Surface Approximation and Visualisation

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University of Canterbury
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Cylindrical Coordinate Representations for Modeling Surfaces of the Cornea and Contact Lenses

Brian A. Barsky
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This work develops four alternatives for modeling surfaces of the cornea and contact lenses. The cornea and contact lenses are generally smooth surfaces with possible discontinuities in circumferential or radial patterns. We define surfaces derived from semi-regular tensor product B-spline surfaces over a polar coordinate domain. The semi-regular partition allows finer control over the patch size, which is desirable for adaptive refinement. In geometric space, several patches meet at the origin, potentially resulting in a discontinuity. This is addressed by either imposing a system of constraints or blending the center region into a continuous function. The representations are fit to sampled data from a range of shapes and compared in terms of overall fit and fidelity at the origin. In the cases where constraint equations are used, their effectiveness and consequences on the resulting accuracy are assessed.

joint work with Lillian Chu and Stanley A. Klein

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Convergence properties of surface spline interpolation on finite uniform grids

Aurelian Bejancu
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Consider interpolation via Duchon's "surface splines" at the vertices of a sequence of uniform grids of decreasing mesh-size over a given bounded domain. We will present two types of results concerning the accuracy of this procedure when the function whose values are interpolated is sufficiently smooth. First, we obtain estimates of the decay of the error over a fixed strictly interior compact subset of the given domain. Second, we assume that the support of the data function is a subset of the interior of the domain and we give uniform error estimates over the whole domain.

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Compliant Components

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A compliant component is characterized by the property that an inextensional deformation of it requires much less energy than an extensional one. Compliant components are essentially surfaces which are flexible until attached to some frame. Thin sheet metal is a paradigm for this type of component. Products such as cars and aeroplanes consist of many such components. These components are usually treated as rigid surfaces in the design phase; however, in the assembly phase it is the flexibility of the surface which mitigates fitting problems arising from minor measurement/production errors. The basic assembly problem is to determine the the most efficient means to attach a compliant component given a certain amount of error in the system. In order to address such a problem it is necessary to determine the available deformations at each stage of assembly.

Compliant components can be modelled by partial differential equations of the Monge- Ampère type, and the characteristics on the solution surface play a significant rôle in the physical interpretation. In this talk we consider a simple Cauchy problem, the extent to which the initial data determine the surface, and approximations of potential small deformations.

joint work with Steve Panton, Dept. Mech. Engineering, University of Auckland

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Radial basis functions of compact support

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In this talk, radial basis functions that are compactly supported and give rise to positive definite interpolation matrices are discussed. There are some restrictions on the dimensionality of the problem (in all cases, an upper bound on the dimension n has to be met), which should be contrasted to the well-known cases such as multiquadric interpolation when there is no such bound. The particular advantage of radial basis functions of compact support, however, is the straightforward solvability of the linear systems that lead to the coefficients, because matrices are banded even when there are large numbers of data. They resemble the finite elements that are used for solving partial differential equations.

Several approaches to compactly supported radial functions are reviewed (for example the piecewise polynomial ones of Wendland and some others that are closely related to the thin-plate spline radial function) and some new classes of radial basis functions with compact support are given.

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Fast evaluation of radial basis functions: Methods for generalised multiquadrics in $\mathcal{R}^n$.

J B Cherrie
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A generalized multiquadric radial basis function is a function of the form

$$s(x) = \sum d_i \phi(x - x_i),$$

where

$$\phi(x) = \left(x^2 + c^2\right)^{k/2},$$

$x \in \mathcal{R}^n$, and $k \in \mathcal{Z}$ is odd. The direct evaluation of an N centre generalized multiquadric radial basis function at m points requires $\mathcal{O}(mN)$ flops which is prohibitive when m and N are large. Similar considerations apparently rule out fitting an interpolating N centre generalized multiquadric to N data points by either direct or iterative solution of the associated system of linear equations in realistic problems.

