RESONANCE QUANTUM SWITCH AND QUANTUM GATE

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Abstract

Solvable models for two- and three-terminal Quantum Switches and Quantum Gates are suggested in form of a quantum ring witha few one-dimensional quantum wires attached to it. In resonance case when the Fermi level in the wires coincides with the resonance energy level on the ring , the magnitude of the governing electric field may be specified such that the quantum current through the switch from up-leading wire to the outgoing wires may be controlled via rotation of the orthogonal projection of the field onto the plane of the device.The working parameters of the switches and gates are defined in dependence of the desired working temperature, the Fermi level [1] and the effective mass of the electron in the wires.

1 Resonance Quantum Switch

Interference of the wave function may serve a base for design of the quantum electronic devices, see [9]. Though the the basic problems of the mathematical design of quantum electronic devices were already formulated in term of quantum scattering by the beginning of nineties, see [11], still the design of most of modern resonance quantum devices, beginning from classical Esaki diode up to modern devices, see for instance [25], [27] are based on the resonance of energy levels rather than on resonance properties of the corresponding wave functions. At the same time modern experimental technique already permits to observe resonance effects caused by details of the shape of the resonance wave functions, see [20], [21], [22]. In frames of the EC-project "New technologies for narrow-gap semiconductors" (ESPRIT-28890 NTCONGS, 1998 - 1999) the problem on mathematical design of a four-terminal Quantum Switch for triadic logic was formulated by Professor G. Metakides and Doctor R.Compano from the Industrial Department of the European Comission. Results of the theoretical part of the project were published in papers [16], [17], [18], [19], [23] where a new design of Resonance Quantum Switches (RQS) was suggested in form of a quantum domain or quantum ring with a few quantum wires (terminals) attached to it. The idea of the new design, as presented in [23] for the device designed in form of a quantum domain (quantum well), is based on properties of the *shape* of the *resonance eigenfunction* which corresponds to the *resonance eigenvalue* [2] of the Schrödinger equation in the quantum well. These resonance properties were observed first, see [6], in the scattering problem for acoustic scattering on a resonator with a

[1]See [5]
[2]Resonance eigenvalue is equal to the Fermi level in the up-leading wires

small opening: an additional term in the scattering amplitude caused by the opening appeared to be proportional to the value of the resonance eigenfunction at the opening (for the Neumann boundary condition on the walls of the resonator) or to the value of it's normal derivative (for the Dirichlet boundary conditions).

We consider a single act of computation as a scattering process. In the corresponding scattering problem for RQS formed as a ring of radius R a few (straight) quantum wires weakly connected to the ring via tunneling of electrons of mass m across the potential barrier power of width l, hight H with electrons charge already included into it in proper units. The potential on the wires is assumed to be equal $V_2 < E_f$, so that the resonance eigenvalue $E_2 = E_f$ on the ring is embedded into continuous spectrum of the Schrödinger operator on the whole graph. If the potential barriers separating the wires from the ring are strong enough then the connection between the wires and the ring may be reduced to the boundary condition with the small parameter

$$\beta = \left(\cosh \frac{\sqrt{2mH}}{\hbar} l\right)^{-1}$$

at the contact points a_s, see below Sections 3 and 4. For weak connection between the ring and the wires the transmission coefficient from one wire to another in the resonance case appears to be proportional to the product of the values of the normalized resonance eigenfunctions $\varphi(a_s)$ at the contact points see [16]. If the renormalized energy $\lambda = k^2 = (E - V_2)\frac{\hbar^2 R^2}{2m}$ proportional to the depth of the quantum wires $E - V_2$ at infinity is close to the renormalized Fermi level $\lambda_f = (E_f - V_2)\frac{\hbar^2 R^2}{2m}$ and β is small,then the following approximate expression in scaled variables, see below Section 4, is true for the transmission coefficient from the wire attached to a_s to the wire attached to a_t :

$$S_{s,t}(\lambda) = \frac{2k|\beta|^2}{k|\beta|^2|\vec{\varphi}|^2 - i(\lambda_f - \lambda)}\varphi(a_s)\varphi(a_t) + O(\beta^2),\ \ s \neq t.$$

where $|\vec{\varphi}|^2$ is the length of the *channel-vector* $(\varphi a_1,\ \varphi a_2,\ \dots \varphi a_4)$ the second term is uniformly small when $\beta \to 0$, but the first one exhibits a nonuniform behaviour in dependence on ratio $(\lambda_f - \lambda)/\beta^2$. The last formula being applied formally to the case $\lambda = \lambda_f$ shows, that the transmission coefficient is approximately equal to

$$S_{s,t}(\lambda) = \frac{2}{|\vec{\varphi}|^2}\varphi(a_s)\varphi(a_t) + O(\beta^2).$$

This looks rather surprising for $\beta \approx 0$ since it would supply a nonzero transmission coefficient for almost zero connection. Actually this means that the transmission coefficients are not continuous with respect to the renormalized energy λ uniformly in β. The physically significant values of the transmission coefficient may be obtained in limit case via averaging over intervals $|E - E_f| < \kappa T$ for *relatively* small and *relatively* large temperature. In the first case we still have:

$$\overline{|S_{ij}(T)|^2} \approx \frac{2|\varphi(a_s)\varphi(a_t)|^2}{|\vec{\varphi}|^4}$$

but in the second case, we have :

$$\overline{|S_{ij}(T)|^2} \approx 4\frac{|\varphi(a_s)\varphi(a_t)|^2}{|\vec{\varphi}|^4}\frac{1}{1 + \frac{\kappa^2 T^2}{\lambda_0 |\beta|^4 |\vec{\varphi}|^4}}.$$

Hence for small β and non-zero temperatures the averaged transmission coefficient is small, according to natural physical expectations.

Nevertheless the above formulae show that in certain range of temperatures the transmission is proportional to the product of values of the resonance eigenfunctions at the contact points. Similar observation takes place for switches based on the quantum well with Neumann boundary conditions ,see [23] and analog of it with normal derivatives of the resonance eigenfunction remains true for the Dirichlet boundary conditions, see [24].

One may obviously construct the *dyadic* RQS basing on this observation. But even *triadic* (four-terminal) RQS may be constructed with minimal alteration of the geometrical construction. For instance, on a circular quantum well $\Omega_0 : |\vec{x}| \leq R$ for a *special choice* of the constant electric field $\mathcal{E}\vec{\nu}$, $|\vec{\nu}| = 1$, and the shift potential V_0, see [23], the corresponding Schrödinger equation

$$-\frac{\hbar^2}{2m} \triangle \psi + (\mathcal{E} e \langle \vec{x}, \vec{\nu} \rangle + V_0) \psi = E\psi \tag{1}$$

may have a resonance eigenfunction with an eigenvalue equal to the Fermi level $E = E_f$ in the wires such that it has only one smooth line of zeroes which crosses the circle dividing it in ratio $1 : 2$. Then, attaching the wires Γ to the domain at the points a_1, a_2, a_3, a_4 characterized by the central angles $0, \pm\pi/3, \pi$ we obtain the Resonance Quantum Switch manipulated by rotating $\vec{\nu} \to \vec{\nu}'$ of the constant electric field $\mathcal{E}\vec{\nu}$ in the plane parallel to the plane of the device. In particular, if for some direction $\vec{\nu}$ the line of zeroes arrives to the boundary exactly at the contact points : a_2, a_3 (or a_3, a_4, or else a_4, a_2) , then the corresponding wires Γ_2, Γ_3 (or respectively Γ_3, Γ_4 , or else Γ_4, Γ_2) are blocked. With two outgoing wires blocked, one up-leading wire Γ_1 and one outgoing wire (respectively Γ_4, or Γ_2, or Γ_3) remain open. Hence the electron current may go across the well from the up-leading wire to the outgoing wire. The corresponding transmission and reflection coefficients may be calculated in course of solution of the corresponding scattering problem, [23].

For fixed contact points the working regime of the RQS is defined by the position of the working point R, $\mathcal{E}$, V_0 in the three-dimensional space of the parameters. This position is uniquely defined, see Section 2, by the desired working temperature T and by the Fermi level E_f in the up-leading quantum wires. Note, that *the position of the working point can't be defined experimentally* just by naive scanning on one of parameters for other parameters fixed at random, since the probability of proper choice of the remaining parameters is zero (proportional to the zero-measure of a point on a $2-d$ plane).

