Patients, queues and hospital beds: modelling and optimisation

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Haere Mai
Welcome
Auckland City Hospital
1
Emergency
Adult & Children
Main Entrance
All Services
P Parking
Entrance B
Transition lounge and Service Vehicles only
ALL PARKING
20

Contributors to the project

- Cardiothoracic Surgical Unit, Auckland City Hospital – heart and thoracic (mainly lung) surgery.
  - Andrew McKee – Director of the Intensive Care Unit (ICU)
  - Pam Freeman – Manager, Cardiovascular Services
  - Andrea Pryce, Steve Withy – Data analysts
  - Nursing staff at unit
- Department of Statistics, University of Auckland
  - Ross Ihaka – co-founder of R
  - Will Chen – BSc(Hons) and Masters projects
  - Kim Frew – BSc(Hons) project

The Cardiothoracic Surgical Unit, Auckland City Hospital

Patients treated

- Heart bypass patients, acute and elective (scheduled)
- Other cardiac patients
- Thoracic (chest) patients
- Other patients (vascular, ...).

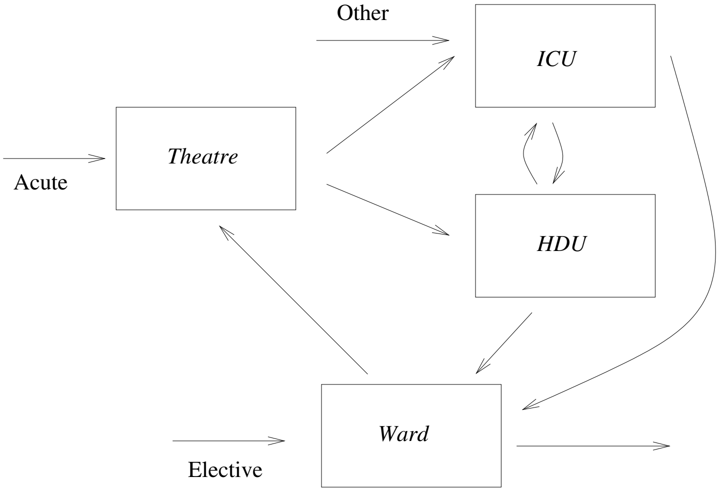
Questions

- Initial question – how to minimize waiting lists and maximize number of patients treated?
- First stage – how many beds should be staffed in intensive care to keep cancellations low? (Need to employ around 5 nurses per bed.)
- Second stage – given the staffing level, what is the optimal roster and number of electives that should be scheduled?
- Third stage – bottlenecks elsewhere.

The system

- 3 operating theatres + 1 for emergencies
- Intensive care unit (ICU) staffs 9-12 beds, up to 16 available.
  - 1 nurse per patient + 2 runners (can look after acute patients) + charge nurse + 2 on-call nurses
- High dependency unit (HDU) has 6 beds
  - 1 nurse per 2 patients + 1 runner + charge nurse
- 52 beds in wards

Patient flow



Data

3,412 admission records ICU and HDU Jan 2006 – Dec 2007,
842 ICU records Feb – Nov 2008

- Admission date and time
- Length of stay (LOS) in minutes
- Type of procedure
- Ward (ICU or HDU)
- Admission type (e.g. elective, acute, vascular, surgical non-bypass, cardiac non-surgical, ECMO, (medical), other)

Additional data on transfers between ICU and HDU, rosters, schedules, elective patients treated...

Cardiovascular Intensive Care Unit (CV-ICU)

Several patient arrival flows 2008 (2006-7)

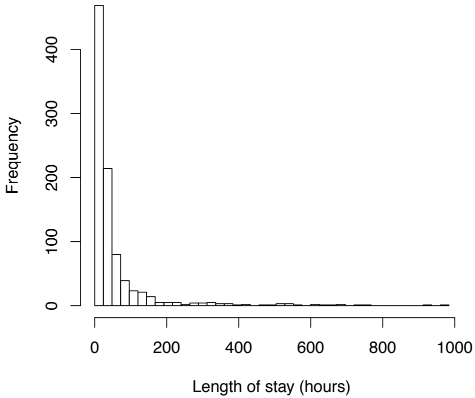
- Acute – emergency patients 4.2% (7%)
- Elective bypass – scheduled cardio-thoracic surgical patients 65.9% (61%)
- Medical, vascular and other patients 29.9% (32%).

Variable admission times for non-elective patients.

Variable lengths of stay 2008 (2006-7)

- 51% (50%) of patients leave ICU within 24 hours,
  94% (96%) within 1 week
  2% (1%) stay longer than 17 days.

