APPROXIMATION AND HARMONIC ANALYSIS

ABSTRACTS

Sigma-Delta quantization and finite frames

John J. Benedetto, University of Maryland

The theory of Sigma-Delta quantization is developed for finite frames for Euclidean space. This theory, including the role of finite frames, is motivated by several recent applications in communication theory and code design. Because of these applications, first order Sigma-Delta quantization schemes are constructed; and optimal quantization search are designed. Error estimates for various quantized frame expansions are derived, and, in particular, it is shown that first order Sigma-Delta quantizers outperform the standard pulse code modulation (PCM) schemes using linear reconstruction. The error estimates are comparable when consistent reconstruction methods are used in conjunction with PCM. Higher order Sigma-Delta error estimates improve on the first order case. Refined estimates in the first order case require the Erdos-Turan and Koksma number theoretic inequalities. Further, the technology requires harmonic analysis, dynamical systems techniques, uniform distribution discrepancy theory, and some methods from algebraic number theory. Higher order schemes also involve tiling methods. The theory is a colloboration with Alex Powell and Ozgur Yilmaz.

Some Remarks on Heisenberg Frames

Len Bos, University of Calgary

A set of d^2 unit vectors v_j in C^d is said to be a Heisenberg frame if their mutual inner products all have the same modulus, i.e., $|v_j^* v_k| = C$, for all j different from k. We discuss some reformulations and equivalent systems of equations and give examples for certain values of d. The general existence problem remains open (to our best knowledge).

Spanning sets in Lebesgue and Hardy spaces

Qui Bui, University of Canterbury

In this talk I will discuss some results related to an open problem posed by Y. Meyer on the spanning property of the affine system generated by the Mexican hat function. The results are joint work with Richard Laugesen (University of Illinois, Urbana).

Signal Reconstruction without Noisy Phase

Peter G. Casazza, University of Missouri

For 50 years Engineers have believed that speech recognition should be independent of phase. We will construct new classes of Parseval frames and use them to show: The Engineers were right! Speech recognition really is independent of phase. As another application, this gives a new method for doing signal reconstruction without having to use “noisy phase” or its estimation - which has resulted in a patent application.

New characterizations of BMO and Morrey-Campanato spaces

Xuan Thinh Duong, Macquarie University

This lecture presents some recent results on new characterizations of BMO and Morrey-Campanato spaces. It is based on the joint works [DY1], [DY2] and [DDY] of the auhor with Donggao Deng and Lixin Yan. A local L^1 function f on $\mathbf{R}^n$ is said to be a BMO function (BMO means bounded mean oscillation) if the average on the ball B of $|f - f_B|$ is uniformly bounded in B, where f_B denotes the average of f on B. In this talk we will give a new characterization of the space BMO by replacing the constant f_B by the quantity $\exp(-t_B\Delta)$ when t_B is scaled to the radius of the ball B. This characterization gives rise to new BMO spaces when the Laplacian operator Δ is replaced by appropriate linear operators with heat kernel bounds. We also give a new characterization of the Morrey-Campanato spaces by using the convolution $\phi_{t_B} * f(x)$ to replace the minimizing polynomial $P_B f$ of a function f in the Morrey-Campanato norm, where $\phi \in \mathcal{S}(\mathbf{R}^n)$ is an appropriate Schwartz function.

References

[DDY] D. Deng, X.T. Duong an d L. Yan, A characterization of the Morrey- Campanato spaces, to appear in Math.Z.

[DY1] X.T. Duong and L. Yan, New function spaces of BMO type, John-Nirenberg inequality, interpolation and applications, to appear in Comm. Pure Appl. Math., 2004.

[DY2] X.T. Duong and L. Yan, Semigroup kernels, Poisson bounds and duality of Hardy space and BMO space, preprint 2004.

The infimum and supremum cosine angles between two finely generated shift-invariant subspaces and applications

Hong Oh Kim, Div. of Appl. Math., KAIST

Asymmetric multi channel sampling and its aliasing error

Kil Hyun Kwon, Div. of Appl. Math., KAIST

We present a general asymmetric(multi-rate) multi-channel sampling formula in the Paley-Wiener space. It is well-known that a function in Paley-Wiener space, i.e., a band-limited signal can be recovered by its equidistant samples. Its frequency bound determines the minimum rate, called Nyquist rate at which the reconstruction process is stable. In the multi-channel sampling it is not necessary to distribute the sampling rates equally among the channels. In this paper, we modify the sampling series so that the sampling densities are weighed in favor of some channels at the expense of other channels. We give a general sampling formula with asymmetric sampling rate by the Riesz basis method. In case of 2-channel derivative sampling, we find condition on the ratio of sampling rates, under which the sampling formula is possible. We also give the aliasing error bound for asymmetric multi-channel sampling formula, when it is applied to non-band limited signal.

Approximation of Gaussian and its Properties

Seng Luan Lee, National University of Singapore

The Gaussian function $G(x) = e^{-x^2/2}$ is the genesis of many branches of mathematics. It is the unique function, up to dilation and shift, that attains the bound of the Heisenberg uncertainty product and is also the unique linear space-scale kernel that has the causality property. The normal approximation by binomial distributions and B-spline distributions is viewed in the more general context of approximation of the Gaussian function by scaling functions and their masks, which enjoy approximately some of the fundamental properties of the Gaussian, such as optimal time-frequency localization and causality.

