# SOME RESULTS ON QUASI–$\sigma$ AND $\theta$ SPACES

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ABSTRACT. In this paper we show that a quasi–$G_\delta^*$–diagonal plays a central role in metrizability.

We prove that: if $X$ is a first–countable $GO$–space, then $X$ is metrizable if and only if $X$ is quasi–$\sigma$–space; a $w\theta$–space is metrizable if and only if it is a quasi–Nagata space with a quasi–$G_\delta^*(2)$–diagonal; a linearly ordered space $X$ with a quasi–$G_\delta^*(2)$–diagonal is a $\Theta$–space; a space $X$ is developable if and only if it is a $w\theta$, $\beta$–space with a quasi–$G_\delta^*(2)$–diagonal.

## 1. INTRODUCTION

Let $(X, \tau)$ be a space, let $g : \mathbb{N} \times X \to \tau$ be a function and $\mathcal{G} = \{g(n, x) : n \in \mathbb{N}, x \in X\}$. We call $\mathcal{G}$ a **graded system of open covers** for $X$ and $g$ is called $COC$–map (= countable open covering map) for a topological space $X$ if the following conditions are satisfied:

1. $x \in \bigcap_{n \in \mathbb{N}} g(n, x)$ for all $x \in X$.
2. $g(n+1, x) \subset g(n, x)$ for all $n \in \mathbb{N}$ and $x \in X$.

Consider the following conditions on $g$.

($A$) If $x \in g(n, x_n)$ for every $n \in \mathbb{N}$, then $x$ is a cluster point of the sequence $\langle x_n \rangle$.

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($B$) If for each $n \in \mathbb{N}$, $\{x, x_n\} \subset g(n, y_n)$, then $x$ is a cluster point of the sequence $\langle x_n \rangle$.

($C$) If for each $n \in \mathbb{N}$, $\{x, x_n\} \subset g(n, y_n)$ and $y_n \in g(n, x)$, then $x$ is a cluster point of the sequence $\langle x_n \rangle$.

($D$) If for each $n \in \mathbb{N}$, $\{x, x_n\} \subset g(n, y_n)$ and the sequence $\langle y_n \rangle$ has a cluster point, then $x$ is a cluster point of the sequence $\langle x_n \rangle$.

($E$) If for each $n \in \mathbb{N}$, $x_n \in g(n, y_n)$ and the sequence $\langle y_n \rangle$ converges in $X$, then the sequence $\langle x_n \rangle$ has a cluster point.

Let ($S$) be any of the conditions ($A$), ($B$), ($C$), ($D$) or ($E$) and ($S^{-1}$) be the statement obtained by formally interchanging all memberships (e.g., ($B^{-1}$) is the condition: If for each $n \in \mathbb{N}$, $y_n \in g(n, x) \cap g(n, x_n)$, then $x$ is a cluster point of the sequence $\langle x_n \rangle$). If the graded system $\mathcal{G} = \{g(n, x) : n \in \mathbb{N}, x \in X\}$ of open covers satisfies condition ($S$)(resp. ($S^{-1}$)) for $S = A$, $B$, $C$, $D$ or $E$, we say that $g$ is an $S$–map (resp. $S^{-1}$–map). If there is an $S$–map (resp. $S^{-1}$-map) for $X$ then we say that $(X, \tau)$ is an $S$-space (resp. $S^{-1}$-space). Corresponding to each of the conditions $S$ (except $E$) above is the weaker condition, denoted by $wS$, in which 'then $x$ is a cluster point of the sequence $\langle x_n \rangle$' is replaced by 'then the sequence $\langle x_n \rangle$ has a cluster point'. If $g$ satisfies $wS$, we say that $g$ is a $wS$–map. If there is a $wS$–map for $X$ then we say that $(X, \tau)$ is a $wS$-space. $wS^{-1}$–maps and $wS^{-1}$-spaces are defined analogously. The following are known, $A =$**semi-stratifiable** space, $B =$**developable** space, $C =\boldsymbol{\theta}$–space, $D =\boldsymbol{\Theta}$–space, $A^{-1} =$**first-countable** space, $B^{-1} =$**Nagata** space, $E^{-1} =$**quasi–Nagata** space, $wA =\boldsymbol{\beta}$–space, $wB =\boldsymbol{w\Delta}$–space, $wC =\boldsymbol{w\theta}$–space, $wD =\boldsymbol{w\Theta}$–space, $wA^{-1} =\boldsymbol{q}$–space.