Length of stay (hours) in ICU Feb–Nov 2008



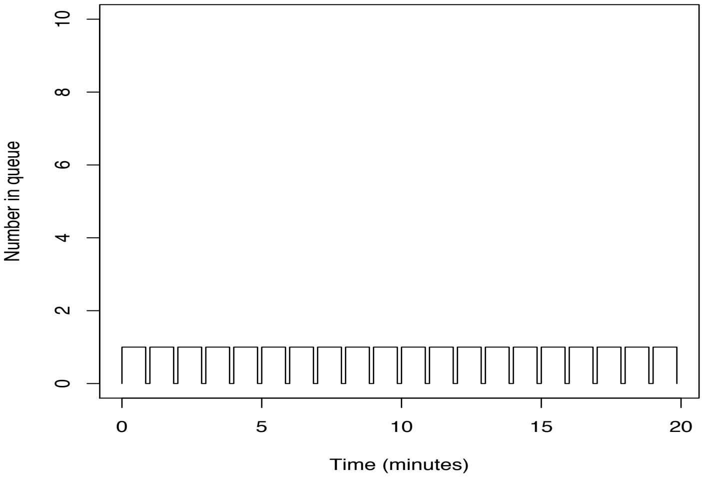
Why is it important to include variability in the model?

Imagine a single-server queue where

- 1 customer arrives at the beginning of each minute
- each customer requires 51 seconds (0.85 minute) service exactly
- so the server is busy 85% of the time

What does a plot of the number in the queue look like?

Deterministic model – arrivals 1 per minute, service time 0.85 minutes (51 seconds)

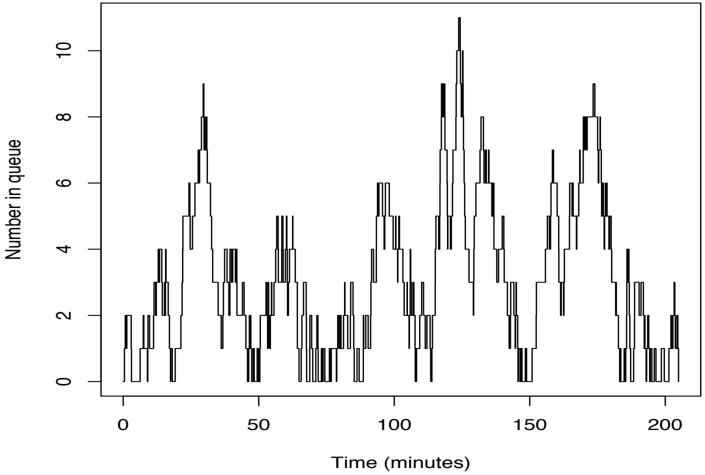


Now let's look at a plot of the number in a single-server queue where

- customers arrive randomly (as a Poisson process) – on average 1 per minute
- customer service times are random (exponentially distributed) – each customer requires 51 seconds service on average
- the server is still busy 85% of the time on average

What does a plot of the number in this queue look like?

Random model – arrivals 1 per minute, service time 0.85 minutes (51 seconds) on average



- The queue buildup is due *just* to variability in the arrival and service processes.
- The greater the variability, the greater the average length of the queue *even though the average arrival and service rates don't change*.
- The variability of the number in the queue increases too, so waiting times are less predictable.
- But the server is no busier, on average, when variability increases.

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- The greater the variability, the greater the average length of the queue *even though the average arrival and service rates don't change*.
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- But the server is no busier, on average, when variability increases.

Conclusion: Deterministic models are not enough to tell you about queue lengths.

ICU Patients Feb-Nov 2008

Important to model different types of patient.

Patient	N	Mean LOS	Median LOS	σ	% occupancy
Bypass elective	555	52.6	23.7	93.2	51.8
Bypass acute	35	163.4	88.0	182.2	10.1
Bypass repeat	74	45.5	23.4	64.6	6.0
Surgical non-bypass	65	58.0	21.6	109.4	6.7
Non-surgical cardiac	80	72.8	35.4	95.3	10.3
ECMO (bypass)	7	348.3	166.5	333.4	8.6
Vascular	68	37.3	20.4	68.2	4.5
Other	32	33.3	20.7	47.9	1.9
Total	916	58.9	23.8	103.7	100

ICU modelling – analytical model

- A suitable model might be a modified G|G|C+2+2|C+2+2 queue.
- Runners and on-call nurses provide buffer – priority reservation for acute bypass and some non-bypass patients.
- Difficulties in modelling arrivals:-
  - Both deterministic and Poisson arrival streams.
  - Arrivals vary with time of day and day of week.
  - Arrival rates vary on a faster time-scale than lengths of stay (unlike emergency departments).

ICU simulation model 24-hour 7-day

- Simulation written in R.
- Simulation splits each day into 5 time periods starting at

  00:00, 7:00, 11:00, 15:00, 19:00.

  State of system at beginning of each time period is given by

$$S = (N,\ \ LOS,\ \ t,\ \ \text{shift},\ \ \text{day of week})$$

$$\begin{aligned} N &= \text{number of patients in ICU} \\ LOS &= \text{vector of residual lengths of stay for patients in ICU} \\ t &= \text{time of day} \\ \text{shift} &= \text{ a.m. or p.m.} \end{aligned}$$

- Arrivals
  - Deterministic arrivals of electives on weekdays at 11:00 and 15:00.

