Electoral Equilibria under Scoring Voting Rules

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Introduction

"For several fundamental reasons, it is particularly appropriate that we should include the analysis of incentives in political institutions as an essential part of the domain of modern economics."

Roger Myerson, Schumpeter Lecture (1999)

The three main reasons:

- markets and politics are substantively interconnected systems
- failures of the political system can affect people's welfare at least as much as failures of the market systems
- there are logical similarities between political competition and market competition

Downs' thesis

Downs suggested that the famous Hotelling (1929) "Main Street" market competition model can be also used to analyse political competition.

"... my central hypothesis: political parties in a democracy formulate policy strictly as a means of gaining votes. "

Anthony Downs (1957)

We should recognize though that each voter's welfare depends on the policies of the candidate who wins the election, not the candidate to whom he gives his vote.

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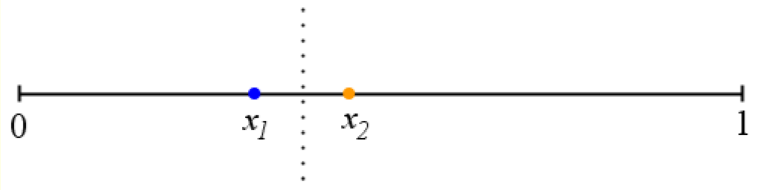

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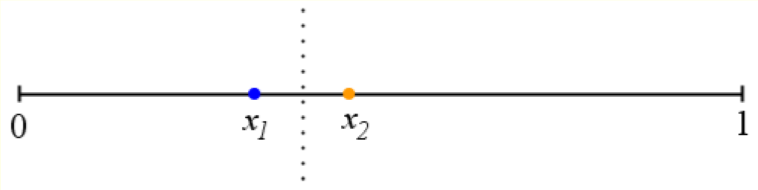

- There are m candidates. A *profile* is an m vector $x = (x_1, \ldots, x_m) \in [0,1]^m$ that specifies each candidate's position: x_i is candidate i's position.

Ideological spectrum in New Zealand

One of the main assumptions of this model is that the ideological spectrum is one-dimensional.

Parties objectives

Stigler (1972) also argued that “political and economic motives will be similar and best modeled by maximization of market or vote share.”

We assume that parties choose their positions on the ideological spectrum in order to maximize their share of the votes.

Questions to answer

In the political context, the main questions are:

- Do equilibria situations exist?
- Will the candidates cluster together, advocating identical or similar policy positions, or will they adopt diverse positions that appeal to different groups of voters?
- Which characteristics of the voting rule force rational candidates to adopt the position of the median voter and which give an incentive to diversify?
- How do the optimal strategies depend on the voting system in use?

Nash equilibrium

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Two kinds of Nash equilibria exist:

- A *convergent* Nash equilibrium (CNE) occurs when all candidates adopt the same ideological position.
- A *non-convergent* Nash equilibrium (NCNE) is when not all candidate positions are the same.

What happens under plurality

Most of the literature on competitive determinants of political policy positions has focused on just one electoral system: plurality voting.

- For $m = 2$ we have CNE (Hotelling 1929) — known as Principle of Minimum Differentiation.
- For $m = 3$ no Nash equlibria exist;
- For $m = 4, 5$ there is unique NCNE;
- For $m \geq 6$ there are infinitely many NCNE (Eaton and Lipsey,1975).

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- This rule belongs to a large class of rules called positional scoring rules.

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- Require that $s_1 \geq \cdots \geq s_m$ and $s_1 > s_m$, i.e., the scores are nonincreasing and first is better than last. For example:
  - Plurality: $s = (1, 0, 0, \ldots, 0, 0)$,
  - Borda: $s = (m-1, m-2, \ldots, 1, 0)$,
  - Antiplurality: $s = (1, 1, 1, \ldots, 1, 0)$,
  - k-Approval: $s = (\underbrace{1, 1, \ldots, 1}_{k}, 0 \ldots, 0)$.