A countable family $\langle \mathcal{G}_n : n \in \mathbb{N} \rangle$ of collections of open subsets of a space $X$ is called a **quasi–$\boldsymbol{G_\delta}$-diagonal** ( **quasi-$\boldsymbol{G^*_\delta}$-diagonal** [11]) [**quasi-$\boldsymbol{G^*_\delta}$(2)-diagonal**], if for each $x \in X$ we have $\bigcap_{n \in c(x)} st(x, \mathcal{G}_n) = \{x\}$ ($\bigcap_{n \in c(x)} \overline{st(x, \mathcal{G}_n)} = \{x\}$) [$\bigcap_{n \in c(x)} \overline{st^2(x, \mathcal{G}_n)} = \{x\}$] where $c(x) =$

$\{n : x \in \text{some } G \in \mathcal{G}_n\}$ [by $st(x, \mathcal{G}_n)$ is meant the union of all sets in $\mathcal{G}_n$ which contain $x$].

A space $X$ has a **quasi–$S_2$–diagonal** if there exists a countable family $\mathcal{G} = \langle \mathcal{G}_n : n \in \mathbb{N} \rangle$ of collections of subsets and for each $x \in X, st(x, \mathcal{G}_n)$ is open for all $n \in \mathbb{N}$ such that for any distinct $x, y \in X$, there exists $n \in \mathbb{N}$ such that $x \in \overline{st(x, \mathcal{G}_n)} \subset X - \{y\}$.

A space $X$ is called **quasi–developable** if $X$ has a countable family $\langle \mathcal{G}_n : n \in \mathbb{N} \rangle$ of collections of open subsets of $X$ such that the nonempty sets of the form $st(x, \mathcal{G}_n)$ form a local base at $x$ for all $x \in X$.

Let $X$ be a space and $\mathcal{G} = \langle \mathcal{G}_n : n \in \mathbb{N} \rangle$ a countable family of collections of open subsets of $X$. Then $\mathcal{G}$ is called a quasi–$w\Delta$–sequence for $X$ if and only if :

(1) for all $x$, $c(x)$ is infinite.
(2) if $\langle x_n : n \in \mathbb{N} \rangle$ is a sequence with $x_n \in st(x, \mathcal{G}_n)$ for all $n \in c(x)$ then $\langle x_n : n \in \mathbb{N} \rangle$ has a cluster point.

If the space $X$ has a such countable family then it said to be a quasi–$w\Delta$–space [11].

If $X$ is a space and $W$ is a relation on $X$ such that for each $x \in X$, $W(x)$ is a neighborhood of $x$, then $W$ is called a **neighbornet** [3].

Recall that a LOTS (=linearly ordered topological space) is a topological space whose topology agrees with the topology induced by some linear ordering $<$. A GO–space (=generalized ordered space) is a subspace of a LOTS. By $\mathcal{G}_n^*$ is meant the union of all sets in $\mathcal{G}_n$.

## 2. Quasi–$\sigma$–spaces

**Definition 1.** A space $X$ is called $\sigma$ (resp. quasi–$\sigma$, resp. weak–$\sigma$) if and only if there exists a $\sigma$–disjoint network $\mathcal{M} = \bigcup_{k=1}^{\infty} \mathcal{M}_k$ such that, for each $k \in \mathbb{N}, \mathcal{M}_k$ is discrete with respect to $X$ (resp. $\mathcal{M}_k$ is discrete with respect to $\overline{\mathcal{M}_k^*}$, resp. $\mathcal{M}_k$ is discrete with respect to $\mathcal{M}_k^*$).

In $COC$–map terms, we can define a $\sigma$–space as follows: A space $X$ is a $\sigma$–space if and only if there is $COC$–map $g$ such that, if for each $n \in \mathbb{N}, x \in g(n, y_n)$ and $y_n \in g(n, x_n)$, then $x$ is a cluster point of the sequence $\langle x_n \rangle$.