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
a.m.	3	3	0	3	3	0	0
p.m.	2	2	2	2	2	0	0

  - Other patients arrive as a Poisson process – rate depends on time of day, day of week and type of patient. Typical marginal rates per hour are:-

start time	00:00	07:00	11:00	15:00	19:00	average
λ	0.0240	0.0304	0.0674	0.0839	0.0579	0.0495

- Length of stay in ICU drawn from empirical distribution of lengths of stay.

- Surgery for elective patients is cancelled if a bed is not available.
- At beginning of each time period
  - Patients who have left ICU during previous time period are removed from list.
  - Acute and other non-elective arrivals are added to the list.
  - If electives are scheduled, a decision is made about whether to continue with surgery.

Inputs to simulation

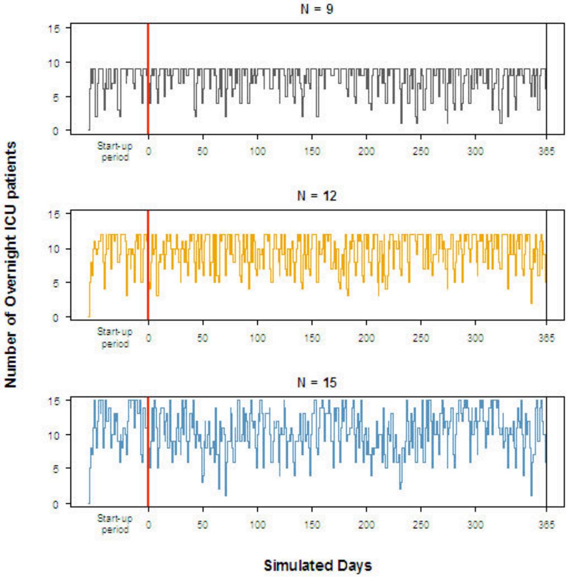
- Elective schedule
- Nursing roster (how many nurses working each shift)
- File of patient data including
  - Day and time of arrival
  - Length of stay
  - Type of patient
- Arrival rates for patient types are calculated from data, but can also be entered manually.

Outputs from simulation

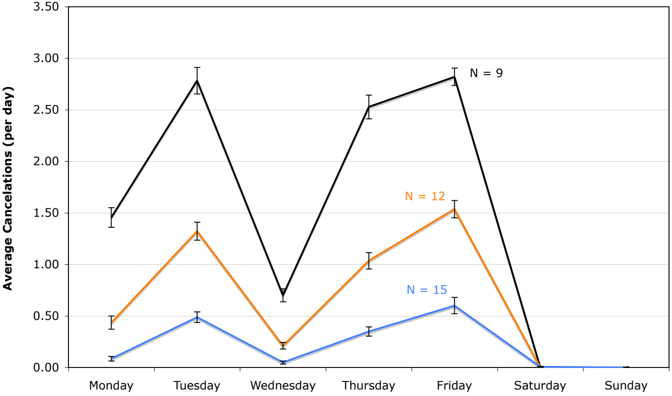
The simulation gives a wide variety of outputs, including estimates of:-

- Number of cancellations per week/shift.
- Number of occupied beds per shift – bed utilisation.
- Number of elective patients treated/admitted per week/shift.
- Number of additional nurses called in per shift.

Simulations of number of occupied beds at 4 p.m.



Estimated number of cancellations per day



What is the optimal roster?

Objective function?

- Minimize number of cancellations.
- Maximize number of electives treated.
- Keep low the number of additional nurses called in.
- A workable roster.

Typical elective operating schedule

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
a.m.	3	3	0	3	3	0	0
p.m.	2	2	2	2	2	0	0

Typical baseline nursing roster, including runners

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
a.m.	8	12	12	11	12	12	10
p.m.	12	12	12	11	12	10	9

12-hour shifts, starting at 7 a.m. and 7 p.m.

Mean cancellations per week 6.66 ± 0.45 (95%CI).

And after searching for improved roster

Starting roster

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
a.m.	8	12	12	11	12	12	10
p.m.	12	12	12	11	12	10	9

Mean cancellations per week 6.66 ± 0.45 (95%CI).

Improved roster

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
a.m.	11	14	12	14	15	10	7
p.m.	12	12	10	11	12	8	7

Mean cancellations per week 2.84 ± 0.30 (95%CI).

Conclusion

Mathematical modelling and optimisation can

- Increase number of patients treated.
- Improve quality of care for patients.
- Improve staffing rosters.
- Improve efficiency of units.

The simulation and optimisation tool described here is a prototype, which has been designed so that it can be applied more generally.

- In other settings (e.g. HDU, Ward)
- To answer other kinds of questions – "what if" questions .....

This is just part of a larger project. Cameron Walker and Mike O'Sullivan (Engineering Science) are working with other units, and we want to extend these ideas to a general simulation and optimisation tool, that can be easily customised for new units, to assist with informed decision making.