Patients, queues and hospital beds: modelling and optimisation

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Joint with Will Chen, Kim Frew, Ross Ihaka and staff at Auckland City Hospital, including Andrew McKee, Pam Freeman, Andrea Pryce, Steve Withy
Contributors to the project

- **Cardiothoracic Surgical Unit, Auckland City Hospital** – heart and thoracic (mainly lung) surgery.
  - Andrew McKee – Director of the Intensive Care Unit (ICU)
  - Pam Freeman – Manager, Cardiovascular Services
  - Andrea Pryce, Steve Withy – Data analysts
  - Nursing staff at unit

- **Department of Statistics, University of Auckland**
  - Ross Ihaka – co-founder of R
  - Will Chen – BSc(Hons) and Masters projects
  - Kim Frew – BSc(Hons) project
The Cardiothoracic Surgical Unit, Auckland City Hospital

Patients treated

- Heart bypass patients, acute and elective (scheduled)
- Other cardiac patients
- Thoracic (chest) patients
- Other patients (vascular, .....).

Questions

- Initial question – how to minimize waiting lists and maximize number of patients treated?
- First stage – how many beds should be staffed in intensive care to keep cancellations low? (Need to employ around 5 nurses per bed.)
- Second stage – given the staffing level, what is the optimal roster and number of electives that should be scheduled?
- Third stage – bottlenecks elsewhere.
The system

- 3 operating theatres + 1 for emergencies
- Intensive care unit (ICU) staffs 9-12 beds, up to 16 available.
  - 1 nurse per patient + 2 runners (can look after acute patients) + charge nurse + 2 on-call nurses
- High dependency unit (HDU) has 6 beds
  - 1 nurse per 2 patients + 1 runner + charge nurse
- 52 beds in wards
Patient flow

- **Acute**
  - *Theatre*
  - *Ward*

- **Elective**
  - *Ward*

- **Other**
  - *ICU*
  - *HDU*

- *Ward*
  - *Theatre*
  - *HDU*

- *ICU*
  - *HDU*

- *HDU*
  - *ICU* (double arrow indicating a cycle)

- *Theatre*
  - *ICU* (double arrow indicating a cycle)
Data


- Admission date and time
- Length of stay (LOS) in minutes
- Type of procedure
- Ward (ICU or HDU)
- Admission type (e.g. elective, acute, vascular, surgical non-bypass, cardiac non-surgical, ECMO, (medical), other)

Additional data on transfers between ICU and HDU, rosters, schedules, elective patients treated...
Cardiovascular Intensive Care Unit (CV-ICU)

Several patient arrival flows 2008 (2006-7)

- Acute – emergency patients 4.2% (7%)
- Elective bypass – scheduled cardio-thoracic surgical patients 65.9% (61%)
- Medical, vascular and other patients 29.9% (32%).

Variable admission times for non-elective patients.

Variable lengths of stay 2008 (2006-7)

- 51% (50%) of patients leave ICU within 24 hours,
  94% (96%) within 1 week
- 2% (1%) stay longer than 17 days.
Length of stay (hours) in ICU Feb–Nov 2008
Why is it important to include variability in the model?

Imagine a single-server queue where

- 1 customer arrives at the beginning of each minute
- each customer requires 51 seconds (0.85 minute) service exactly
- so the server is busy 85% of the time

What does a plot of the number in the queue look like?
Deterministic model – arrivals 1 per minute, service time 0.85 minutes (51 seconds)
Now let’s look at a plot of the number in a single-server queue where

- customers arrive randomly (as a Poisson process) – on average 1 per minute
- customer service times are random (exponentially distributed) – each customer requires 51 seconds service on average
- the server is still busy 85% of the time on average

What does a plot of the number in this queue look like?
Random model – arrivals 1 per minute, service time 0.85 minutes (51 seconds) on average
• The queue buildup is due *just* to variability in the arrival and service processes.

• The greater the variability, the greater the average length of the queue *even though the average arrival and service rates don’t change*.

• The variability of the number in the queue increases too, so waiting times are less predictable.

• But the server is no busier, on average, when variability increases.
• The queue buildup is due *just* to variability in the arrival and service processes.

• The greater the variability, the greater the average length of the queue *even though the average arrival and service rates don’t change*.

• The variability of the number in the queue increases too, so waiting times are less predictable.

• But the server is no busier, on average, when variability increases.

Conclusion: Deterministic models are not enough to tell you about queue lengths.
### ICU Patients Feb-Nov 2008

**Important to model different types of patient.**

<table>
<thead>
<tr>
<th>Patient</th>
<th>N</th>
<th>Mean LOS</th>
<th>Median LOS</th>
<th>σ</th>
<th>% occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bypass elective</td>
<td>555</td>
<td>52.6</td>
<td>23.7</td>
<td>93.2</td>
<td>51.8</td>
</tr>
<tr>
<td>Bypass acute</td>
<td>35</td>
<td>163.4</td>
<td>88.0</td>
<td>182.2</td>
<td>10.1</td>
</tr>
<tr>
<td>Bypass repeat</td>
<td>74</td>
<td>45.5</td>
<td>23.4</td>
<td>64.6</td>
<td>6.0</td>
</tr>
<tr>
<td>Surgical non-bypass</td>
<td>65</td>
<td>58.0</td>
<td>21.6</td>
<td>109.4</td>
<td>6.7</td>
</tr>
<tr>
<td>Non-surgical cardiac</td>
<td>80</td>
<td>72.8</td>
<td>35.4</td>
<td>95.3</td>
<td>10.3</td>
</tr>
<tr>
<td>ECMO (bypass)</td>
<td>7</td>
<td>348.3</td>
<td>166.5</td>
<td>333.4</td>
<td>8.6</td>
</tr>
<tr>
<td>Vascular</td>
<td>68</td>
<td>37.3</td>
<td>20.4</td>
<td>68.2</td>
<td>4.5</td>
</tr>
<tr>
<td>Other</td>
<td>32</td>
<td>33.3</td>
<td>20.7</td>
<td>47.9</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>916</td>
<td>58.9</td>
<td>23.8</td>
<td>103.7</td>
<td>100</td>
</tr>
</tbody>
</table>
ICU modelling – analytical model