**Lemma 2.1.** *Let $\mathcal{G} = \{G_\lambda : \lambda \in \Lambda\}$ be a collection of subsets of $X$. If $\mathcal{G}$ is discrete with respect to $\overline{\mathcal{G}^*}$, then $\bigcup_{\lambda\in\Lambda}\overline{G_\lambda} = \overline{\bigcup_{\lambda\in\Lambda} G_\lambda}$.*

*Proof.* Let $x \in \overline{\bigcup_{\lambda\in\Lambda} G_\lambda}$. There exists a neighborhood $U(x)$ of $x$ such that $U(x)$ meets exactly one member $G_0 \in \mathcal{G}$. We show that $x \in \overline{G_0}$. If $x$ does not belong to $\overline{G_0}$, then there exists a neighborhood $V(x)$ such that $V(x) \cap G_0 = \emptyset$. Since $U(x) \cap V(x)$ is a neighborhood of $x$, we have $(U(x) \cap V(x)) \cap (\bigcup_{\lambda\in\Lambda} G_\lambda) \neq \emptyset$. This implies $(\bigcup_{\lambda\in\Lambda}(U(x) \cap V(x) \cap G_\lambda) \neq \emptyset$. It is a contradiction. Hence $\overline{\bigcup_{\lambda\in\Lambda} G_\lambda} \subset \bigcup_{\lambda\in\Lambda}\overline{G_\lambda}$. The converse is obvious. ■

**Theorem 2.2.** *Every regular quasi–$\sigma$–space is perfect.*

*Proof.* Let $U$ be a non–empty open set of $X$. By the assumption, for any $x \in U$, there exists a neighborhood $V_x$ of $x$ such that $\overline{V_x} \subset U$. Let $\mathcal{M} = \bigcup_{k=1}^{\infty}\mathcal{M}_k$ be a $\sigma$–disjoint network such that $\mathcal{M}_k$ is discrete with respect to $\overline{\mathcal{M}_k^*}$ for each $k \in \mathbb{N}$. There exist $k \in \mathbb{N}$ and $M \in \mathcal{M}_k$ such that $x \in M \subset V_x$. Thus $\overline{M} \subset \overline{V_x} \subset U$. Set $F_k = \bigcup\{\overline{M} : M \in \mathcal{M}_k, \overline{M} \subset U\}$. Then $F_k$ is a closed set by Lemma 2.1, and $F_k$ is contained in $U$. Therefore $U = \bigcup_{k=1}^{\infty} F_k$. Hence $X$ is perfect. ■

**Theorem 2.3.** *If $X$ is a first–countable $GO$–space, then $X$ is metrizable if and only if $X$ is quasi–$\sigma$–space.*

*Proof.* Apply [1, Theorem 7] (If $X$ is a first–countable $GO$–space, then $X$ is quasi–developable if and only if $X$ is weak–$\sigma$–space) and

Theorem 2.2. Since a space $X$ is developable if and only if it is quasi–developable and perfect; every quasi–$\sigma$–space is weak–$\sigma$–space; and every developable $GO$–space is metrizable, the proof is complete. ■

From [1, Corollary 7.1](A $GO$–space is quasi–developable if and only if it is a weak–$\sigma$–space with a quasi–$G_\delta$–diagonal) and Theorem 2.2 we have the following corollary:

**Corollary 2.4.** *A $GO$–space is metrizable if and only if it is a quasi–$\sigma$–space with a quasi–$G_\delta$–diagonal.*

## 3. $\theta$–SPACES

**Theorem 3.1.** *A linearly ordered space $X$ with a quasi–$G_\delta$–diagonal is a $\theta$–space.*

*Proof.* Let $\{\mathcal{G}_n\}_{n\in\mathbb{N}}$ be a sequence of families satisfying the condition for a quasi–$G_\delta$–diagonal. Without loss of generality we assume that $\mathcal{G}_n \subset \mathcal{G}_{n+1}$ and $st(x,\mathcal{G}_m)$ refines $st(x,\mathcal{G}_n)$ if $m > n$ and $m,n \in c(x) = \{k : x \in \mathcal{G}_k^*\}$ and that $st(x,\mathcal{G}_n)$ is an open interval that contains $x$.