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- The candidates' overall scores are then calculated by integrating across all voters.

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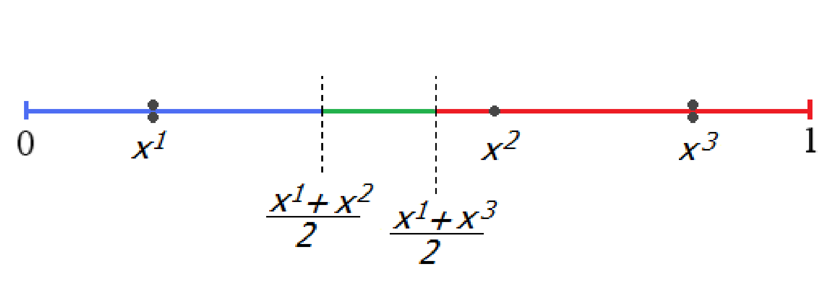
- For example, if Borda rule is used:

Ranking	Points received
A	6
B	5
$C \sim D \sim E$	3
F	1
G	0

Workings of a positional scoring rule



The score of a candidate positioned at x^1 would be

$$\frac{s_1+s_2}{2}\frac{x_1+x_2}{2}+\frac{s_2+s_3}{2}\frac{x_3-x_2}{2}+\frac{s_4+s_5}{2}\left(1-\frac{x_1+x_3}{2}\right).$$

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- The issue space is a road through an urban area, with customers distributed along it.
- Firms choosing locations so as to maximise their share of the market.
- $s = (s_1, \ldots, s_m)$ is a vector of probabilities, s_i being the probability a customer buys from i-th nearest firm.

Economics question

- Plurality rule cannot explain for $m > 2$ the tendency of firms to cluster together.
- Can we explain this tendency by a more general model?

Convergent equilibria

Theorem (Cox, 1987). For m candidates and scoring rule s, a profile $x = (x^*, \ldots, x^*)$ is a CNE if and only if

$$c(s,m) \leq x^* \leq 1 - c(s,m), \tag{1}$$

where $c(s,m) = \dfrac{s_1 - \bar{s}}{s_1 - s_m}$ is the *c-value* (with $\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i$) .

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- If $c(s, m) > 1/2$ (best-rewarding rule[1]), the inequality (1) cannot hold. So no CNE exist.
- If $c(s, m) \leq 1/2$ (worst-punishing or intermediate rule), any x^* in $[c(s, m), 1 - c(s, m)]$ is a CNE.

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Note. Borda is intermediate.

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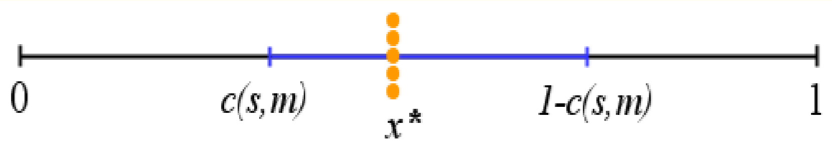
A picture of a convergent equilibria



- The blue interval is the set of valid equilibrium platforms.
- The size of the blue interval depends on the rule. The stronger the incentive to place first, the smaller it is.
- If $c(s, m) > 1/2$ then there is no blue interval – no CNE exist.

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  - If $c(s, 3) \leq 1/2$ only CNE exist;
  - If $c(s, 3) > 1/2$ no equilibria exist.
- If $m = 4$, can we characterize the rules for which NCNE exist? This is the first challenge.

The four-candidate case

Theorem (C.-S., 2011). In a four-candidate election under scoring rule $s = (s_1, s_2, s_3, s_4)$, NCNE exist iff both the following conditions are satisfied:

- a) $s_1 > s_2 = s_3$;
- b) $c(s, 4) > 1/2$.

