Manipulation of Elections in Age of Twitter

(based on joint work with Shaun White)

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Elections

Election is:

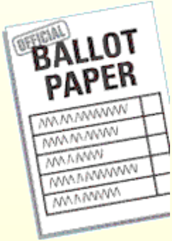
- is an important tool that is used whenever a group of agents needs to make a joint decision that in some way accommodates preferences of all the participants.
- an indispensable part of the democratic process.
- at the heart of the election there is voting rule.

We deal with single-winner elections and rules.

Preferences

There are several types of structures that can represent preference. Most often we use linear orders. Preferences are submitted in the form of ballots:

1. International Herald Tribune
2. Financial Times
3. USA Today
4. Le Figaro
5. Frankfurter Algemaine Zeitung



Voters' types

Voters with identical preferences are said to be of the same type.

With three alternatives a, b, c we will have only six types:

1	2	3	4	5	6
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

For m alternatives we have $m!$ types.

Profiles

Collection of preferences of all participants is called the profile (of the given society).

As an example: Let us consider a society of 19 voters deciding on 4 candidates a, b, c, d to an office.

4	4	5	6
a	b	c	d
c	a	a	b
b	c	b	a
d	d	d	c

Who should be elected to the office?

Plurality

Most commonly used rule is Plurality.

4	4	5	6
a	b	c	d
c	a	a	b
b	c	b	a
d	d	d	c

It counts the number of first preferences:

$$\mathrm{Sc}(a) = \mathrm{Sc}(b) = 4,\ \mathrm{Sc}(c) = 5,\ \mathrm{Sc}(d) = 6$$

and awards a candidate with the highest score. So d wins.

Plurality with runoff

We may follow France (as well Louisiana and Georgia) and legislate a second round.

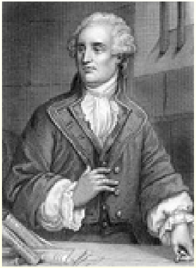
$$\begin{array}{cccc} 4 & 4 & 5 & 6 \\ \hline a & b & c & d \\ c & a & a & b \\ b & c & b & a \\ d & d & d & c \end{array} \quad \rightarrow \quad \begin{array}{cc} 13 & 6 \\ \hline c & d \\ d & c \end{array}$$

In the second round *c* wins.

Condorcet

His method was to organise pairwise comparisons of all candidates.

4	4	5	6
a	b	c	d
c	a	a	b
b	c	b	a
d	d	d	c



We find that the Condorcet winner is *b* since

$$b \text{ beats } a \quad 10:9$$

$$b \text{ beats } c \quad 10:9$$

$$b \text{ beats } d \quad 13:6$$

Borda

His method was to award points for positions of candidates on ballots.

points	4	4	5	6
3	a	b	c	d
2	c	a	a	b
1	b	c	b	a
0	d	d	d	c

We find that the Borda winner is *a* since

$$\mathrm{Sc}(a) = 36, \quad \mathrm{Sc}(b) = 33, \quad \mathrm{Sc}(c) = 27, \quad \mathrm{Sc}(d) = 18.$$

Dictatorial

In a dictatorial election one voter determines the outcome. All other votes are ignored.

Information field

Elections are not held in vacuum. Public opinion research companies constantly monitor the state of the society:

- Colmar Brunton in NZ,
- Gallup Poll (in 140 countries).

Media plays in election a very important role. In particular,

- informs voters about platforms of candidates so that they can form their preferences;
- publishes opinion polls;

Ideological and strategic voters

Voters are partitioned into:

- ideological voters: interested in stating their preference, report it no matter what.
- strategic voters: interested in optimizing the outcome rather than in stating their own preference.

Strategic voters:

- need as much information as possible.
- their voting intentions might differ from their true preferences.

Election of 2000 in Florida

This is the final count in of votes in Florida in 2000 elections when a Republican candidate George W. Bush defeated Democratic candidate Al Gore by just 537 votes. All voters who voted for Green's candidate Ralph Nader were ideological.

Republican	2,912,790
Democratic	2,912,253
Green	97,488
Natural Law	2,281
Reform	17,484
Libertarian	16,415
Workers World	1,804
Constitution	1,371
Socialist	622
Socialist Workers	562
Write-in	40

Social networks enrich the information field

In the past there were no means for extensive voter-to-voter communications. This is now changing fast.

Classical assumption of Social Choice:

- voters' sincere preferences is a public knowledge
- their voting intentions are their private information

These may no longer be realistic as people share their voting intentions on social networks.

The Theorem of Gibbard and Satterthwaite

Theorem

Suppose that the voting rule in use is:

- *fully deterministic;*
- *always select a single winner;*
- *has at least three alternatives in its range;*
- *nondictatorial.*

Then at a certain profile of sincere preferences a single strategic voter can submit an insincere preference and improve on the outcome when everyone else is sincere.