Define $h : \mathbb{N} \times X \to \tau$ as follows:

$$h(n,x) = \begin{cases} st(x,\mathcal{G}_n) & \text{if } x \in \mathcal{G}_n^*. \\ X & \text{if } x \notin \mathcal{G}_n^* \ . \end{cases}$$

We show that $h$ is a $\theta$–map. Suppose that for each $n \in \mathbb{N}, y_n \in h(n,x), \{x, z_n\} \subset h(n, y_n)$ and $x$ is not a cluster point of $\{z_n\}$. Then there is an open interval $(a,b)$ about $x$ and an $M \in \mathbb{N}$ such that if $n > M$, then $z_n \notin (a,b)$. Moreover, without loss of generality there is a subsequence $\{z_{j_n}\}$ of $\{z_n\}$ such that $z_{j_n} \leq a$ for all $n \in \mathbb{N}$. Then for each $n \in \mathbb{N}, a \in [z_{j_n}, x] \subset h(j_n, y_{j_n}) \subset st(x,\mathcal{G}_{j_n}) \subset st(x,\mathcal{G}_n)$.

It follows that $a \in \bigcap_{n\in c(x)} st(x,\mathcal{G}_n) = \{x\}$, a contradiction. ■

**Lemma 3.2.** *A $q$–space with quasi–$S_2$–diagonal is first countable.*

*Proof.* Let $f$ be a $q$–map and $\langle \mathcal{G}_n : n \in \mathbb{N} \rangle$ a quasi–$S_2$–sequence on $X$. Define $g$ by

$$g(n,x) = \begin{cases} st(x, \mathcal{G}_n) & \text{if } x \in \mathcal{G}_n^*. \\ X & \text{if } x \notin \mathcal{G}_n^* . \end{cases}$$

For each $x \in X$ and $n \in \mathbb{N}$, let $h(n,x) = f(n,x) \cap g(n,x)$. Then $h$ is a first countable map. Let $x_n \in h(n,x)$. Then $\langle x_n \rangle$ has a cluster point, say $y$ (because $f$ is $q$–map). For all $n \in \mathbb{N}, y$ is a cluster point of $\langle x_m : m \geq n \rangle$, so $y \in \overline{h(n,x)}$ as $x_m \in h(n,x)$ for all $m$. Thus $y \in \bigcap_{n\in\mathbb{N}} \overline{h(n,x)} \subset \bigcap_{n\in\mathbb{N}} \overline{st(x, \mathcal{G}_n)} = \{x\}$, so $y = x$ and $x$ is a cluster point of $\langle x_n \rangle$. ■

**Theorem 3.3.** *Every $w\theta$–space with a quasi–$G_\delta^*$–diagonal is a $\theta$–space.*

*Proof.* Every $w\theta$–space $X$ is $q$–space, so by Lemma 3.2, $X$ is first countable. By [2, Proposition 2.2] [a space $X$ is a $\theta$–space if and only if it is a first countable $w\theta$–space that satisfies condition $I$ ($I$ = there is a sequence $\langle U_n \rangle$ of neighbornets such that if $K$ is a compact set and $x \notin K$, then there is an $n \in \mathbb{N}$ such that $U_n^*(x) \cap K = \emptyset$, where $U_n^*(x) = \bigcup\{U_n(y) : y \in U_n(x) \text{ and } x \in U_n(y)\}$)], we need to prove that $X$ satisfies condition $I$.

Let $\{\mathcal{G}_n\}_{n\in\mathbb{N}}$ be a quasi–$G_\delta^*$–diagonal sequence. Without loss of generality we may assume that $st(x, \mathcal{G}_m)$ refines $st(x, \mathcal{G}_n)$ if $m > n$ and $m, n \in c(x) = \{k : x \in \mathcal{G}_k^*\}$.

Define $h : \mathbb{N} \times X \to \tau$ as follows:

$$h(n,x) = \begin{cases} st(x, \mathcal{G}_n) & \text{if } x \in \mathcal{G}_n^*. \\ X & \text{if } x \notin \mathcal{G}_n^* . \end{cases}$$

For each $(n,x) \in \mathbb{N} \times X$ define $V_n(x) = \bigcap_{i=1}^n h(i,x)$. Let $z \in X$. Then $V_1(z) = h(1,z) = V_1^*(z)$ (from the definition of $V_n(x)$ and because $z \in st(x, \mathcal{G}_n)$ if and only if $x \in st(z, \mathcal{G}_n)$). Suppose that for $1 \leq j \leq n, V_j(z) = V_j^*(z)$. Then $V_{j+1}(z) = h(j+1, z) \cap V_j(z) =$

$h(j+1,z)\cap V_j^*(z)=V_{j+1}^*(z)$. Therefore, by induction for each $z\in X$ and $n\in\mathbb{N}$, $V_n(z)=V_n^*(z)$. We note that for each $n\in\mathbb{N}, V_{n+1}\subset V_n$. Suppose that $K$ is a compact set, $p\in X-K$ and for each $n\in\mathbb{N}, V_n^*(p)\cap K\neq\emptyset$. Then for each $n\in\mathbb{N}$, there is a point $x_n\in V_n(p)\cap K$. There is a point $q\in K$ that is a cluster point of $\langle x_n\rangle$. For each $n\in\mathbb{N}$ such that $p\in\mathcal{G}_n^*, q\in\overline{\{x_i : i\geq n\}}\subset\overline{V_n(p)}\subset\overline{st(p,\mathcal{G}_n)}$.