Polynomial operators and local approximation of solutions of pseudo-differential equations on the unit sphere

Q.T. Le Gia, School of Mathematics, University of New South Wales

We study the solutions of an equation of the form $Lu = f$, where L is a pseudo-differential operator defined for functions on the Euclidean unit sphere, f is a given functions, and u is the desired solution. We give conditions under which the solution exists, and deduce local smoothness properties of u given corresponding local smoothness properties of f, measured by local Besov spaces. We study the global and local approximation properties of the spectral solutions, describe a method to obtain approximate solutions using values of f at scattered sites on the sphere and polynomial operators. We also describe the global and local rates of approximation provided by our polynomial operators.

The Riesz energy of counting measures on the sphere

Paul Leopardi, University of New South Wales

Let $\{\mu_N\}$ be a sequence of normalized counting measures on the sphere S^d in R^{d+1} which satisfies a natural separation condition and which converges in a weak-star sense to the normalized area measure on S^d. Then, for $0 < s < d$, the normalized Riesz r^{-s} energy of each measure converges to the energy integral.

Moving least-squares for surfaces

David Levin, Tel Aviv University

The moving least-squares (MLS) approach, first presented by McLain, is a method for approximating multivariate functions using scattered data information. The method is based upon local polynomial approximations, incorporating weight functions of different types. Some weights, with certain singularities, induce C^∞ interpolation in $\mathbb{R}^n$. It is also possible to attain Hermite type interpolation, namely, interpolation to derivatives' data as well. The MLS approach also serves as a basis for some recent algorithms for surface reconstruction from an unstructured point-set data. The method is based upon a projection strategy, namely, defining a projection operator taking points near the data set onto an infinitely smooth surface. The projection involves a first stage of defining a local reference domain and a second stage of constructing an approximation with respect to the reference domain. Both stages are performed via the MLS approach. The degree of approximation depends, of course, upon the degree of polynomials used in the second stage. The main issues in the definition of the MLS projection will be reviewed. In particular, the possibility of achieving surface interpolation will be described.

Representations of Quasi-Interpolants as Differential Operators and their Approximation

Detlef H. Mache, University of Applied Sciences TFH Bochum

Quasi-Interpolants are practical and effective approximation operators with remarkable properties. Here we present the differential forms of these linear isomorphism and of their inverses for different generalized methods and extensions. Therefore the polynomial sequences (and the intermediate types) can be computed by recurrence relations, which allow us to study the approximation properties. For the polynomial coefficients of the associated linear differential methods one can give a connection to orthogonal polynomials.

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Quadrature over the Sphere

Alvise Sommariva, University of New South Wales

Consider integration over the unit sphere in $\mathbb{R}^3$ especially when the integrand has singular behaviour in a polar region. In an earlier paper [1], a numerical integration method was proposed that uses a transformation that leads to an integration problem over the unit sphere with an integrand that is much smoother in the polar regions of the sphere. The transformation uses a *grading parameter* q. The trapezoidal rule is applied to the spherical coordinates representation of the transformed problem. The method is simple to apply and it was shown in [1] to have convergence of order $O(h^{2q})$ or better for integer values of $2q$. In this talk, we show how to extend those results to non-integral values of $2q$. We also examine superconvergence that was observed when $2q$ is an odd integer. The overall results agree with those of [3], although the latter is for a different but related class of transformations.

References

[1] K. Atkinson, *Quadrature of singular integrands over surfaces*, Electronic Transactions on Numerical Analysis 17 (2004), pp. 133-150.

[2] K. Atkinson and A. Sommariva, *Quadrature over the sphere*, Electronic Transactions on Numerical Analysis, in press.

[3] A. Sidi, *Class $\mathcal{S}_m$ variable tranformations and applications to numerical integration over smooth surfaces of* $\mathbb{R}^3$, submitted.

Equilibrium measures and polynomials

Vilmos Totik, University of Szeged and University of South Florida

Potential theoretical methods have turned out to be useful in different areas of mathematical analysis. In the last 20 years or so they have revolutionized several branches of approximation theory and orthogonal polynomials. In many problems not only the proofs, but even proper formulation of the results require the use of such concepts as Green's functions, equilibrium measures, logarithmic potentials etc. In the present talk we offer a glimpse of how potential theory works in approximation theory and in polynomial inequalities. We shall present a quick introduction to equilibrium measures and their modification when an external field is present. Then apply this modification of Frostman's theory to a Weierstrass-type approximation that is at the core of several results for orthogonal polynomials and rational approximation. We shall also present a recent method of transferring results from an interval to compact sets which is based on taking polynomial inverse images. This method is applied for polynomial inequalities, and for establishing asymptotics for Christoffel functions and best approximation. At the heart of the method is a density theorem of polynomial inverse images of intervals, which turns out to be connected with the open mapping property of some elementary monotone systems.

The Kadison-Singer Problem and the Feichtinger Conjecture

Janet C. Tremain, University of Missouri

We will give several easily stated problems in frame theory which are equivalent to the infamous 1959 Kadison-Singer Problem. We will also show some recent results surrounding the Kadison-Singer problem and the Feichtinger Conjecture for Toplitz operators.

Some Innovations in Computing Best Fit Features in Metrology

Daniel S. Zwick, Wilcox Associates

In many practical measurement situations, we would like to fit a geometric element or other feature to measurement data for which we are given not only the coordinates of the center of the probe tip at each point, but also the tip radius used in the measurement and, often, the direction of approach for the measurement. In this talk, I will describe some modifications to the usual fitting procedures that take this additional information into account. In addition, I will discuss ODR alignment, an improvement on ICP, sparse matrix methods for speeding up spline fitting, and the importance of noniterative fitting methods in computing initial estimates for iterative methods.