Discussing the RQS based on a quantum well in [23] we assumed that the connection between the wires and the quantum well was defined by some small parameter β in the corresponding boundary conditions (Section 2). This boundary parameter was interpreted in [23] as an "exponential power" of the potential barrier separating the wires from the well. At the moment of submitting of the paper the authors did not have any idea how the power of the potential barrier may be controlled practically. It appeared that actually not the *hight of the barrier* but the *width of the the part of the up-leading channel* connecting the wire and the well may be controlled by a special nano-electronic construction – the *split-gate*. The power of the potential barrier inside the split-gate may be manipulated by the classical electric field applied to the brush of boron's dipoles sitting on the shores of the channel. This construction was suggested in experimental papers [20], [21], [22], [26]. The hight of the barrier, and hence the power of it, is defined by the position of the lowest energy level in the cross-section of the channel of variable width.

The plan of our paper is the following. In the second Section we suggest a procedure of selection of size of the device in form of a circular quantum well in dependence on desired working temperature. In the third Section we show, that for weak connection between the wires and the ring the scattering matrix near the resonance eigenvalue may be approximated by an analytic matrix function with two poles only. Then we calculate the life time of resonances and suggest conditions of elimination of the Coulomb blockade. In the fourth Section we discuss the realistic boundary conditions at the contact points defined by the adjacent split-gates and estimate the position of the working point of the RQS modeled as a Quantum Ring, assuming that the working temperature and the Fermi level are fixed. We omit essential part of reasoning concerning the spectral and the scattering problem for the Schrödinger operator on the Quantum Ring, see [16], but derive the boundary conditions for the scattered waves at the Fermi level “from the first principles” and define the life time of the corresponding resonance for the selected working point in dependence of the power of the potential barrier. The life time defines both the speed of switching and the minimal current through the device. In the last Section we calculate the working parameters of the ring-based three-terminal Quantum Gate, manipulated by the single-hole charging of electrodes situated inside the ring. We show, that the problem of calculating of the working parameters of the Gate may be reduced to the situation discussed in Section 4.

Note that the derivation of the boundary condition at the contact points "from the first principles" is actually a part of the general program of replacement of the partial Schrödinger equation on the quasi-one-dimensional structures with properly chosen extensions on a one-dimensional Schrödinger operators, see also [10], [8], [7], [14], [15]. Using of this approach in mathematical design of nano-electronic devices permits predict qualitative features of devices and gives preliminary estimation of their working parameters.

Note that the next two important problems of the mathematical design of the Resonance Quantum Switches and Gates are : calculation of the Voltage-Current characteristics and estimation of an affordable precision of the geometrical details of it. These problems are not discussed yet here: we just estimate the minimal current eliminating the Coulomb Blockade and assume that both the ring and the wires have the ideal geometry. Since the technologically affordable deviation of geometrical parameters is now circa 2 nm, it is clear that the device should work at the nitrogen temperature 77 K , but it is not clear yet if this precision is sufficient to guarantee the stability of the working regime of the switches and gates at room temperature.

2 High-temperature triadic RQS

Consider a RQS constructed in form of a quantum domain - a circular quantum well - with four terminals - quantum wires - attached to it at the contact points a_1, a_2, a_3, a_4, selected as suggested above. To choose the working point of the switch in dependence of desired temperature we consider first the *dimensionless* Schrödinger equation

$$- \triangle u + \epsilon \langle \vec{\xi}, \vec{\nu} \rangle u = \lambda u \tag{2}$$

in the unit disc $|\vec{\xi}| < 1$ with Neumann or Dirichlet boundary conditions at the boundary. The dimensionless Schrödinger equation may be obtained from the original equation by scaling $\vec{x} = R\vec{\xi}$:

$$- \triangle_\xi u + \frac{2me\mathcal{E}R^3}{\hbar^2} \langle \vec{\xi}, \vec{\nu} \rangle u = \frac{2mR^2(E - V_0)}{\hbar^2} u. \tag{3}$$

Here $\mathcal{E}$ is the magnitude of the selected electric field and the unit vector $\vec{\nu}$ defines it's direction, e is the absolute value of the electric charge of the electron and R is the radius of the circular well. Selecting $\epsilon = \dfrac{2me\mathcal{E}R^3}{\hbar^2} = 3.558$ for Neumann boundary conditions one may see, [23], that the eigenfunction corresponding to the second lowest eigenvalue $\mu_2 = 3.79$ of the dimensionless equation (2) has only one smooth zero line in the unit disc which crosses the unit circle at the points situated on the ends of radii forming the angles $\pm\frac{\pi}{3}$ with the electric field $\epsilon\vec{\nu}$. The minimal distance δ_0 of μ_2 to the nearest eigenvalues (the spacing of eigenvalues at μ_2), depending on boundary condition on the border of the well, may be between 2 and 10. For Dirichlet or Neumann boundary conditions the eigenfunctions of the spectral problem for the above Schrödinger equation (2) are even or odd with respect to reflection in the normal plane containing the electric field $\epsilon\vec{\nu}$. In particular for the Neumann boundary conditions the nearest eigenvalues corresponding to *even* eigenfunctions are equal $\mu_1 = -0.79$ and $\mu_3 = 9.39$, that it the spacing between μ_2 and other eigenvalues of the *even* series may be estimated as $\delta_0 := \min\{|\mu_2 - \mu_1|, |\mu_2 - \mu_3|\} \approx 4$. Generally for the circular domain the spacing between the second lowest eigenvalue μ_2 and other eigenvalues (of both even and odd series) may be estimated from below as $\delta_0 \geq 2$. The working regime of the switch will be stable if the bound states corresponding to the neighboring eigenvalues will not be excited at the temperature T :

$$\kappa T \, \frac{2mR^2}{\hbar^2} \leq \frac{\delta_0}{2}. \tag{4}$$

This condition may be formulated in terms of the *scaled temperature* $\Theta = \dfrac{2mR^2 T}{\hbar^2}$ as

$$\kappa \Theta < \frac{\delta_0}{2}. \tag{5}$$

The temperature which fulfils the above condition we call *low* temperature for the given device. If the radius R of the corresponding quantum well is small enough , then it may work at the (absolutely) high temperature, which correspond to the *low* scaled temperature. It may take place if the radius of the well is sufficiently small. Importance of developing technologies of producing devices of small size with rather high potential barriers is systematically underlined when discussing the prospects of nano-electronics, see for instance [27].

We assume that the effective depth V_f of the bottom value V_2 of the potential on the wires from the Fermi-level E_f in the wires is positive $V_f = E_f - V_2 > 0$, and the De-Broghlie wavelength on Fermi level is defined as

$$\Lambda_f = \frac{h}{\sqrt{2mV_f}}.$$

Then we obtain the estimate of the radius R of the domain from (4) as:

$$\frac{R}{\Lambda_f} \leq \sqrt{\frac{V_f}{\kappa T}} \sqrt{\frac{\delta_0}{8\pi^2}}. \tag{6}$$

For fixed radius R, the shift potential V_0 may be defined from the condition

$$\frac{2mR^2[E_f - V_0]}{\hbar^2} = \mu_2.$$

For instance, if we choose the radius R of the domain as $R^2 = \dfrac{\delta_0 \hbar^2}{4m\kappa T}$, we obtain :

$$V_0 = E_f - \frac{\hbar^2 \mu_2}{2mR^2} = E_f - 2\kappa T \frac{\mu_2}{\delta_0}.$$

Finally, the electric field $\mathcal{E}$ may be found from the condition

$$\epsilon = 3.8 = e\mathcal{E} \frac{2mR^3}{\hbar^2},$$

where e is the absolute value of the electron charge. Hence for the value of R selected above we have :

$$e\mathcal{E}R = \epsilon \frac{\hbar^2}{2mR^2} = \frac{2\epsilon\kappa T}{\delta_0}.$$

Hence the switch may work even at room temperature if the radius R of the quantum well is small enough and the geometrical details are exact.

Similar calculations may be done for the circular quantum well with Dirichlet boundary conditions. It appeared that for the dimensionless equation with the potential factor $\epsilon = 18.86$ the eigenfunction with a single zero-line dividing the unit circle into ratio 1 : 2 corresponds to the second smallest eigenvalue $\mu_2 = 14.62$. The lowest eigenvalue which corresponds to the *even* eigenfunction is $\mu_1 = 2.09$ and the spacing between μ_2 and other eigenvalues of both even and odd series is estimated as before, $\delta_0 \geq 2$. This gives proper base for calculation of the radius of the quantum well,the intensity of the electric field and the shift potential subject to given temperature and the Fermi level. Similar calculation may be done for RQS based on quantum ring, see below Section 4.