In this talk we will develop far field expansions, error estimates, recurrence relations for efficient formation of the expansions, and translation formulas, for generalised multiquadric radial basis functions in n-variables. These pieces are combined in a hierarchical fast evaluator requiring only $\mathcal{O}((m+N)\log N)$ flops for evaluation of an N centre generalised multiquadric at m points. This flop count compares very favourably with the cost of the direct method. Moreover, as outlined above, the approach provides a basis for fast fitting routines based on iterative solution of the associated linear systems.

joint work with R K Beatson and G N Newsam

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Best interpolating triangulations of 3D data-sets

Ruud van Damme
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When reconstructing a surface from irregularly spaced data, sampled from a closed surface in 3D, we need to decide how to identify a good triangulation. As a measure of quality we consider various differential geometrical properties, such as integral Gaussian curvature, integral mean curvature and area. We furthermore study a non-functional approach, which is based on a mapping procedure. A locally optimal triangulation is then identified as a fixed point under the map. The optimisation methods all require an initial triangulation as a starting point. To find an initial triangulation, we look at growing and shrinking approaches.

joint work with Dejana Djokovic and Gertjan Kloosterman

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Abstract Realization and Bernstein–type theorems

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joint work with K. Runovskii

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Hermite subdivision schemes for smooth interpolation of functions, curves and surfaces

Nira Dyn
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Subdivision schemes for the smooth interpolation of a univariate functions, based on refinements of function values and derivative values at a set of equidistant points is presented. For bivariate functions the initial data of function and gradient values are at the vertices of a triangulation. The schemes considered are interpolatory in the sense that the limit function obtained by repeated refinements interpolates the initial Hermite data.

The univariate case can be used also for the design of curves given locations and tangents. For the case of surfaces we consider initial data of locations and normals and a non-functional subdivision scheme is presented.

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A handheld 3D surface digitiser using magnetic trackers

Rick Fright
(*Applied Research Associates NZ Ltd*)

The aim of the hand-held laser scanner project has been the development of an extremely flexible scanner suitable for rapidly digitising irregularly shaped objects, such as a model or actual human body, and capable of a modest resolution and accuracy concomitant with the level of detail in these kinds of surfaces.

The scanner is essentially a hand-held laser range-finder that incorporates a magnetic tracker (electromagnetic spatial locator) as a 3D frame-of-reference. The special requirements that determine the design of the range-finding optics are described. Our choice of spatial locator and techniques to optimise range, resolution and accuracy are explained. The computer algorithms developed for processing the data are discussed. Examples from actual applications of the scanner are presented.

joint work with Bruce McCallum, Mark Nixon and Brent Price

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Fast Computation of the Entries of the Bezout and Dixon Resultants

Ron Goldman
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The Dixon resultant for three bivariate polynomials of bidegree (m,n) is the most commonly used resultant formulation for investigating tensor product surfaces. But while in principle the Dixon resultant is easy to construct from Cayleys determinant device, in practice the entries of the Dixon resultant are complicated expressions in the coefficients of the original three polynomials and explicit formulas for these expression are difficult to compute. Here we provide a simple recursive algorithm for calculating the entries of the Dixon resultant. We introduce this procedure first for the Bezout resultant of two univariate polynomials, and then show how this technique extends to three bivariate polynomials of bidegree (m,n).

joint work with Eng-Wee Chionh and Ming Zhang

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Theory and Application of a Nonconforming Finite Element Method for the Efficient Approximation of Thin Plate Splines

Markus Hegland
(*Australian National University*)

The interpolation problem for thin plate splines in more than 1 dimension leads to a symmetric indefinite dense linear system of equations. Thus the direct solution of the interpolation equations is of complexity $O(n^3)$ and very large interpolation problems are not feasible even on high performance computers. Various methods have been investigated in the past in order to deal with this computational curse. These algorithms are based on conjugate gradients, multipole expansion and Lagrange interpolation. The fastest of these approaches allow the solution of interpolation problems with more than 10,000 points on modern workstations especially if the interpolation points are on regular grids.

However, in the case of the smoothing problem one often encounters even larger numbers of data points, around 10^6 or more in data mining applications. Unlike in the case of the interpolation problem one is not free to choose the location of these points which are typically scattered irregularly over the domain.