Moreover, the NCNE is unique and symmetric with two candidates at

$$x_1 = \frac{s_1}{4(s_1 - s_2)}$$

and two at

$$x_2 = 1 - x_1 .$$

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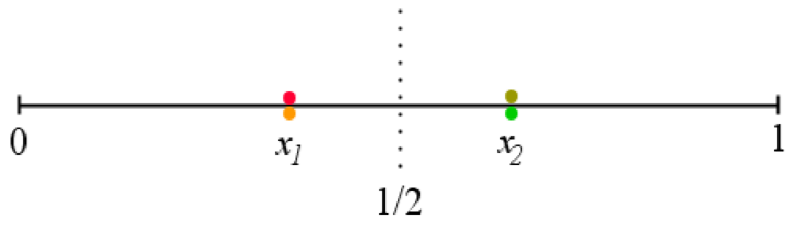

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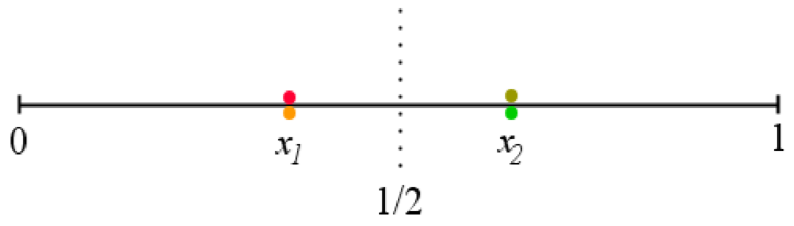

- The distance between the pairs of candidates in NCNE depends on the rule. As $c(s, 4) \to 1/2$, the positions converge to the half-way point.
- When $c(s, 4)$ falls below 1/2, only CNE exist, given by the previous theorem.
- If $c(s, 4) > 1/2$ but $s_2 \neq s_3$ then no NE of either kind exist.

The five-candidate case

Theorem (C.-S., 2011). In a five-candidate election under scoring rule $s = (s_1, s_2, s_3, s_4, s_5)$, NCNE exist iff both the following conditions are satisfied:

- a) $s_1 > s_2 = s_3 = s_4$;
- b) $c(s, 5) > 1/2$.

Moreover, the NCNE is unique and symmetric, with equilibrium profile $x = ((x^1, 2), (1/2, 1), (x^2, 2))$, where

$$x^1 = \frac{1}{6}\left(\frac{s_1 + s_2}{s_1 - s_2}\right) \quad \text{and} \quad x^3 = 1 - x^1.$$

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Note. For both $m = 4$ and $m = 5$ CNE and NCNE cannot coexist together.

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E.g., if $c(s, m) > 3/4$, then $q \geq 3$. For the plurality $c(s, m) > 1 - 1/m$ so we have at least $m/2$ occupied positions.

Repeated highest scores

Theorem. (C.-S. 2011) Given a scoring rule s, let $1 \leq k \leq m-1$ be such that $s_1 = \cdots = s_k > s_{k+1}$. Then a necessary condition for NCNE is $\min(n_1, n_q) > k$.

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Example.) If m is odd, consider k-approval with $k = (m-1)/2$. That is, $s = (1, \ldots, 1, 0, \ldots, 0)$, where the first k positions are ones. Then

$$c(s, m) = 1 - \frac{1}{m}\left(\frac{m-1}{2}\right) = \frac{1}{2} + \frac{1}{2m} > \frac{1}{2}.$$

So the rule is best-rewarding but by Corollary it has no NCNE.

Concave scores

We say that the score vector $s = (s_1, \ldots, s_m)$ is concave if

$$s_1 - s_2 \geq s_2 - s_3 \geq \cdots \geq s_{n-1} - s_n \geq s_n - s_m.$$

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Theorem. (C.-S. 2011) Let s be a scoring rule $s = (s_1, \ldots, s_n, s_{n+1}, \ldots, s_m)$, with

$$s_{n+1} = s_{n+2} = \cdots = s_m$$

for some $1 \leq n < m$. If s is concave then there are no NCNE, unless the subrule $s' = (s_1, \cdots, s_n, s_{n+1})$ is Borda and $n + 1 \leq \lfloor m/2 \rfloor$ (i.e., more than half the scores are constant).