3 Scattering matrix: two-poles approximation, resonances and estimation of the speed of switching

Consider RQS based on quantum ring. It is constructed in form of a graph constituted of a *circular quantum ring* and four quantum wires attached to it at the contact points selected as suggested above. We choose the boundary condition connecting the boundary values at the end-point of the wire Γ_s similarly to the choice made in [16],that is the jump of the derivative $[\psi_s']$ and the value of the wave-function ψ_s at the corresponding contact point a_s on the ring :

$$\begin{pmatrix} [\psi'] \\ \psi_s \end{pmatrix} = \begin{pmatrix} 0 & \beta \\ \beta & 0 \end{pmatrix} \begin{pmatrix} \psi \\ -\psi_s' \end{pmatrix}. \tag{7}$$

Later in Section 4 we shall suggest arguments approving the choice.

We find the scattered waves as solutions of a system of Schrödinger equations on the graph:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \left(\mathcal{E}e\langle x,\nu\rangle + V_0\right)\psi = E\psi,$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_s}{dx^2} + V_2\psi_s = E\psi_s \tag{8}$$

with the above boundary conditions (7). It is convenient to reduce the system (8) to the dimensionless form with respect to the new coordinate $\xi = x/R$ and the new spectral parameter $k^2 = R^2\dfrac{2m(E-V_2)}{\hbar^2}$:

$$-\psi_{\xi\xi} + R^3\frac{2m\mathcal{E}e}{\hbar^2}\psi + R^2\frac{2m(V_0-V_2)}{\hbar^2}\psi = k^2\psi,$$

$$-\frac{d^2\psi_s}{d\xi^2}\psi_s = k^2\psi_s,\ s = 1,2,3,4. \tag{9}$$

The scattered wave iniciated by the input from the wire Γ_1 may be found as smooth solutions of the system (9) on the graph which have the standard form on the wires (with positive values of the spectral parameter k)

$$\psi_s = e^{ik\xi}\delta_{s1} + S_{s1}e^{-ik\xi} := \left\{\left(e^{ik\xi}I + e^{-ik\xi}S\right)\vec{e}_1\right\}_s.$$

Here $\vec{e}_1$ is the vector $(1,0,0,0)$ in the 4-dimensional " channel-space" of the system (9) - the space, where the results of the scattering processes are observed . The component of the scattered wave on the ring is presented as a linear combination of Green functions $G(x,\ a_s,k^2)$ of the equation on the ring :

$$\psi(\xi) = \sum_{s=1}^{4} u_s G(\xi,\ a_s/R,k^2) := G\vec{u}$$

which satisfies the boundary condition $[\psi'](a_s/R) = -u_s$. The Ansatz $\{\psi,\, \psi_1,\, \psi_2,\, \psi_3,\, \psi_4\}$, which obviously satisfies the equation, may be inserted into boundary conditions and used for calculation of the scattering matrix :

$$\begin{pmatrix} -\vec{u} \\ (I+S)e_1 \end{pmatrix} = \begin{pmatrix} 0 & \beta \\ \beta & 0 \end{pmatrix} \begin{pmatrix} G\vec{u} \\ -ik(I-S)\vec{e}_1 \end{pmatrix}. \tag{10}$$

Using the notation $\{G(a_s/R,\, a_t/R,\, k^2)\} = Q_{st}(\lambda),\ \lambda = k^2$, we obtain the explicit formula for the scattering matrix S of the dimensionless Schrödinger equation, see also [16], [23],

$$S = -\frac{\frac{I}{ik\beta^2} - Q}{\frac{I}{ik\beta^2} + Q}. \tag{11}$$

In the case of quantum domain, see [23], the role of 4×4 matrix Q is played by the matrix combined of the *regularized* values of the Green function at the contact points a_s. In the case of the quantum ring the matrix Q admits the spectral representation in terms of the orthogonal and normalized eigenfunctions ϕ_l of the dimensionless Schrödinger equation on the ring. This matrix is an analytic function of the spectral parameter $\lambda = k^2$, and has a positive imaginary part in upper half-plane $\Im\lambda > 0$ and simple poles on the real axis at the eigenvalues μ_l, $l = 1, 2\dots$ of the Schrödinger equation on the ring:

$$Q_{st}(\lambda) = \{G(a_s/R,\, a_t/R,\, \lambda)\}_{s,t} =$$

$$\left\{\sum_{l=1}^{\infty} \frac{\varphi_l(a_s/R)\,\varphi_l(a_t/R)}{\mu_l - \lambda}\right\}_{s,t}.$$

It is easy to see that the zeroes of the numerator of the scattering matrix (11) sit in the lower half-plane of the spectral parameter $k = \sqrt{\lambda}$ symmetrically with respect to the reflection in the imaginary axis of k and are complex conjugate to the poles (zeroes of the denominator) in the upper half-plane.

Assuming that the normalized eigenfunctions are uniformly bounded [3] one may derive the approximate expression for the matrix $Q(\lambda)$ when $\lambda = k^2$ sits on the real axis close to the resonance eigenvalue μ_2 , $|\mu_2 - \lambda| < \delta_0/2$. Using the notation $\vec{\varphi}_l = \{\varphi_l(a_1),\, \varphi_l(a_2), \dots \varphi_l(a_4)\}$ for the vector of the channel space we obtain :

$$Q(\lambda) = \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2 - \lambda} + \sum_{l\neq 2}^{\infty} \frac{|\vec{\varphi}_l|^2 P_l}{\mu^l - \lambda} :=$$

$$\frac{|\vec{\varphi}_2|^2 P_2}{\mu_2 - \lambda} + K,$$

where P_l is the orthogonal projection in R_N onto $|\vec{\varphi}_l|^{-1}\vec{\varphi}_l$ and K admits an estimate outside the spectrum $\{\mu_l\}$, $l \neq 2$ as an operator in 4-dimensional complex euclidean space with the square of the Hilbert-Schmidt norm estimated as

$$||K||^2 \leq \sup_l |\vec{\varphi}_l|^2 \sum_{l\neq 2} \frac{1}{|\mu_l - k^2|}.$$

This suggests considering the additional term K as a weak perturbation. A convenient technique for estimation of errors appearing from neglecting additional terms was developed in series of papers by R. Mennicken concerning spectral problems for matrix operators see, for instance [13]. In particular one may derive with use of this technique an approximation for the scattering matrix in the small neighborhood

[3] This fact may be derived for the Schrödinger operator on the Quantum Ring with smooth potential.

of the resonance eigenvalue μ_2 *embedded* into the absolutely continuous spectrum of the Schrödinger operator on the graph:

$$S = -\frac{\frac{I}{ik\beta^2} - \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2-\lambda} - K}{\frac{I}{ik\beta^2} + \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2-\lambda} + K}.$$

Assertion. *The subordinate terms $\pm K$ in the numerator and in the denominator in the above expression for the scattering matrix are dominated by the leading terms*

$$M(k) = \frac{I}{ik\beta^2} \pm \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2 - \lambda}$$

on the boundary of a "small" $\frac{|\vec{\varphi}_2|^2\beta^2}{4}$ - neighborhoods $\omega_\pm$ of the zeroes $k_\pm$ of the group $M(k)$ the leading terms. In particular

$$sup_{k\in\partial\omega_s}||M^{-1}K(k)|| < 3\alpha\beta^2||K|| < 1, \tag{12}$$

if $\frac{|\vec{\varphi}_2|^2\beta^2}{4} < 1$.

Rather technical verification of the Assertion may be found in the Appendix 1.

If the functions $M\pm$ have zeroes $k_\pm$ with orthogonal projections $P_\pm$ onto the corresponding zero-subspaces, then zeroes and projections onto the zero-subspaces of the perturbed operator-functions $M_\mp$ may be characterized with a help of the matrix-valued version of Rouche theorem, see [2]. The analytic operator function $M(k) + K(k)$ which fulfills the domination condition on the boundary $\partial\omega_0$ of a neighborhood ω_0 of the (vector) zero k_0 of the analytic function $M : M(k_0)P_0 = 0$ with some maximal orthogonal projection P_0,

$$\sup_{k\in\partial\omega} ||M_\mp^{-1}K(k)|| < 1,$$

has inside the neighborhood vector zeroes $\tilde{k}_l$ with the total multiplicity $\text{dim}P_0$

$$\left[M(\tilde{k}_l) + K(\tilde{k}_l)\right] P_l = 0$$

with the sum of multiplicities equal to the multiplicity of the zero of the leading term:

$$\sum_l \text{dim}\tilde{P}(\tilde{k}_l) = \text{dim}P_0.$$

If the vector-zeroes of the leading terms are simple, then the root - vectors of the scattering matrix are simple as well and the corresponding root-vectors are close to the corresponding root - vectors of the approximate scattering matrix

$$S_{approx} = -\frac{\frac{I}{ik\beta^2} - \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2-\lambda} - K}{\frac{I}{ik\beta^2} + \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2-\lambda} + K}. \tag{13}$$