The approach discussed here uses finite elements to approximate the thin-plate splines. The elements are tensor products of piecewise linear functions in the variables. They are computed from a new variational characterisation of the thin-plate splines on compact domains which does not require high-order derivatives. The smoothing parameter is determined with a fast GCV method based on Krylov spaces. Applications to data mining and generalisations to higher dimensions using additive models will also be presented.

This is joint work with S. Roberts, R. Sidje, I. Altas, K. Burrage, N. Potter and O. Nielsen.

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Constructing surfaces using alpha-shapes and homology

John Hudson
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The approximation of surfaces can be tricky when they bound objects with holes through them. This study investigates the use of alpha-shapes, homology and simplicial collapsing to construct a topologically correct surface through a collection of points.

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Twist Method for Surface Approximation

Ye Jieping
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A method for fairing the 'twist vectors' at the grid points for an interpolating surface over a rectangular network of curves is given. It is the continumm of our previous work [Qu, R. and Ye, J., 1999], where the generalized form of the thin plate model is investigated. But this model assumes small deflections of the surface curvatures and this restricts its applications. In this paper, a new method based on twist dependent quadratic functional is used to fair the twist vector. This twist vector is determined iteratively by minimizing this functional and can be obtained by solving a well defined linear system.

joint work with Li Qi

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Multigrid Convergence of Surface Approximations

Reinhard Klette
(*University of Auckland*)

This report deals with multigrid approximations of surfaces. Surface area and volume approximations are discussed for regular grids (3D objects), and surface reconstruction for irregular grids (terrain surfaces). Convergence analysis and approximation error calculations are emphasized.

The complete TR is on http://tcs.auckland.ac.nz/Research/Reports/CITR-TR-25.html

joint work with Feng Wu and Shao-zheng Zhou

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Modified Adaptive Approximation in R^d

Kirill Kopotun
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Several results dealing with adaptive approximation by piecewise polynomials in the Besov spaces as well as spaces of bounded "L_p-variation", $V^r_{s,p}$ will be presented in this talk. Some results concerning interpolation spaces between $L_p([a,b]^d$, certain Besov spaces and $V^r_{s,p}$ will also be discussed.

joint work with Y. K. Hu (Georgia Southern University) and X. M. Yu (Southwest Missouri State University)

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General Approach to Discrete Splines

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The tools of generalized splines and generalized B-splines (GB-splines for short) are widely used in solving problems of shape preserving approximation. By introducing various parameters into the spline structure, one can preserve characteristics of the initial data such as convexity, monotonicity, presence of linear and planar sections, etc. Here, the main challenge is to develop algorithms that choose parameters automatically. Recently, in [1,2] a difference method for constructing shape preserving hyperbolic tension splines as solutions of multipoint boundary value problems was developed. Such an approach permits us to avoid the computation of hyperbolic functions and has substantial other advantages. However, the extension of a mesh solution will be a discrete hyperbolic tension spline.

The contents of this paper is as follows. In section 1 we give a general definition of a discrete generalized spline and prove sufficient conditions for its existence and uniqueness. Next, we construct a minimum length local support basis of the new splines, denoted as discrete GB-splines; see section 2. The local approximation properties are discussed in section 3, while in section 4 we consider recurrence formulae for calculations with discrete GB-splines. The properties of GB-spline series are summarized in section 5. Section 6 provides some examples of defining functions that conform to the sufficient conditions derived earlier in the paper. We conclude in section 7 with graphical examples to illustrate the tension features of discrete generalized splines and to discuss their possibility in applications.

[1] Costantini, P., B. I. Kvasov, and C. Manni, Difference methods for constructing hyperbolic tension splines, Rapporto Interno 341/1998, Università di Siena. 22 p.

[2] Kvasov, B. I., On tension spline construction by difference method, in: *Proceedings of International Conference on Computational Mathematics*, Chulalongkorn University, Bangkok, ISBN 974-637-467-2, 1997, pp. 204–211.