Since $\{\mathcal{G}_n\}_{n\in\mathbb{N}}$ be a quasi–$G_\delta^*$–diagonal sequence, $p=q$, a contradiction. Therefore $\langle V_n\rangle$ satisfies condition $I$. ■

**Theorem 3.4.** *A $w\Theta$–space is metrizable if and only if it is a quasi–Nagata space with a quasi–$G_\delta^*(2)$–diagonal.*

*Proof.* Necessity. Every metrizable space is Nagata [6] and has a quasi–$G_\delta^*(2)$–diagonal, and every Nagata space is a quasi–Nagata.

Sufficiency. Let $X$ be a quasi–Nagata, $w\Theta$–space. Then $X$ has a quasi–Nagata–map $h$ and a $w\Theta$–map $k$. Define $g$ such that $g(n,x)=h(n,x)\cap k(n,x)$ for each $(n,x)\in\mathbb{N}\times X$. Then $g$ is a COC–map. We now prove that $g$ is a $w\Delta$–map. For any two sequences $\langle x_n\rangle$ and $\langle y_n\rangle$ let $\{p,x_n\}\subset g(n,y_n)$ for each $n\in\mathbb{N}$. Consider $\langle p\rangle$ as a constant sequence. We have that $p\in g(n,y_n)$ and $\langle p\rangle$ is converges to $p$. Since $g$ is a quasi–Nagata–map, $\langle y_n\rangle$ has a cluster point. Now, we have that $\{p,x_n\}\in g(n,y_n)$ and the sequence $\langle y_n\rangle$ has a cluster point. Since $g$ is a $w\Theta$–map, the sequence $\langle x_n\rangle$ has a cluster point, and hence $X$ is $w\Delta$–space. By [10, Corollary 2.3] (a space $X$ is $w\Delta$ and has a quasi–$G_\delta^*$–diagonal if and only if it is developable), $X$ is developable. From [11, Theorem 4.10] (a space $X$ with quasi–$G_\delta^*(2)$–diagonal is Nagata if and only if it is a $q$–space and a quasi–Nagata–space ), $X$ is a Nagata–space. Thus $X$ is metrizable by the well–known fact that every developable, Nagata space is metrizable. ■

**Theorem 3.5.** *A space $X$ with a quasi–$G_\delta^*(2)$–diagonal is $\Theta$ if and only if it is $w\theta$.*

*Proof.* Every $\Theta$–space is $w\theta$. Now suppose that $X$ is a $w\theta$–space with a quasi–$G_\delta^*$–diagonal. From Theorem 3.3, $X$ is $\theta$–space.

We know that every $\theta$–space is a $q$–space. Since the space $X$ has a quasi–$G^*_\delta$–diagonal, by Lemma 3.2, $X$ is a first countable space. Let $f$ be a $\theta$–map. Define $g : \mathbb{N} \times X \to \tau$ as follows:

$$g(n,x) = \begin{cases} st(x, \mathcal{G}_n) & \text{if } x \in \mathcal{G}^*_n. \\ X & \text{if } x \notin \mathcal{G}^*_n \ . \end{cases}$$

Let $h(n,x) = \bigcap_{i=1}^n g(i,x)$. Set $k(n,x) = f(n,x) \cap h(n,x)$. We show that $k$ is a $\Theta$–map for $X$. Let $\{x_n, p\} \in k(n, y_n)$ and suppose $q$ is a cluster point of the sequence $\langle y_n \rangle$. Since $X$ is first countable, there is a convergent subsequence $\langle y_{j_n} \rangle$ of $\langle y_n \rangle$ such that for each $n \in \mathbb{N}, y_{j_n} \in k(n,q)$. Then $\{x_{j_n}, p\} \in k(j_n, y_{j_n}) \subset k(n, y_{j_n}) \subset f(n, y_{j_n})$. If $p = q$, it follows from the definition of $\theta$–map that $p$ is a cluster point of $\langle x_n \rangle$. Suppose that $p \neq q$.