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In case of the weak connection $\beta << 1$ the numerator of the scattering matrix has zeroes $\tilde{k}_\pm$ in "small" neighborhoods $\omega_\pm$ of zeroes of the corresponding leading terms. Due to the symmetry principle for the unitary operator functions the zeroes of the denominator are situated in the complex-conjugated points $\bar{\tilde{k}}_\pm$. These zeroes are usually interpreted as resonances, see below. These resonances are caused by the embedded eigenvalue $\mu_2 = \alpha^2 > 0$ of the Schrödinger operator on the ring. Using the root-decomposition of the polynomial $k^2 + ik\beta^2|\vec{\varphi}_2|^2 - \mu_2 = (k - k_+)(k - k_-)$ one may derive the "two-pole approximation" which corresponds to the two poles $\bar{k}_\pm$ and two zeroes $k_\pm$ of the approximate scattering matrix in k-plane :

$$S_{approx}(k) = -\frac{(k-k_+)(k-k_-)}{(k-\bar{k}_+)(k-\bar{k}_-)}P_2 - (I - P_2)$$

In this case under condition of domination of the matrix K by the leading terms one may derive from the Rouche theorem quoted above, that there exist zeroes $\tilde{k}_\pm$ of the original Scattering matrix which sit in a small neighborhood of zeroes $k_\pm$ of it's two-poles approximation S_{approx} and have root-vectors $\tilde{e}$, $S(\tilde{k})\tilde{e} = 0$ which are close to the corresponding root-vectors $\vec{\varphi}_2$ of the approximate S-matrix. One may derive from it that the original scattering matrix may be presented near the point $\tilde{k}_+$ in form

$$S(k) = S_0^+(k)\left[\frac{k-\tilde{k}_+}{k-\bar{\tilde{k}}_+}\tilde{P}_2^+ - (I-\tilde{P}_2^+)\right],$$

and near $\tilde{k}_-$ in form

$$S(k) = S_0^-(k)\left[\frac{k-\tilde{k}_-}{k-\bar{\tilde{k}}_-}\tilde{P}_2^- - (I-\tilde{P}_2^-)\right],$$

where the projections $\tilde{P}_2^\pm$ onto root-vectors of the scattering matrix at the vector-zeroes $\tilde{k}_\pm$,are close to $P_2^\pm$ and the factors $S_0^\pm$ are analytic invertible functions near the zeroes $\tilde{k}^\pm$ respectively. The operators $P^\pm$ are commuting and the operators $\tilde{P}_2^\pm$ are almost commuting, we may use on the real axis k near the points $\pm\alpha$ matrix any of three representations for the scattering - with small errors :

$$S(k) = S_0^\pm(k)\left[\frac{k-\tilde{k}_-}{k-\bar{\tilde{k}}_-}\tilde{P}_2^- - (I-\tilde{P}_2^-)\right]\left[\frac{k-\tilde{k}_-}{k-\bar{\tilde{k}}_-}\tilde{P}_2^- - (I-\tilde{P}_2^-)\right] + o(1) =$$

$$S_0^\mp(k)\left[\frac{k-\tilde{k}_-}{k-\bar{\tilde{k}}_-}\tilde{P}_2^- - (I-\tilde{P}_2^-)\right]\left[\frac{k-\tilde{k}_+}{k-\bar{\tilde{k}}_+}\tilde{P}_2^+ - (I-\tilde{P}_2^+)\right] + o(1) =$$

$$S_{00}(k)\left[\frac{k-k_-}{k-\bar{k}_-}P_2^- - (I-P_2^-)\right]\left[\frac{k-k_+}{k-\bar{k}_+}\tilde{P}_2^+ - (I-P_2^+)\right] + o(1)$$

with analytic invertible near $k_\pm$ factors $S_0^\pm$, $S_0^\mp$, S_{00}. These representations play the role of an asymptotical two-poles representation for the S-matrix near the resonance eigenvalue.

The imaginary part $-\dfrac{\beta^2|\vec{\varphi}_2|^2}{2}$ of resonances in the plane of the spectral parameter k is usually interpreted as a inverse life time of the resonance state (with respect to the scaled time variable in our case, see below). Being correct for acoustic equation, see [1], which contains the second derivative on time, this interpretation should be reconsidered for the non-stationary Schrödinger equation. We follow here the classical analysis of the problem via contour deformation described in [3], but add few details concerning specific two-poles approximation of the Scattering matrix on the graph for the Schrödinger equation with proper boundary condition at the contact points:

$$\frac{1}{i}\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + q(x)\psi, \tag{14}$$

where $q(x) = (\mathcal{E}e\langle\vec{x}, \vec{\nu}\rangle + V_0)$ on the ring and $q(x) = V_2$ on the wires. Introducing the scaled coordinate $\xi = x/R$ and scaled time variable $\tau = \dfrac{\hbar^2 t}{2mR^2}$ we may rewrite the non-stationary equation in form

$$\frac{1}{i}\frac{\partial\psi}{\partial\tau} = -\frac{\partial^2\psi}{\partial\xi^2} + \frac{2mR^2}{\hbar^2}q(\xi R)\psi,$$

and represent the solution of Cauchy problem with given initial data u_0 in spectral form via decomposition in scattered waves. In particular the diffracted wave may be represented using the above two-pole approximation and substituting $k_{\pm}$, P_2 for $\tilde{k}(\pm)$, $\tilde{P}_2^{\pm}$ respectively:

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik^2\tau} S_0(k)\frac{(k-k_+)(k-k_-)}{(k-\bar{k}_+)(k-\bar{k}_-)}\langle u_0, \psi(k)\rangle e^{-ik\xi} 2k dk.$$

If the initial data have a compact support, then all terms of the integrand are analytic functions near the real axis and the integral may be deformed to the new path passing the origin along the direction $\arg k = \pi/4$ (the direction of the most rapid descent of the function $e^{ik^2\tau}$ for positive τ) and approaching infinity along the line $\Im k = h,\ k \to +\infty + ih,\ h > \Im \bar{k}_{\pm}$. Then the residue calculated at the pole $\bar{k}_+$ gives the exponentially decreasing term

$$ie^{i\bar{k}_+^2\tau}\frac{(\bar{k}_+-k_+)(\bar{k}_+-k_-)}{\bar{k}_- - \bar{k}_+}\langle u_0, \psi(\bar{k}_+)\rangle 2\bar{k}_+ e^{-i\bar{k}_+\xi},$$

which is essential for relatively small time before the non-exponential terms begin play a role. The real part of the coefficient in front of the physical time $t = \tau\frac{2mR^2}{\hbar^2}$ in the exponent

$$\sqrt{\alpha^2 - \frac{\beta^4|\vec{\varphi}_2|^4}{4}}\ \frac{\beta^2|\vec{\varphi}_2|^2\hbar^2}{mR^2} \approx \frac{\alpha\beta^2|\vec{\varphi}_2|^2\hbar^2}{2mR^2}$$

plays the role of the *inverse life time* of the resonance state for the non-stationary Schrödimger equation. It differs from the imaginary part of the resonance by the presence of the factor $\sqrt{\alpha^2 - \frac{\beta^4|\vec{\varphi}_2|^4}{4}} \approx \sqrt{\mu_2}$ if $\beta|\vec{\varphi}_2| << 1$. More accurate estimate of the inverse life-time taking into account realistic boundary conditions at the contact points see in the next Section, (23). The life time shows how long an electron may stay in the resonance state and hence defines how fast is the switching process. On the other hand, if one electron already sits in this resonance state , another electron in the same spin state can't appear in the same position. This situation is interpreted as Coulomb blockade: to pass the quantum well, the next electron has to tunnel under the higher potential barrier defined by the spacing between μ_2 and the nearest bigger eigenvalue. The product of the charge of electron by the inverse life-time gives an estimate for the minimal current through the device. It is essentially defined by the value of "small " parameter β in the boundary conditions, by the size R of the ring, by the value of the effective mass of the electron m, and by the "geometrical" parameter $|\vec{\varphi}_2|^2$.

4 Boundary conditions at the contact points.

In this Section we obtain "from the first principles" the parameter β in the boundary conditions for the RQS based on the quantum ring with few terminals. We assume that the potential barrier separating the ring from the quantum wire at the contact point may be controlled by the split-gate described above, Section 1, see also [21], [22].