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Convergence of Cascade Algorithms and Subdivision Schemes

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A *cascade algorithm* is a Picard-type iteration for the solution of equations of the form

$$\phi(x) = \sum_{j \in \mathbf{Z}^s} 2^s \ h(j) \ \phi(2x - j), \quad x \in \mathbf{R}^s,$$

called a *refinement equation.* Here, h is assumed to be a finitely suppor ted sequence that sums to 1. The cascade algorithm starts with a chosen functi on ϕ_0, and defines a sequence ϕ_n iteratively by

$$\phi_n(x) = \sum_{j \in \mathbf{Z}^s} 2^s \ h(j) \ \phi_{n-1}(2x - j), \quad x \in \mathbf{R}^s.$$

The limit of the cascade algorithm if it exists is the solution of the corresponding refinement equation. Solution of refinement equations has been studied extensively in conjunction with the construction of wavelets. However, interest in cascade algorithms is not confined only to wavelet analysis, but also extends to geometric modelling and computer graphics because of its close connection with subdivision schemes. In this talk we consider the convergence of cascade algorithms and the corresponding subdivision schemes, and their extensions to nonstationary and nonuniform cases.

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Boundary Penalty Finite Element Method for Blending Surfaces, II. Superconvergence, Stability and Applications.

Zi-Cai Li
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This paper is a continuity of study in [1] by using partial differential equations for blending surfaces. This paper consists of two parts. In Part I, the biharmonic equations are chosen, and the boundary penalty finite method (FEM) using piecewise cubic Hermite elements is employed to seek their approximate solutions, in particular satisfying the derivative and periodical boundary conditions. Theoretical analysis is made to discover that when the penalty power $\sigma = 2$, 3.5 and 1.5, optimal convergence rate, superconvergence and optimal stability can be achieved, respectively. Moreover, the derivative and periodical conditions of the numerical solutions have, at least, $O(h^4)$ of convergence rates, where h is the maximal boundary length of quasi-uniform elements. Moreover, a new transformation for the nodal variables used is given to improve numerical stability significantly. To compromise accuracy and stability, $\sigma = 2 \sim 3$ is suggested. By the techniques proposed in this paper, the finite elements may not be necessarily chosen to be small due to high convergence rates.

In Part II, the blending surfaces in 3 dimensions (3D) are taken into account by parameters, $x(r,t)$, $y(r,t)$ and $z(r,t)$. The boundary penalty techniques are well suited to the complicated tangent (i.e., derivative) in engineering blending. The corresponding theoretical analysis can be obtained from Part I. Moreover, the stiff analysis of 3D blending is given to conduct the linear algebraic equations. Several interesting samples of 3D blending surfaces are provided, to display the effectiveness of the new techniques in this paper, and the advantages: optimal and unique solutions of blending surfaces, ease in handling the complicated boundary constraint conditions, and less CPU time needed.

[1] Z.C. Li, Boundary Penalty Finite Element Methods For Blending Surfaces, I. Basic Theory, accepted by J. Comp. Math.

joint work with Chia-Shen Chang

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Trigonometric Splines; A Survey with New Results.

Tom Lyche
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Over the years many generalizations of the classical univariate polynomial splines have been proposed, see the book by Schumaker and references thereinn. However, further development of these generalized univariate splines has for the most part been slow, since there seem to be few problems that cannot be solved as well with polynomial splines as with any of the generalized splines. Lately it has been demonstrated that certain problems can be better solved by a class of splines known as trigonometric splines, than by the classical polynomial splines. A specific example is data-fitting on the sphere. As shown by Schumaker and Traas[1991], by using trigonometric splines we can use fast and simple tensor product methods which can handle the problem with smoothness at the poles. The resulting surface has a continuous tangent plane everywhere and can be decomposed in a wavelet fashion, see Lyche and Schumaker [1999]. Another application is to the class of so called Pythagorean-Hodograph curves which are rational curves with rational offsets. In addition trigonometric splines have been suggested for CAM design see Neamtu, Pottmann, and Schumaker[1998], and trajectory generation ,see Srinivasan, L., Rastegar[1997]. Trigonometric splines can be used to define single valued curves in polar coordinates,(Sanchez-Reys[1990,1992]), circular Bernstein Bézier polynomials (Alfeld, Neamtu, Schumaker[1998]), and piecewise rational curves with rational offsets (Neamtu, Pottmann, Schumaker[1997]). There is also an interesting connection with circle splines, see Goodman Lee[1984].

