Profinite groups

Dan Segal, Oxford University

1 Where do they come from, what are they good for?

- Galois groups, tree automorphisms, p-adic groups.

- Completions of abstract groups.

- Free groups

Topological group structure.

2 Case study: analytic pro-p groups

Finitely generated pro-p groups; derived group and Frattini subgroup.
Finite rank and powerful pro-p groups.
p-adic linear groups.
Coclass theory.

3 Algebraic properties

Topological-group properties of a profinite group vs. group-theoretic properties of its finite quotients.

- number of generators;

- verbal width.

Closed normal subgroups.
Subgroups of finite index and strong completeness in finitely generated profinite groups.
Power subgroups.

4 Further topics

Applications to compact topological groups.
'Small' vs. strongly complete.
Criteria for strong completeness; open questions.