- A suitable model might be a modified $G|G|C+2+2|C+2+2$ queue.
- Runners and on-call nurses provide buffer – priority reservation for acute bypass and some non-bypass patients.
- Difficulties in modelling arrivals:
  - Both deterministic and Poisson arrival streams.
  - Arrivals vary with time of day and day of week.
  - Arrival rates vary on a faster time-scale than lengths of stay (unlike emergency departments).
ICU simulation model 24-hour 7-day

- Simulation written in R.
- Simulation splits each day into 5 time periods starting at 00:00, 7:00, 11:00, 15:00, 19:00.

State of system at beginning of each time period is given by

\[ S = (N, \ LOS, \ t, \ \text{shift}, \ \text{day of week}) \]

\[ N = \text{number of patients in ICU} \]
\[ LOS = \text{vector of residual lengths of stay for patients in ICU} \]
\[ t = \text{time of day} \]
\[ \text{shift} = \text{a.m. or p.m.} \]
• Arrivals

  – Deterministic arrivals of electives on weekdays at 11:00 and 15:00.

  \[
  \begin{array}{|c|c|c|c|c|c|c|c|}
  \hline
  \text{start time} & \text{Mon} & \text{Tue} & \text{Wed} & \text{Thur} & \text{Fri} & \text{Sat} & \text{Sun} \\
  \text{a.m.} & 3 & 3 & 0 & 3 & 3 & 0 & 0 \\
  \text{p.m.} & 2 & 2 & 2 & 2 & 2 & 0 & 0 \\
  \hline
  \end{array}
  \]

  – Other patients arrive as a Poisson process – rate depends on time of day, day of week and type of patient. Typical marginal rates per hour are:-

  \[
  \begin{array}{|c|c|c|c|c|c|c|}
  \hline
  \text{start time} & 00:00 & 07:00 & 11:00 & 15:00 & 19:00 & \text{average} \\
  \lambda & 0.0240 & 0.0304 & 0.0674 & 0.0839 & 0.0579 & 0.0495 \\
  \hline
  \end{array}
  \]

• Length of stay in ICU drawn from empirical distribution of lengths of stay.
• Surgery for elective patients is cancelled if a bed is not available.

• At beginning of each time period
  – Patients who have left ICU during previous time period are removed from list.
  – Acute and other non-elective arrivals are added to the list.
  – If electives are scheduled, a decision is made about whether to continue with surgery.
**Inputs to simulation**

- Elective schedule
- Nursing roster (how many nurses working each shift)
- File of patient data including
  - Day and time of arrival
  - Length of stay
  - Type of patient
- Arrival rates for patient types are calculated from data, but can also be entered manually.
**Outputs from simulation**

The simulation gives a wide variety of outputs, including estimates of:

- Number of cancellations per week/shift.
- Number of occupied beds per shift – bed utilisation.
- Number of elective patients treated/admitted per week/shift.
- Number of additional nurses called in per shift.
Simulations of number of occupied beds at 4 p.m.

Number of Overnight ICU patients

Simulated Days
Estimated number of cancellations per day

Average Cancellations (per day)

- Monday: N = 9
- Tuesday: N = 12
- Wednesday: N = 15
- Thursday: N = 9
- Friday: N = 12
- Saturday: N = 15
- Sunday: N = 9
What is the optimal roster?

Objective function?

- Minimize number of cancellations.
- Maximize number of electives treated.
- Keep low the number of additional nurses called in.
- A workable roster.
Typical elective operating schedule

<table>
<thead>
<tr>
<th></th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.m.</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>p.m.</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Typical baseline nursing roster, including runners

<table>
<thead>
<tr>
<th></th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.m.</td>
<td>8</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>p.m.</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

12-hour shifts, starting at 7 a.m. and 7 p.m.

Mean cancellations per week $6.66 \pm 0.45$ (95%CI).
And after searching for improved roster

Starting roster

<table>
<thead>
<tr>
<th></th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.m.</td>
<td>8</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>p.m.</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

Mean cancellations per week 6.66 ± 0.45 (95% CI).

Improved roster

<table>
<thead>
<tr>
<th></th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.m.</td>
<td>11</td>
<td>14</td>
<td>12</td>
<td>14</td>
<td>15</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>p.m.</td>
<td>12</td>
<td>12</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

Mean cancellations per week 2.84 ± 0.30 (95% CI).


Conclusion

Mathematical modelling and optimisation can

- Increase number of patients treated.
- Improve quality of care for patients.
- Improve staffing rosters.
- Improve efficiency of units.

The simulation and optimisation tool described here is a prototype, which has been designed so that it can be applied more generally.

- In other settings (e.g. HDU, Ward)
- To answer other kinds of questions – "what if" questions ..... 

This is just part of a larger project. Cameron Walker and Mike O’Sullivan (Engineering Science) are working with other units, and we want to extend these ideas to a general simulation and optimisation tool, that can be easily customised for new units, to assist with informed decision making.