Fix $n \in c(x)$. Then there are infinitely many integers $m \geq n$ such that $x_m \in k(n,q)$. Let $m \geq n$ with $x_m \in k(n,q)$. Then $x_m \in g(n,q) = st(q, \mathcal{G}_n)$. Thus $\{y_m : m \geq n\} \subseteq st^2(q, \mathcal{G}_n)$ for all $n \in c(x)$. So, $p \in \overline{\{y_m : m \geq n\}} \subseteq \overline{st^2(q, \mathcal{G}_n)}$ for all $n \in c(x)$. It follows that $p \in \bigcap_{n \in c_G(x)} \overline{st^2(q, \mathcal{G}_n)} = \{q\}$. Thus $p = q$, and $k$ is a $\Theta$–map. ■

From Theorem 3.4 and Theorem 3.5 we get the following result:

**Theorem 3.6.** *A $w\theta$–space is metrizable if and only if it is a quasi–Nagata space with a quasi–$G^*_\delta(2)$–diagonal.*

From Theorem 3.1 and Theorem 3.5 we get the following result:

**Theorem 3.7.** *A linearly ordered space $X$ with a quasi–$G^*_\delta(2)$–diagonal is a $\Theta$–space.*

**Theorem 3.8.** *A space $X$ is developable if and only if it is a $w\theta$, $\beta$–space with a quasi–$G^*_\delta(2)$–diagonal.*

*Proof.* From Theorem 3.5, $X$ is $\Theta$–space. It is easily verified that a space is developable if and only if it is a $\beta$, $\Theta$–space. ∎

The relationships between the classes of spaces considered in this section can be summarized in the following diagram:

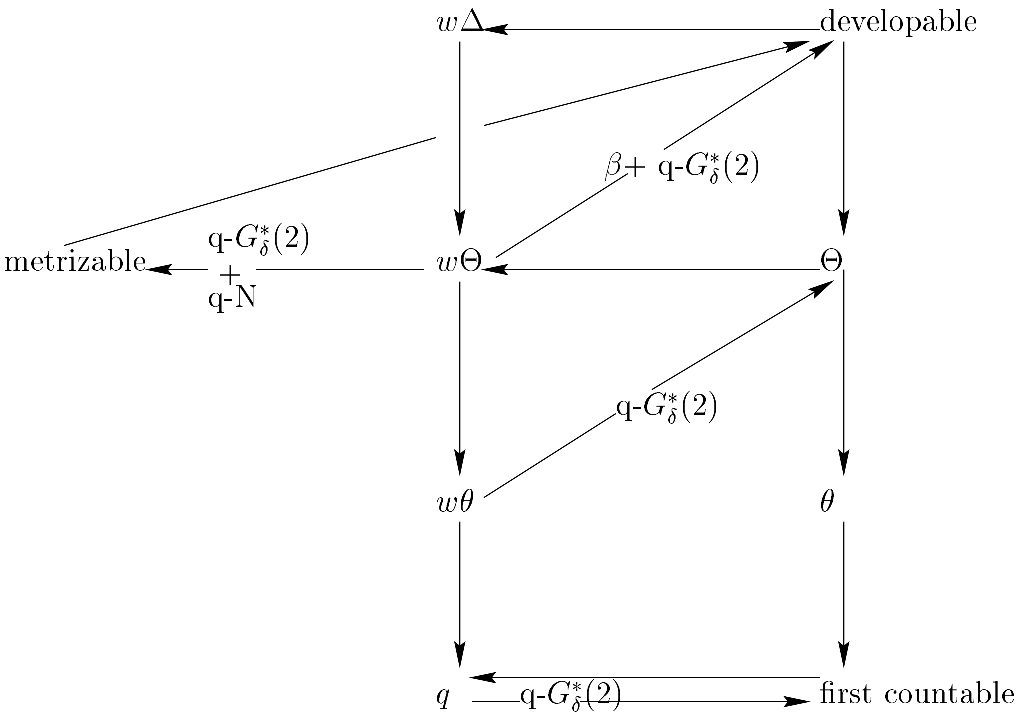


FIGURE 1. Relationships between the classes of spaces considered in this section.

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