Manipulability

Any profile where one voter can change the result to her advantage is called now manipulable.

It is often believed that if voters are "rational" a manipulable profile, if occurs, will be manipulated.

Is this the right concept?

What if two voters are strategic?

We assume that there are only two voters and three alternatives $\mathcal{A} = \{A, B, C\}$ and the following voting rule F used:

F	ABC	L
ABC	B	A
M	A	C

(here L and M are arbitrary linear orders different from ABC).

Suppose two voters Alice and Bob

- believe that A is better than B which is better than C.
- both are risk averse and this is a common knowledge.

How should they vote?

A hierarchy of beliefs

		Bob	
	F	ABC	L
Alice	ABC	B	A
	M	A	C

Alice: I know Bob is risk-averse so he won't deviate from *ABC* (or we may get C).

Alice: As I now believe that Bob will not deviate, then I should deviate and we will get A.

Alice: Hey, but Bob also knows that I am risk-averse so he will then also believe that I will not deviate and will deviate himself so I should not deviate.

Alice: ...

Pattanaik's concerns

Conjecture (Pattanaik, 1975)

When the number of voters tend to infinity, the probability of a manipulable profile tends to zero.

A voter is pivotal if she can single-handedly change the result of the election (ceteris paribus).

Theorem (Slinko, 2004)

For all classical voting rules the probability of a profile where someone is pivotal is in order of $\frac{1}{\sqrt{n}}$, where n is the number of voters.

Coalitional manipulability

This concept was introduced by Y. Murakami (1968) in his book "Logic and Social Choice."

Given a voting rule, a profile is called coalitionally manipulable if a group of strategic voters, by voting insincerely, could secure a more preferred outcome for each of them.

There are several things to emphasize:

- manipulating coalition may include voters of different types,
- they may submit different insincere ballots.

Difficulties with the concept

Theorem (Slinko, 2005)

For all classical voting rules the only coalitions of size $O(\sqrt{n})$ can manipulate with nonzero probability, where n is the number of voters.

- a manipulating coalition of this size must be somehow formed.
- this group must include a coordination centre who calculates the manipulation and then privately communicates instructions to coalition members.
- all the coalition members must obey the instructions of the centre but there does not seem to be obvious ways to reinforce the discipline.

How a coalition can naturally be formed?

Strategic voters vote strategically even not being pivotal.

This suggests that voters are cognisant of the preferences and incentives of others.

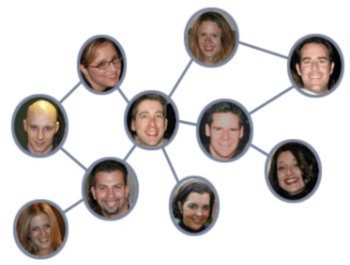
They feel that they belong to a type and that jointly they can change the result.

A natural step is to allow the whole type to be strategic. But how will they coordinate?

Social networks

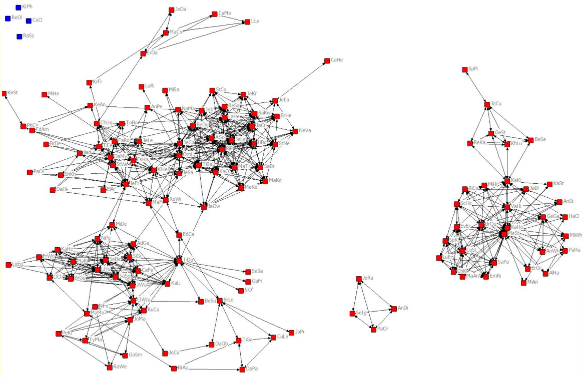
Social networks is an obvious answer. Voters can:

- Discuss manipulation strategies in chat rooms.
- Announce their voting intentions by sending tweets to their followers.
- Call upon their followers to vote in a certain way.



Real social networks

Real networks have few very influential nodes. Many links are one-way.



So only public figures or celebrities can make a difference.

Some statistics from Twitter

On 19 Dec 2010 on Twitter:

Celebrity	Followers
Lady Gaga	7,276,241
Britney Spears	6,361,290
Justin Bieber	6,252,121
Ashton Kutcher	6,089,878

These have real power.

Harward Business Review (2011) reported:

The surprising result in the recent (2010) elections in Korea can be partially explained by the fact that Korean movie stars and pop idols pushed voters to the polling booths.

- Cult novelist, Lee Oi-Soo, moved his almost 170,000 followers (mostly young) into action by tweeting, “If you give up your right to vote, it is as worthless as trash.”