Consider a quantum switch constructed in form of a circular ring of quasi-one-dimensional quantum wire Γ_0 with a few straight radial up-leading wires $\Gamma_s = \Gamma_{s1} \cup \Gamma_{s2}$ attached to it orthogonally at the contact points a_s, $s = 1, 2, \ldots 4$. The Schrödinger equation on the ring Γ_0 is defined by some smooth potential $q(x) + V_0$, and the Schrödinger equations on the wires $\Gamma_s = \Gamma_{s1} \cup \Gamma_{s2}$ have piecewise constant potentials

$$V_s(x) = \begin{cases} V_1, \text{ if } & x \in \Gamma_1 : -l < x < 0, \\ V_2, \text{ if } & x \in \Gamma_2 : 0 < x < \infty \end{cases}$$

The hight $H = V_1 - E_f$ of the potential over the Fermi level on the initial part of the quantum wire (within the split-gate $x \in (-l, 0)$) is controlled by the electric field orthogonal to the wire , which may change the width of the the channel via turning the boron's dipoles sitting on the shore of it. We assume, that the Fermi level in the wires lies between V_1, V_2: $H > 0 > V_2$, the boundary condition at the point of contact is chosen in Kirchhoff form [4]:

$$[u_0'](a_s) + u_s'(a_s) = 0, \tag{15}$$

and the solutions $u_s = \{u_{s1},\, u_{s2}\}$ of the Schrödinger equations on $\Gamma_s = \{\Gamma_{s1},\, \Gamma_{s2}\}$ are smooth functions on the joint interval $(-l, 0) \cup (0, \infty) = \Gamma_1 \cup \Gamma_2$ for which the matching conditions are fulfilled

$$u_{s1}(-0) = u_{s2}(0),\ u_{s1}'(-0) = u_{s2}'(0).$$

For "low" temperature $\kappa\Theta < \frac{\delta_0}{2}$ one may assume that the dynamics of electrons is described as the restriction of the evolution defined by the non-stationary Schrödinger equation (14) onto the spectral interval length κT near the Fermi level (that is near the corresponding resonance eigenvalue on the ring). Practically we should calculate the scattering matrix on the graph for values of energy inside this interval. We may do it in different ways: beginning with the Green function $G_0(x, y, \lambda)$ on the ring with the smooth potential $V(x)$

$$Lu = -\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + V(x)u = Eu + \delta(x - y),$$

defined by the macroscopic electric field, or beginning with the Green function $\tilde{G}_0(x, y, \lambda)$ of the perturbed problem on the ring which already takes into account the wires attached to the ring at the contact points. We choose the second option of the two equivalent possibilities, which gives better approximation for solutions in case of weak connection between the terminals and the quantum ring. The Kirchhoff conditions for the solutions ψ, ψ_s of the Schrödinger equation on the ring and the Schrödinger equation on the wires at the contact points

$$[\psi'] + \psi_s'|_{a_s} = 0,\ \psi|_{a_s} = \psi_s|_{a_s},$$

may be simplified at the resonance $E = E_f$ due to the assumption that the potential barrier on the initial part of the wire is strong enough, so that the solution of the Schrödinger equation on the initial interval of the quantum wire may be presented just as a solution of the equation with the constant potential V_1 :

$$\psi_s = Ce^{-\frac{\sqrt{2m(V_1 - E_f)}}{\hbar}(x+l)}.$$

Then eliminating the Cauchy data of the decreasing solution on the wire we obtain from the above Kirchhoff condition the the jumping boundary condition for the wave-function on the ring at the contact points :

$$[\psi'] - \frac{\sqrt{2m(V_1 - E_f)}}{\hbar}\psi\ |_{a_s} = 0.$$

This boundary condition may be also presented in form of an additional singular potential :

$$V(x) \longrightarrow V(x) + \sum_{s=1}^{4} \delta(x - a_s)\frac{\hbar\sqrt{2m(V_1 - E_f)}}{2m} := \tilde{V}(x).$$

[4]In fact one may show that the boundary condition connecting the solutions of the differential equations on the wires and on the ring at the contact points depends on local geometry of the joining. We consider the Kirchhoff condition as a zero-order approximation for the realistic boundary conditions.

The formally introduced "perturbed" Schrödinger operator

$$\tilde{L} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi + \sum_{s=0}^{4}\delta(x-a_s)\frac{\hbar\sqrt{2m(V_1-E_f)}}{2m}\psi$$

on the ring with the new potential $\tilde{V}$ serves as a convenient step to construct the scattered waves on the whole graph. It is exactly the operator which substitutes asymptotically the operator L on the ring for energy close to Fermi-level when the power of the potential barrier is growing, $l \to \infty$. Using of the Green function $\tilde{G}(x,y,\lambda)$ of the "perturbed" operator $\tilde{L}$ instead of the Green function $G(x,y,\lambda)$ of the operator L with the smooth potential $V(x)$ is more practical, because it suggests a convenient Ansatz for the component of the scattered wave on the ring with resonance energy and proper asymptotic behaviour for strong barriers.

Note that the Green function $\tilde{G}$ of the perturbed Schrödinger operator $\tilde{L}$ may be easily calculated as a linear combination of the Green functions of the Schrödinger operator L with the smooth potential:

$$\tilde{G}(x,y,\lambda) = G(x,y,\lambda) + \sum_{s=1}^{4} g_s(y)G(x,a_s,\lambda),\ \ [\frac{d\tilde{G}(x,y,\lambda)}{dx}]_{x=y} = -1.$$

where the coefficients g_s may be found from the linear system defined by the boundary conditions at the contact points :

$$-\frac{\hbar^2}{2m}[\tilde{G}'(a_t,y,\lambda)] = g_t(y) = -\left[G(a_t,y,\lambda) + \sum_{s=1}^{4} g_s(y)G(a_t,a_s,\lambda)\right]\frac{\hbar\sqrt{2m(V_1-E_f)}}{2m}.$$

Denoting by $\mathbf{G}$ the matrix $G(a_t,a_s,\lambda)$ combined of the values of the non- perturbed Green-function and by $\vec{G}(y)$ the vector $(G(a_1,y,\lambda), G(a_2,y,\lambda), G(a_3,y,\lambda), G(a_4,y,\lambda))$ in thje channel- space, one may represent the solution $\vec{g} := (g_1,g_2,g_3,g_4)$ of the above linear equation in form

$$\vec{g} = -\frac{\hbar\sqrt{2m(V_1-E_f)}}{2m}(\mathbf{I} + \frac{\hbar\sqrt{2m(V_1-E_f)}}{2m}\mathbf{G})^{-1}\vec{G}(y).$$

We use, further, the perturbed Green function and the corresponding matrix when combining the Ansatz for the scattered waves of the Schrödinger equation at the Fermi level.

Note that the Kirchhoff boundary condition imposed at the point of contact a, which has the coordinate $x = -l$ may be transformed to the point $x = 0$ of the wire with use of the corresponding transfer-matrix for the Schrödinger equation on the wire Γ_s for the corresponding component of the scattered wave:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_s}{dx^2} + (V_1-V_2)\psi_s = (E_f-V_2)\psi_s.$$

We shall use the scaled equation on the scaled wire:

$$\tilde{l}_s\tilde{\psi}_s = -\frac{d^2\tilde{\psi}_s}{d\xi^2} + \frac{2mR^2}{\hbar^2}(V_1-V_2)\tilde{\psi}_s = \frac{2mR^2}{\hbar^2}(E_f-V_2)\tilde{\psi}_s := k^2\tilde{\psi}_s,$$

and on the scaled quantum ring with $V(\xi R) = \epsilon R\cos(\xi,\nu) + V_0 - V - 2$:

$$\tilde{l}\tilde{\psi}_0 = -\frac{d^2\tilde{\psi}_0}{d\xi^2} + \frac{2mR^2}{\hbar^2}V(\xi R)\tilde{\psi}_0 + \sum_{s=0}^{4}\delta(\xi - \frac{a_s}{R})R\frac{\sqrt{2m(V_1-E_f)}}{\hbar}\tilde{\psi}_0 = k^2\tilde{\psi}_0,$$

Then Cauchy data of the solution $\tilde{\psi}_s(\xi) = \cosh Rr\xi\ \tilde{\psi}_s(0) + \frac{\sinh Rr\xi}{Rr}\ \tilde{\psi}'_s(0)$, at the points $\xi = -l/R$ and $\xi = 0$ are connected by the transformation which leaves the boundary form $\psi'\bar{\varphi} - \psi\bar{\varphi}'$ invariant :

$$\begin{pmatrix} \tilde{\psi}'_s(-l/R) \\ \frac{d\tilde{\psi}_s}{d\xi}(-l/R) \end{pmatrix} = \begin{pmatrix} \cosh rl & -\frac{\sinh rl}{Rr} \\ -Rr\sinh rl & \cosh rl \end{pmatrix} \begin{pmatrix} \tilde{\psi}'_s(0) \\ \frac{d\tilde{\psi}'_s}{d\xi}(0) \end{pmatrix}. \tag{16}$$