Symmetric bipositional NCNE

Theorem. Suppose m is even and s is a scoring rule with c-value $c(s, m) \leq 1/2$ and $s_{m/2} < \bar{s}$. Then any profile $x = ((x_1, m/2), (x_2, m/2))$ satisfying

$$\frac{1}{2} - \frac{\bar{s} - s_{m/2}}{s_1 - s_{m/2}} \leq x_1 < \frac{1}{2} \quad \text{and} \quad x_2 = 1 - x_1$$

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Example. Let $m = 6$. Suppose $s = (2, 2, 1, 1, 1, 0)$. We have $c(s, m) = 5/12 < 1/2$, so the rule is indeed worst-punishing. Also, $s_3 = 1 < 7/6 = \bar{s}$. So any profile with half the candidates one each side such that $1/3 \leq x_1 < 1/2$ and $x_2 = 1 - x_1$ is a NCNE.

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Dichotomy fails. Both types of equlibria can coexist together!

Multipositional NCNE

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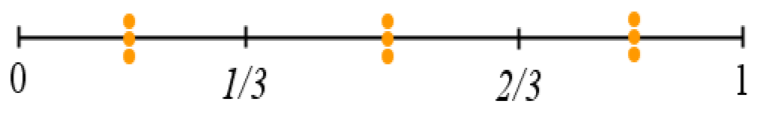
Theorem. Let there be $m = qr$ candidates, $q \geq 2$. Let

$$s = (s_1, \ldots, s_{r-1}, 0, \mid \underbrace{0, \ldots, 0}_{r} \mid \cdots \mid \underbrace{0, \ldots, 0}_{r})$$

be a scoring rule. Divide the interval into q equally sized subintervals. Then the profile in which r candidates locate at the half-way point of each subinterval is a NCNE if and only if $c(s', r) \leq 1/2$, where $s' = (s_1, \ldots, s_{r-1}, 0)$.



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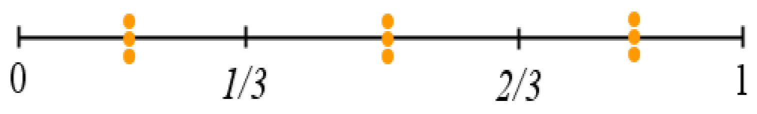

Example: $m = 9$ candidates and $q = r = 3$. NCNE for rules: 2-approval, $s = (1, 1, 0, 0, 0, 0, 0, 0, 0)$ or $(2, 1, 0, 0, 0, 0, 0, 0, 0)$.

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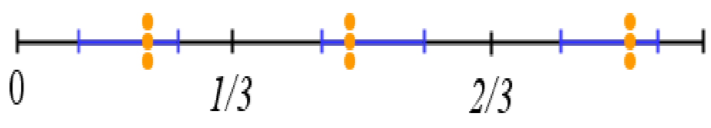

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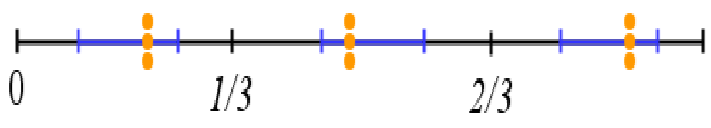

- Finding a NCNE reduces to finding a CNE on each subinterval.

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And this is not an equilibrium since $s'' = (1, 0, 0)$ is best-rewarding:

Firms locations revisited

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- Our results can be seen a confirmation for the Principle of Local Clustering.

- **Principle of Local Clustering** (Eaton-Lipsey, 1975). When a new firm enters a market, or when an existing firm relocates, there is a strong tendency for that firm to locate as close as possible to another firm. This behaviour tends to create local clusters of firms in many equilibrium and disequilibrium situations.

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- Plurality, $s = (1, 0, \ldots, 0)$ does not explain the Principle of Local Clustering but more general scoring rules do.

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- Suppose a rule does not have any type of equilibria. Will the clustering effect still be observed to some extent?