Further up the thermocouple tube, this balance will shift (varying roughly as z^2), so that for example half-way up the tube the ceiling temperature will have four times as much weight as the steel strip temperature.

This suggests that two thermocouples in each steel tube, one set at the very tip and one rather higher, might be a useful measurement tool, for added confidence, allowing in principle an estimation of the steel strip temperature, as well as of furnace temperature. More modelling work, backed by numerical integration for the geometry of the actual furnace, would be useful here.

6. Conclusions and recommendations

We have developed a simple model of the furnace–strip system which captures the main features of the system. More importantly, we have laid out the important mathematical and physical principles from which a more detailed model may be constructed. Such a model may be developed by approximating the surfaces within the furnace as a large number of isothermal surfaces which exchange thermal radiation with each other. Obviously such an undertaking is not a trivial task because it involves detailed calculations of view factors between portions of the many different surface types within the furnace–strip system, and separate calculations have to be done for different strip dimensions. There are a number of benefits for developing such a model:

1. It would allow accurate calculation of strip temperature in steady and in changing furnace conditions. This would allow calculation of furnace settings for the annealing process and it would provide a tool for optimising the running of the furnace, especially during transient periods of operation.
2. It would provide knowledge of what the furnace thermocouples are measuring. The thermocouples are used to estimate the furnace temperature in each zone of the furnace. They play a vital role, as they are used to control the power fed to the heating elements. These thermocouples are housed in tubes which are exchanging radiation with all of the surfaces within the furnace and so their temperature readings depend on the temperatures of every surface in the furnace. Thus, only a reasonably detailed model of the furnace will tell us what these thermocouples are measuring.
3. It would allow accurate calculation of strip temperature *across the strip's width*. In particular, it would allow calculation of temperature along the edges of the strip. NZS has identified the edges of the strip as being more susceptible to over–heating, which influences the annealing. Such overheating can cause a wavy pattern along the product's edges. Being able to monitor the temperature of the strip's edges will allow the company to reduce waste due to edges overheating.

We should mention here that our simple model assumed that all surfaces are grey and diffuse. This is a good approximation for refractory brick, but perhaps not such a good approximation for the steel strip. At least for radiation in the visual spectrum, the angle of reflected radiation appears to be randomly clustered around the angle of incidence of the radiation, i.e. the surface is partly *specular*. It is not difficult to model this feature of the steel. Indeed, a series of papers by Pérez-Grande et al [3], Sauermann et al [4], Teodorczyk and Januszkiewicz [7], involve an electric furnace model for crystal formation. The crystal, of course, is highly specular.

This same series of papers seems to be the only modern literature involving the modelling of a specific electric radiant furnace. While the perfect cylindrical symmetry of the furnace under study simplifies the problem of modelling the furnace, the main principles of the work apply to the NZS furnace.

Finally, we have also conducted a preliminary investigation into the meaning of temperatures recorded in thermocouples suspended in steel tubes in each section of the furnace. These thermocouples are used in practice to set desired furnace operating temperatures via a feedback control system, and to measure how far from these desired setpoint temperatures the furnace is operating at any moment in time. We note that the temperatures recorded by these thermocouples may be sensitive to the temperature of cold steel strip passing through that section of the oven. Further modelling of the thermocouple tube temperatures would be very useful, and promises better control of furnace and steel temperatures.

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