Here the notation $r = \frac{\sqrt{2m(V_1 - E_f)}}{\hbar}$, is used. We shall transmit the Kirchhoff boundary condition, via transform-matrix on the wire for the scaled equation, from the point $\xi = -l/R$ to the point $\xi = 0$. This way we connect the boundary values $\left\{[\frac{d\tilde{\psi}_0}{d\xi}](a_s),\, \tilde{\psi}_0(a_s)\right\}$ of the solution of the scaled Schrödinger equation on the ring with the boundary values $\left\{\psi_s(0), -\frac{d\psi_s}{d\xi}(0)\right\}$ of the solution of the scaled Schrödinger equation on the wire:

$$[\frac{d\tilde{\psi}_0}{d\xi}](a_s/R) = Rr\sinh rl\ \tilde{\psi}_s(0) - \cosh rR\ \frac{d\tilde{\psi}_s}{d\xi}(0),$$

$$\tilde{\psi}_0(a_s/R) = \cosh rl\ \tilde{\psi}_s(0) - \frac{\sinh rl}{Rr}\frac{d\tilde{\psi}_s}{d\xi}(0).$$

Eliminating the function $\tilde{\psi}_s(0)$ from the first equation we obtain the connection between symplectic variables on the ring at the contact point and on the wire at the point $\xi = 0$ in hermitian form :

$$\begin{pmatrix} [\frac{d\tilde{\psi}_0}{d\xi}](a_s/R) \\ \tilde{\psi}_s(0) \end{pmatrix} == \begin{pmatrix} Rr\tanh rl & \frac{1}{\cosh rl} \\ \frac{1}{\cosh rl} & \frac{-1}{Rr}\tanh rl \end{pmatrix} \begin{pmatrix} \tilde{\psi}_0(a_s/R) \\ -\frac{d\tilde{\psi}_s}{d\xi}(0) \end{pmatrix}. \tag{17}$$

Denote by $\tilde{g}(\xi,\, \eta)$ the Green function of the scaled perturbed Schrödinger equation on the ring. Then the jump of the derivative of it is equal to -1 at all points on the unit circle, except contact points, where the jump is calculated as:

$$[\tilde{g}']_{\xi = a_s/R} - rR\tilde{g}(a_s/R,\, a_s/R) = -1.$$

We choose an Ansatz for the component $\tilde{\psi}_0(\xi)$ of the scattered wave on the ring in form

$$\tilde{\psi}_0(\xi) = \sum_{s=1}^{4} \tilde{g}(\xi,\, a_s/R)u_0^s = \tilde{\mathbf{g}}(\xi)\vec{u}_0.$$

Inserting this Ansatz into the boundary condition (17) we obtain the equation

$$-u_0^s = Rr(\tanh rl - 1)\tilde{\psi}_0(a_s) + \frac{1}{\cosh rl}(-\tilde{\psi}'_s(0)),$$

$$\tilde{\psi}_s(0) = \frac{1}{\cosh rl}\ \tilde{\psi}_0(a_s) - \frac{1}{Rr}\ \tanh rl\,(-\tilde{\psi}'_s(0)).$$

We may substitute now into the first equation the corresponding component of the Ansatz $\tilde{\mathbf{g}}(a_s/R)\vec{u}_0 := \{\tilde{\mathbf{g}}\vec{u}_0\}_s$ instead of $\tilde{\psi}_0(a_s)$, and the s-component of the standard Ansatz $\left[e^{ik\xi} + S(k)e^{-ik\xi}\right]\vec{e}$ of the scattered wave for $\tilde{\psi}_s$:

$$\tilde{\psi}_s(0) = ([I+S]\vec{e})_s := u_s(0), \quad \tilde{\psi}'_s(0) = ik([I-S]\vec{e})_s := u'_s(0),$$

then the equations may be rewritten in vector form as:

$$\begin{pmatrix} -\vec{u}_0 \\ \vec{u}(0) - \frac{1}{Rr}\vec{u}'(0) \end{pmatrix} = \begin{pmatrix} Rr\frac{e^{-rl}}{\cosh rl} & \frac{1}{\cosh rl} \\ \frac{1}{\cosh rl} & -\frac{1}{Rr}\frac{e^{-rl}}{\cosh rl} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{g}}\vec{u}_0 \\ -ik[I-S]\vec{e} \end{pmatrix}, \tag{18}$$

where $\tilde{\mathbf{g}}$ is a matrix combined of values of the perturbed scaled Green function $\tilde{g}(a_s/R,\, a_t/R)$ at the contact points. The scattering matrix may be found from these equations via eliminating of the variables $\vec{u}_0$ on the ring.

We shall assume now, that the ratio $1/\cosh rl$ plays a role of the small parameter. Then the diagonal elements of the matrix in the right-hand side of the last equation have the second exponential order, meanwhile the anti-diagonal elements are of the first exponential order.

Seems natural that for $\cosh^2 rl >> 1$ one may cancel the diagonal elements thus obtaining in the above boundary condition a matrix similar to the matrix used in (10) for calculation of the scattered waves. But now we already have an explicit expression for the parameter $\beta = (\cosh rl)^{-1}$. Notice, that the role of the non-perturbed operator is played by the Schrödinger operator with a special boundary conditions (or singular potentials) at the contact points. These potentials take into account the presence of the potential barrier hight $V_1 - E_f$ over the Fermi level on the initial interval $(-l, 0)$ of the quantum wire. If we assume $\cosh rl >> 1$ and use the corresponding simplified version of the boundary conditions

$$\begin{pmatrix} -\vec{u}_0 \\ \vec{u}(0) - \frac{1}{Rr}\frac{d\vec{u}}{d\xi}(0) \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\cosh rl} \\ \frac{1}{\cosh rl} & 0 \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{g}}\vec{u}_0 \\ -\frac{d\vec{u}}{d\xi}(0) \end{pmatrix}, \tag{19}$$

then the calculation of the scattering matrix may follow the pattern suggested in the previous Section 3. Taking into account that $\vec{u}(0) = \frac{ik(I-S)}{\cosh rl}\vec{e}$ we may solve the equation (19) with respect to the $S\vec{e}$ for any 4-vector $\vec{e}$ and then obtain an expression for the scattering matrix in form

$$S(k) = \frac{\frac{\tilde{\mathbf{g}}(\lambda)}{\cosh^2 rl} + \frac{1}{Rr} - \frac{1}{ik}}{\frac{\tilde{\mathbf{g}}(\lambda)}{\cosh^2 rl} + \frac{1}{Rr} + \frac{1}{ik}}. \tag{20}$$

Here $\tilde{\mathbf{g}}(\lambda)$ is a matrix combined of values at the contact points of the Green- function of the scaled perturbed equation on the ring: $\{\tilde{\mathbf{g}}(\lambda)\}_{st} = \tilde{g}(a_s/R,\, a_t/R,\, \lambda)$, $\lambda = k^2$. Now, similarly to analysis done in Section 3, we may select the leading terms in the numerator and denominator of the matrix function $\tilde{\mathbf{g}}(\lambda)$ near the resonance eigenvalue μ_2 of the perturbed operator :

$$\tilde{\mathbf{g}} = \frac{|\vec{\varphi}_2|^2 P_2}{\mu_2 - \lambda} + K.$$

Here P_2 is the orthogonal projection in 4-dimensional euclidean space onto the vector $\vec{\varphi}_2$ formed of the values of the resonance eigenfunction φ_2 at the contact points; the non-singular addend K may be estimated similarly as in Section 2. If the condition of domination of the non-singular term K by the group of the leading terms,

$$\cosh^2 rl \left(\frac{1}{ik} - \frac{1}{Rr}\right)(I - P_2) + \left[\cosh^2 rl \left(\frac{1}{ik} - \frac{1}{Rr}\right) - \frac{|\vec{\varphi}_2|^2}{\mu_2 - \lambda}\right] P_2 := M(k),$$

is fulfilled,

$$||KM^{-1}(k)|| << 1,$$

in a small real neighborhood of the resonance eigenvalue μ_2 then calculating of an approximate expression for the scattering matrix in this neighborhood one may pertain the leading terms only:

$$S_{approx}(k) = \frac{\frac{|\vec{\varphi}_2|^2}{\mu_2-\lambda}P_2 + \cosh^2 rl(\frac{1}{Rr} - \frac{1}{ik})}{\frac{|\vec{\varphi}_2|^2}{\mu_2-\lambda}P_2 + \cosh^2 rl(\frac{1}{Rr} + \frac{1}{ik})}. \tag{21}$$