The third option

There are three actions at their disposal:

- vote sincerely;
- vote strategically;
- vote strategically and call upon their like-minded followers to vote like they.

However

- Issuing a call to supporters the public figure will not know exactly how many supporters will follow her example and vote as she recommends.

Example

Suppose the Borda rule is used.

17	15	18	16	14	14
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

Then $\mathrm{Sc}(a) = 96$, $\mathrm{Sc}(b) = 99$, $\mathrm{Sc}(c) = 87$. The winner of this election is b.

This profile is not manipulable from GS Theorem point of view but calls to vote strategically can be made.

Example continued

The *acb* types are unhappy.

17	15	18	16	14	14
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

$$\begin{bmatrix} a \\ c \\ b \end{bmatrix} \xrightarrow{13} \begin{bmatrix} c \\ a \\ b \end{bmatrix}$$

makes $\mathrm{Sc}(a) = 83$, $\mathrm{Sc}(b) = 99$, $\mathrm{Sc}(c) = 100$. The winner of this election is c.

If a smaller number of *acb* types switch, nothing happens. It is safe for an *acb* type to call to vote *cab*.

Example continued

The *abc* types are not completely happy.

17	15	18	16	14	14
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \xrightarrow{4-8} \begin{bmatrix} a \\ c \\ b \end{bmatrix}$$ makes a the winner.

But

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \xrightarrow{>8} \begin{bmatrix} a \\ c \\ b \end{bmatrix}$$ makes c the winner.

The call for *abc* types to vote *acb* is unsafe (may overshoot).

The Geometry of Example

We define the normalised Borda score of the alternative a is given by:

$$Scn(a) = \frac{Sc(a)}{Sc(a_1) + \ldots + Sc(a_m)}.$$

We get the vector of normalised scores

$$Scn = (Scn(a_1), Scn(a_2), \ldots, Scn(a_m)),$$

where

$$Scn(a_1) + Scn(a_2) + \ldots + Scn(a_m) = 1.$$

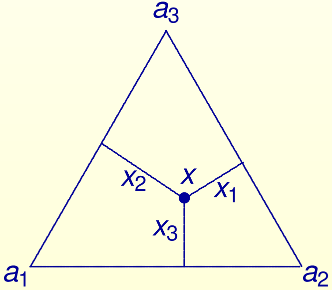
Geometric representation of scores

The vector of normalised scores Scn can be represented as a point $\mathbf{x}$ of the m-dimensional simplex S^{m-1}:

$$\mathbf{x} = (x_1, \ldots, x_m), \quad x_1 + \ldots + x_m = 1,$$

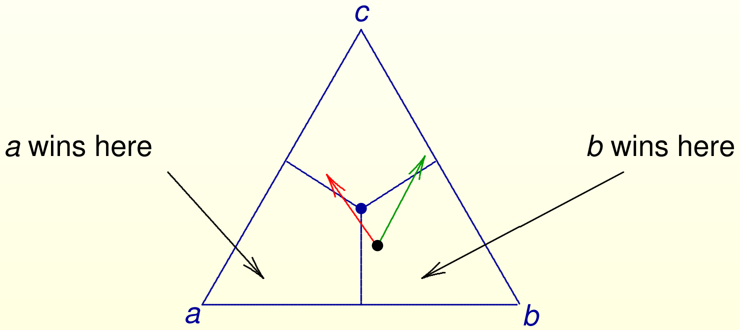
where $x_i = Scn(a_i)$ is the normalised score of the ith alternative.

We treat $x_1, \ldots, x_n$ as the homogeneous barycentric coordinates of $\mathbf{x}$.



Geometry of the two strategic calls

The simplex S^2 is divided into three zones: where the candidates a, b and c win, respectively.



The green arrow is the safe manipulation and the red arrow is the unsafe one.

Main Result 1

Theorem

Suppose that the voting rule in use is:

- *fully deterministic;*
- *always select a single winner;*
- *has at least three alternatives in its range;*
- *nondictatorial.*

Then at a certain profile of sincere preferences a voter may issue a safe strartegic call.

Main Result 2

Theorem (Extention of the GS Theorem)

Suppose that the voting rule in use is:

- *fully deterministic;*
- *always select a single winner;*
- *has at least three alternatives in its range;*
- *nondictatorial.*

Then at a certain profile of sincere preferences a single strategic voter can safely manipulate.

Matters of concern

An individual voter's influence is strictly regulated in the offline world, not so online.

Celebrities, in particular, have acquired a considerable influence and can play (if they wish) a major role in any election.

Finding a safe manipulation for Borda, Approval etc. is easy (Hason & Elkind, 2010; Ianovski, Yu, Elkind, Wilson, 2011).

The probability of existence of a safe strategic call is not known but may not tend to zero.

Thank you for your attention!