We present a fairly complete survey of trigonometric splines. This includes a new elementary introduction to trigonometric B-splines, knot insertion- and removal algorithms, blossoming, quasi-interpolants, total positivity, variation diminishing properties, and L^p-stability. There are new results on knot insertion and removal, and total positivity of trigonometric B-splines normalized to form a partition of unity.

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Case Studies in Visualisation

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Three case studies are presented which are ongoing research projects. Each arise from a different discipline, but in each case the use of visualisation is demonstrated as a tool for assisting in the understanding of the underlying physical and computational processes. The three studies arise from:

1. A model of pituitary cell calcium dynamics.
2. Simulation of solute transport in underground aquifers.
3. Monitoring local cache memory behaviour in a client-server computing system.

joint work with Clare Churcher, Don Kulasiri, Lincoln University and Roger Jarquin, Aoraki Corporation

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Interpolation - as bad as it gets

Terry Mills
(*La Trobe University*)

Just over 60 years ago, G. Grünwald and J. Marcinkiewicz discovered a divergence phenomenon pertaining to Lagrange interpolation polynomials based on the Chebyshev nodes of the first kind. This talk will review the place of this classical result in approximation theory. The main new result presented in this talk is an extension of their theorem. In particular, we will show that this divergence phenomenon occurs for odd higher order Hermite-Fejér interpolation polynomials of which Lagrange interpolation polynomials form one special case.

joint work with P. Vértesi

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Fast fitting of radial basis functions: Methods based onpreconditioned GMRES iteration

Cameron Mouat
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Solving large radial basis function (RBF) interpolation problems with non-customised methods is computationally expensive and the matrices that occur are typically badly conditioned. The usual direct methods require $\mathcal{O}(N^2)$ storage, and $\mathcal{O}(N^3)$ operations, so that solving large problems with $10,000$ or more centres by such non-customised methods is prohibitively expensive.

In this talk we present preconditioning strategies which, in combination with a fast multiply and GMRES iteration, make the solution of large RBF interpolation problems orders of magnitude less expensive in storage and operations. Typically, the preconditioning results in dramatic clustering of eigenvalues and improves the condition numbers of the interpolation problem by several orders of magnitude. As a result of the eigenvalue clustering the number of GMRES iterations required to solve the preconditioned problem is of the order of 10 to 20. Taken together, the combination of a suitable approximate cardinal function preconditioner, the GMRES iterative method, and existing fast matrix-vector codes for RBFs reduce the computational cost of solving an RBF interpolation problem to $\mathcal{O}(N)$ storage, and $\mathcal{O}(N \log N)$ operations.

joint work with Rick Beatson and Jon Cherrie

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The decreasing rate of the nonstationary conjuagate quadrature filters

Li Qi
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In the paper, we study the convergent rate of the coefficients of the polynomials

solutions q_k of the equations

$$p_k(x)q_k(1-x) + p_k(1-x)q_k(x) = 1, k \in,$$

and the convergent rate of the nonstationary conjugate quadrature filters.
Our main tool is the divided difference and the Riesz factorization.

joint work with Tang Wai Shing

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Approximation and expansion of rotation invariant kernels on spheres and Euclidean spaces

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On Compactly Supported Strictly Positive Definite Functions on the Sphere

Andrei Reztsov
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Several examples of compactly supported positive definite functions on the sphere S^2 were considered. As it was mentioned in recent paper by N. Dyn, F. Narkowich and J. Ward such functions could be constructed as a restriction to the sphere of compactly supported radial basis functions on m-dimensional Euclidean space, $m \geq 4$. As a initial functions (for this restriction) we considered the modification of Wendland's functions. Some ideas of the paper mentioned above and ideas of R. Schaback and H. Wendland helped us to find the restriction of the modified Wendland's function from 3-dimensional Euclidean space that is STRICTLY positive definite.

joint work with Prof Ian Sloan and Dr Michael Hirschhorn

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Numerical Experiments in Surface Approximation

Robert Schaback
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For surface construction by radial basis functions from large sets of scattered data, we first look at the numerical behavior of iterative algorithms. Inspired by recent results of Faul & Powell we prove linear convergence of a modified algorithm. Numerical experiments with large systems show certain effects that should be investigated further, e.g. into the direction of multigrid/multilevel techniques. We show a series of examples and try to compare different techniques.