To obtain the corresponding two-poles approximation for the scattering matrix and estimate the decay of the resonance terms in solution of the non-stationary Schrödinger equation it suffice to calculate zeroes of the leading term in the numerator assuming that $\cosh rl >> 1$. Using the notation $\alpha = \pm\sqrt{\mu_2}$, we obtain two zeroes in lower half-plane :

$$k \approx \alpha + \frac{irR}{i\alpha - rR}\ \frac{|\vec{\varphi}_2|^2}{2\cosh^2 rl}. \tag{22}$$

One may derive from it an analog of the two-poles approximation, see the previous section, and define the inverse life time $\tilde{\gamma}$ of the resonances. The scaled time τ corresponding to the scaled equation and the real time t are connected by the formula $k^2\tau = (E - V_2)t$, or $t = \frac{2mR^2}{\hbar^2}\tau$. The exponential decay of the resonance states of both scaled and the non-scaled equations is defined by the behaviour of he exponential factor $e^{ik^2\tau} = e^{i\Re k^2\tau}e^{-\Im k^2\tau}$. The decreasing exponential factor may be rewritten with respect to real time as $e^{\Im k^2 \frac{\hbar^2}{mR^2}t}$. Hence the role of the real inverse lifetime is played by $\Im k^2\frac{\hbar^2}{mR^2}$ and may be calculated approximately for $\cosh rl >> 1$ as

$$\frac{\alpha r^2\hbar^2|\vec{\varphi}_2|^2}{2m(\alpha^2 + r^2R^2)\cosh^2 rl}. \tag{23}$$

In fact even substituting the above Ansatz for scattered waves into the non-simplified boundary conditions (18) we actually may obtain a slightly inconvenient, but still exact expression for the scattering matrix in form

$$S(k) = \frac{\left(1 - \frac{rR}{ik} + \frac{e^{-rl}}{\cosh rl}\right)e^{-rl}\cosh rl + 1}{\left(1 + \frac{rR}{ik} + \frac{e^{-rl}}{\cosh rl}\right)e^{-rl}\cosh rl + 1}\ \frac{\cosh^2 rl\frac{1-\frac{rR}{ik}+\frac{e^{-rl}}{\cosh rl}}{\left(1-\frac{rR}{ik}+\frac{e^{-rl}}{\cosh rl}\right)e^{-rl}\cosh rl+1} + Rr\tilde{g}}{\cosh^2 rl\frac{1+\frac{rR}{ik}+\frac{e^{-rl}}{\cosh rl}}{\left(1+\frac{rR}{ik}+\frac{e^{-rl}}{\cosh rl}\right)e^{-rl}\cosh rl+1} + Rr\tilde{g}}. \tag{24}$$

For intermediate values of $\cosh^2 rl >> \frac{1}{\sqrt{1+\frac{r^2R^2}{\mu_2}}}$ one may simplify the expressions in both terms of the previous formula for the scattering matrix neglecting e^{-2rl} compared with $\sqrt{1+\frac{r^2R^2}{\mu_2}}$. Then we obtain more convenient approximate expression for the *approximate scattering matrix* near the resonance eigenvalue μ_2:

$$S_{approx}(k) = \frac{3ik - Rr}{3ik + Rr}\ \frac{2\cosh^2 rl\frac{Rr-ik}{Rr-3ik} + Rr\tilde{g}}{2\cosh^2 rl\frac{Rr+ik}{Rr+3ik} + Rr\tilde{g}}.$$

There two zeroes which coincide with the approximate zeroes (22) calculated above. S_{approx} gives more accurate two-poles approximation for the scattering matrix of the switch, than the expression (21) derived from the simplified boundary condition. The comparison of the explicit expression (24) for the scattering matrix and the approximate scattering matrix shows that under some natural domination conditions the scattering matrix has also three zeroes in small neighborhoods of zeroes of the approximate scattering matrix. Complete analysis of this alterantive will be done in another publication.

We shall discuss now a version of RQS based on a circular *quantum ring* Γ_0 of radius R with three *outgoing* straight radial quantum wires Γ_s, $s = 1, 2, 3$ attached to it at the points $\varphi = \pm\pi/3$, π via tunneling across the potential barriers controlled by the split-gates. We assume, that the up-leading quantum wire is supplied with so high potential barrier that the jump of the derivative of the wave-function on the ring at this point may be neglected when calculating the eigenvalues and eigenfunctions of the perturbed operator $\tilde{L}$. Still we pertain the jumps at the contact points of the outgoing channels $\varphi = \pm\pi/3$, π, characterized by the potential barriers width l and hight $H = V_1 - E_f$ over the Fermi level E_f in the radial quantum wires Γ_s. In this Section we assume that RQS is manipulated by the constant macroscopic electric field $\mathcal{E}\vec{\nu}$ which generates the potential $\mathcal{E}eR\langle\vec{\nu}, \vec{x}\rangle + V_0$ in the Schrdinger equation on the ring $\vec{x} = R\vec{\xi}$, $|\vec{\xi}| = 1$. We assume as before that the influence of the field on the quantum wires is eliminated by some additional construction, so that the potential on the wires produced by the macroscopic field is equal to zero. It means that the Schrödinger equation on the network combined of the up-leading wire, the ring Γ_0 and outgoing wires Γ_1, Γ_2, Γ_3 may be written as a system of Mathieu equation on the ring Γ_0

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + (\mathcal{E}e\langle\nu,\, x\rangle + V_0)\, u = Eu, \;\; u = u_0,$$

and the Schrödinger equation with the step-wise potential

$$V_s(x) = \begin{cases} H + E_f, & \text{if } -l < x < 0, \\ E_f + V, & \text{if } 0 \le x < \infty,\ V < 0 \end{cases} :$$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + V_s(x)u = Eu, \;\; u = u_s,$$

on the outgoing wires Γ_s, $s = 1, 2, 3$ and on the incoming wire Γ_4. We assume also, that the incoming wire Γ_4 is attached to the quantum ring at some point a different from the above points a_s. The connection of the outgoing wires with the quantum ring is characterized by the "small" parameter $\beta = (\cosh \frac{\sqrt{2m(V_1-E_f)}}{\hbar} l)^{-1}$.

An important engineering problem is actually *the proper choice of the macroscopic electric field* $\mathcal{E}$ such that the corresponding differential operator on the ring has an eigenfunction with special distribution of zeroes: the zeroes of the eigenfunction corresponding to the second smallest eigenvalue should divide the ring in ratio $1:2$. We assume that the potential barrier at the contact point a_4 with the incoming wire is so high that we may neglect the jump of the derivative of the perturbed operator $\tilde{L}$ eigenfunction at this point. Then the whole potential of $\tilde{L}$ on the ring is combined of the smooth potential defined by the macroscopic field $\mathcal{E}$ and an additional singular potential appearing from the Kirchhoff's conditions of smooth matching of the solution ψ at the contact points a_s, $s = 1, 2, 3$ on the ring with proper solutions of the equations on the wires when the energy is fixed on the Fermi level $E = E_f$:

$$[\psi_0'] - \frac{\sqrt{2m(V_1 - E_f)}}{\hbar}\psi\,\Big|_{a_s} = 0,$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \mathcal{E}e\langle\vec{x},\vec{\nu}\rangle + \sum_{s=1}^{N}\delta(x - a_s)\frac{\hbar\sqrt{2m(V_1 - E_f)}}{2m}\psi + (V_0 - V_2)\,\psi = E\psi.$$

We select the field $\mathcal{E}$ such that the resonance eigenfunction for $E = E_f$ would have, for certain direction of the unit vector $\vec{\nu}$, two zeroes on the ring sitting at the points $\varphi = \pm\pi/3$. When using the standard form of the scaled Mathieu equation with properly renormalized coefficients $q = \frac{4m\mathcal{E}eR^3}{\hbar^2}$, $a = frac8mR^2(E - V_0 + V_2)\hbar^2$,

$$y'' + (a - 2q\;\; \cos(2z))y = 0, \tag{25}$$

we should pass from the angular (scaled) variable $\xi = x/R$ to the new variable $z = \frac{x}{2R}$ which is changing on the interval $-\pi/2$, $\pi/2$. We have found that if the vector $\vec{\nu}$ is directed toward $z = 0$, and the solution ψ, we are looking for, is an even (cosine-type) solution of the Mathieu equation on the scaled ring $-\pi/2 < z < \pi/2$ with a positive value at the point $z = 0$, and zeroes at $z = \pm\pi/6$, then it is smooth at the contact points with $z == \pm\pi/6$ and has a jump of the derivative $[\tilde{\psi}']_\pi = \frac{R\sqrt{2m(V_1-E_f)}}{\hbar}\tilde{\psi}(\pi)$ at the point $\xi = \pm\pi/2$. Hence, $y = \tilde{\psi}(2z)$ satisfies the Mathieu equation on the interval $(0, \pi/2)$ and the boundary conditions

$$y'(0) = 0,$$

$$\frac{dy}{dz}(\pi/2) + \frac{R\sqrt{2m(V_1 - E_f)}}{\hbar} y(\pi/2) = 0. \tag{26}$$

The dimensionless Mathieu equation in standard form (25) with properly scaled variable z , $-\pi/2 < z < \pi/2$, was analyzed with Mathematica in dependence of the re-normalized electric field and the parameter $\gamma = \dfrac{R\sqrt{2m(V_1 - E_f)}}{\hbar}$ in the boundary condition $[\frac{d\tilde{\psi}}{d\xi}] - \gamma\tilde{\psi} = 0$ at the contact points. It was found that for the following values of the parameters q, γ the resonance eigenfunction with two zeroes at $z = \pm\pi/6$ exists, for instance :

$$\gamma = 10, \ q = -1.98, \ a = 5.24.$$

For the parameter q selected as shown above, there exist an eigenfunction u of the Mathieu equation perturbed by the δ-potentials attached to the points a_s with weight γ such that the zeroes of u divide the unit circle in ratio 1 : 2. These eigenfunctions may play a role of the resonance eigenfunctions for the corresponding triadic Resonance Quantum Switch. Being normalized by the condition $\varphi(0) = 1$ this function has square L_2-norm 3.5234. The spacing between the resonance eigenvalue $\mu_2 = \frac{a}{4} = 1.30$ on the unit ring and the nearest eigenvalue $\mu' = \frac{a'}{4} = 1.49$ (from the odd series of the eigenfunctions) is estimated as 0.19. Now the working temperature of the switch may be estimated as in Section 3: $\kappa T \leq \frac{0.19\,\hbar^2}{2mR^2}$. For quantum rings with radius 10 nm the switching time estimated from the life time of the corresponding resonance may be circa 10^{-17} sec.

5 Resonance Quantum Gate

The Resonance Quantum Switch manipulated by the macroscopic electric field is actually a *classical device for manipulating the quantum current.* It can't be used as a detail of a quantum network since the macroscopic electric field can't be used , generally , in the quantum network, since it would affect simultaneously all elements of the network neighboring with the switch. In this Section we consider a completely quantum device manipulated by a single electron or hole. Mathematical modeling of this device requires solution of a two-electron problem on a network, similar to one solved in the simplest case in [12]. In this note we consider a one-body version of the problem, assuming that a single hole is sitting inside the circular ring at the hight h over some point b_s on the continuation of one of the radii corresponding to the contact point a_s with outgoing wires, at the distance $|b_s| = b$ from the center of the ring. We may have three electrodes inside the ring, and hence three possible potentials to be used for redirecting of the electron current to different outgoing wires. The Coulomb potential on the ring, produced by the single charge sitting on one of electrodes, is equal to

$$\frac{e}{\sqrt{R^2 + h^2 + b^2 - 2bR\cos(\theta - \theta_s)}}.$$

If the condition $\frac{2bR}{R^2+h^2+b^2} << 1$ is fulfilled, then using the Taylor expansion we may find the approximate expression for the potential energy of the Schrödinger equation on the ring in form :

$$V_s^C(x) \approx -\frac{e^2}{\sqrt{R^2 + h^2 + b^2}} -$$

$$\frac{e^2 bR}{(R^2 + h^2 + b^2)^{3/2}} \cos(\theta - \theta_s) + O(\frac{e^2 b^2 R^2}{(R^2 + h^2 + b^2)^{5/2}}). \tag{27}$$

Neglecting the comparatively small second addend we obtain the renormalized harmonic potential in form

$$V_s(x) = -Q\cos(\theta - \theta_s) - A,$$

and thus we arrive again to the Mathieu equation of the same type as described in the previous Section. The only difference is that after introducing the new variable $z = 1/2\ (\theta - \theta_s)$ the coefficients of the Mathiew equation in standard form (25) are calculated as

$$a = \frac{8mR^2}{\hbar^2}\,(E + \frac{e^2}{\sqrt{R^2+h^2+b^2}}),$$

$$q = -\frac{8me^2bR^3}{\hbar^2(R^2+h^2+b^2)^{3/2}}.$$

Calculation of the working parameters may be accomplished similarly to one in previous Sections 3,4.

The above one-body approximation may be used if the life time of the single hole on the electrode is greater than the life time of the resonance, but still small enough to provide necessary speed of switching. If this condition is not fulfilled, then the corresponding scattering problem should be analysed in two-body approximation similarly to [12].

6 Appendix 1: Verification of the Assertion.

Denoting μ_2 by α^2 and assuming that $\frac{|\vec{\varphi}_2|^2\beta^2}{\alpha} \leq 1$, we may calculate the vector zeroes of the leading term $M_+ := M$,

$$M(k) = \frac{1}{ik\beta^2}(I - P_2) + \left(\frac{1}{ik\beta^2} - \frac{|\vec{\varphi}_2|}{\alpha^2 - k^2}\right) P_2$$

$$M(k)e = 0.$$

Due to the orthogonality of the projections P_2, $I - P_2$ we see that the root- vectors may lie in the subspace $P_2\mathcal{H}$ of the channel-space $\mathcal{H}$ only if the values of the spectral parameter k fulfil the equation $\frac{1}{ik\beta^2} - \frac{|\vec{\varphi}_2|}{\alpha^2-k^2} = 0$. Then we may find this values approximately as

$$k_\pm \approx \pm\alpha - i\frac{|\vec{\varphi}_2|^2\beta^2}{2}.$$

This implies some useful estimates for the zeroes :

$$|k_\pm \mp \alpha| \approx \frac{|\vec{\varphi}_2|^2\beta^2}{2},$$

$$|k_- - k_+| \approx 2\alpha,$$

$$2\alpha - \frac{|\vec{\varphi}_2|^2\beta^2}{2} \leq |k_\pm \pm \alpha| \leq 2\alpha + \frac{|\vec{\varphi}_2|^2\beta^2}{2}$$

and the estimates of distances of points k'_{pm} on the boundaries $\partial\omega_{pm}$ of the neighborhoods ω_{pm} of the zeroes :

$$|k - k_{pm}| = \frac{|\vec{\varphi}_2|^2\beta^2}{4}$$

from $\pm\alpha$:

$$\frac{|\vec{\varphi}_2|^2\beta^2}{4} \leq |k'_{pm} \mp \alpha| \leq \frac{3|\vec{\varphi}_2|^2\beta^2}{4},$$

from $k_\mp$

$$2\alpha - \frac{3}{4}|\vec{\varphi}_2|^2\beta^2 \leq |k'_\pm - k_\mp| \leq 2\alpha + \frac{3}{4}|\vec{\varphi}_2|^2\beta^2,$$

and from $k_\mp$:

$$2\alpha - \frac{3|\vec{\varphi}_2|^2\beta^2}{4} \le |k'_\pm - k_\mp| \le 2\alpha + \frac{3|\vec{\varphi}_2|^2\beta^2}{4}.$$

The above estimates imply the estimate of the ratio

$$\frac{(k-k_+)(k-k_-)}{(k-\alpha)(k+\alpha)}$$

on the boundaries of the $\frac{|\vec{\varphi}_2|^2\beta^2}{4}$ - neighborhoods $\omega_\pm$ of the zeroes $k_\pm$:

$$\frac{2-\frac{|\vec{\varphi}_2|^2\beta^2}{4\alpha}}{2+\frac{|\vec{\varphi}_2|^2\beta^2}{4\alpha}} \le \left|\frac{(k'-k_+)(k'-k_-)}{(k'-\alpha)(k'+\alpha)}\right| \le \frac{2+\frac{|\vec{\varphi}_2|^2\beta^2}{4\alpha}}{2-\frac{|\vec{\varphi}_2|^2\beta^2}{4\alpha}}$$

Hence the norm of the inverse of the operator function M

$$M^{-1} = ik\beta^2\left[(I-P_2) + \frac{(k-\alpha)(k+\alpha)}{(k-k_+)(k-k_-)}P_2\right]$$

for $\frac{|\vec{\varphi}_2|^2\beta^2}{\alpha} < 1$ may be estimated on the boundaries $\partial\omega_\pm$ of the neighborhoods $\omega_\pm$ as

$$||M^{-1}|| \le \alpha\beta^2\frac{2+\frac{|\vec{\varphi}_2|^2\beta^2}{4\alpha}}{2-\frac{|\vec{\varphi}_2|^2\beta^2}{4\alpha}} < 3\alpha\beta^2$$

and hence the domination condition is fulfilled on the boundaries $\partial\omega_\pm$:

$$||M^{-1}K|| \le 3\alpha\beta^2 \sup_{k\in\omega_\pm} ||K(k)|| < 1.$$

for β small enough.

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