The second topic concerns choosing the proper scale of a compactly supported radial basis function when applied to interpolate a surface. Experiments show that there is good numerical support for multilevel techniques that are comparable to stationary methods on regular grids. Linear convergence with respect to the level index can be observed in many cases.

If time permits, we finally report on moving least squares approximations to surfaces. This will include a simple proof of an optimal approximation order and some numerical experiments.

joint work with Holger Wendland

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Recent Advances in Macro-Element Methods for Fitting Scattered Data

Larry L. Schumaker
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Many of the most successful methods for fitting surfaces to bivariate scattered data are based on piecewise polynomials defined over either a rectangular or triangular partition of the domain of interest. These types of methods are particularly effective when the interpolation process can be described *locally*, i.e., the polynomial piece of the spline associated with a given subset T can be constructed from data associated with points in (or near) T. Such methods are often referred to as *macro element methods*. Among the best-known are the classical polynomial, Clough-Tocher, and Powell-Sabin elements. In this presentation we show how recent results in the theory of splines over triangulations can be used to construct new macro elements and improve existing ones in terms of smoothness, accuracy, degrees of freedom, and amount of derivative data required.

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Affine Frames in $L_2(\mathbf{R}^d)$

Zuowei Shen
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What are frames? Why and how are frames used in applications? How are affine frames related to wavelets? The first part of this talk is devoted to answer above questions.

The second part of the talk focuses on affine frames. We completely unravel the structure of the affine frame with the aid of a new notion *a quasi-affine system*. This leads to a characterization of affine frames. The induced characterization of *tight* affine frames is in terms of exact orthogonality relations that the wavelets should satisfy on the Fourier domain. Moreover, this characterization suggests a very simple construction of tight frames.

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Curves Design by Bi-Conics

Liu Song Tao
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In the note, we discussed the construction of the bi-conics, which interpolate the given data points, the associated unit tangent vectors and the curvatures. With this sort of bi-conics, we can use planar curves to approximate the curves in R^3. In planar case, the bi-conics will be GC^2 continuous curve or quasi-GC^2 continuous curves.

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The eigenfunctions of the multivariate Bernstein operator

Shayne Waldron
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It is shown that B_n the Bernstein operator of degree n for a simplex in R^s is diagonalisable, with eigenvalues

$$\lambda_k^{(n)} := \frac{n!}{(n-k)!}\frac{1}{n^k}, \quad k = 1, \ldots, n, \qquad 1 = \lambda_1^{(n)} > \lambda_2^{(n)} > \cdots > \lambda_n^{(n)} > 0.$$

The $\lambda_k^{(n)}$–eigenspace consists of polynomials of exact degree k that are uniquely determined by their leading term (i.e., the eigenspace is isomorphic to the space of homogeneous polynomials of degree k). These eigenspaces are described in terms of the barycentric coordinates (for the underlying simplex) and their substitution into *elementary eigenfunctions*. In contrast to the univariate case there are eigenfunctions of every degree k which are common to each B_n, $n \geq k$ for sufficiently large s. Time permitting we will discuss the *limiting eigenfunctions* and their connection with orthogonal polynomials of several variables, and the connection of the diagonal form of B_n with the failure of Lagrange interpolation of degree n at the simplex points to converge for all continuous functions.

joint work with Shaun Cooper

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Minimax ODR Fitting of Geometric Elements

Daniel S. Zwick
(*double star Research*)

We consider the fitting of geometric elements, such as lines, planes, circles, cones, and cylinders, in such a way that the maximal distance from the element to the data points is minimized. We refer to this kind of distance-based fitting as *orthogonal distance regression* or *ODR*.

We present an algorithm for minimax ODR fitting of geometric elements. The algorithm is iterative and allows the element to be given in either implicit form $f(x, b) = 0$ or in parametric form $x = f(s, b)$, where b is the vector of shape parameters, x is a 2- or 3-vector, and s is a vector of location parameters. The algorithm may even be applied in cases, such as with ellipses, in which a closed form expression for the distance is either not available or is difficult to compute.

joint work with Hans-Peter Helfrich, University of Bonn, Germany

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