Towards realism in modeling ocean wave behavior in marginal ice zones

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Abstract. The model of *Meylan and Squire* [1996], which treats solitary ice floes as floating, flexible circular disks, is incorporated into the equation of transport for the propagation of waves through a scattering medium, assumed to represent open ice pack in a marginal ice zone. The time-independent form of the equation is then solved for homogeneous ice conditions allowing for dissipation due to scattering, together with extra absorption from interactions between floes, losses in the water column, and losses arising from the inelastic character of the sea ice including local brash. The spatial evolution of spectra as they progress through the pack is investigated, with the aim of explaining the field data of *Wadhams et al.* [1986] and other observations that relate to the modification of directional structure, and specifically to the change towards isotropy experienced by waves as they travel into the ice interior. In accord with observations, directional spread is found to widen with penetration until eventually becoming isotropic, the process being sensitive to wave period. The effect of absorption on the solution is investigated.

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Introduction

Many aspects of the marginal ice zone (MIZ), defined here as the segment of seasonal sea ice zone that is affected significantly by open ocean processes, are dominated by the presence of ocean waves. Yet several field experiments have produced data that are not explained by contemporary theoretical models. In particular, features that relate to the evolution of directional structure as a sea or swell progresses into and through pack ice are not described well, and neither are reflections of waves at the ice margin.

Pragmatically, there is an obvious way to proceed, namely, to consider the MIZ to be composed of a large number of discrete solitary floes and then to integrate the effect of these floes. This is the preferred option, and is the method of this paper, but an alternative approach, whereby the MIZ is characterized in some fashion by a continuous surface boundary condition, also has some history. The difficulty with synthesizing contributions from many floes is that the outcomes are only as good as the basic floe model. The basic floe model has recently been improved by allowing floe flexure in the work by *Meylan* [1995] and *Meylan and Squire* [1996] (hereafter MS). In this work a solitary ice floe model is developed that computes scattering by allowing the compliant floe to respond properly to the local sea. It is shown by MS, for example, that bending of the ice floe by the waves alters its scattering function significantly, and that this should be taken into account when MIZ models are devised. Although MS show that modeling floes as rigid floating disks as done by *Masson and LeBlond* [1989] is imprecise unless the wavelengths are much larger than the diameter of the floe, the qualitative agreement between observations and theoretical conclusions in *Masson and LeBlond* [1989] is pleasing. Indeed an observed trend towards isotropy with increased penetration is predicted. To correctly represent an ice field under a broad range of wave conditions and floe morphologies, however, requires the use of the best solitary floe model available, namely, one that can flex as well as display the degrees of freedom associated with a completely rigid floating body.

Accordingly, in this paper we assimilate the MS model of the solitary, flexible circular ice floe into a transport equation that describes wave propagation in a scattering medium. Initially a brief account is given of the model reported in full by MS. This is followed by the main theoretical development of the paper, where an ice field composed of many such floes is assembled with the purpose of reproducing the progression of waves through the MIZ. Finally various examples of the analogue MIZ are given in the context of explaining some field observations, e.g., *Wadhams et al.* [1986], that hitherto have not been entirely realistically modeled.

Masson and LeBlond

The work in this paper is closely related to the work of *Masson and LeBlond* [1989]. In this paper a scattering model was developed using multiple scattering theory as opposed to our approach of starting with the equation of transport. In many ways these differences are cosmetic and *Masson and LeBlond* [1989] end up with a scattering equation which is almost identical to ours. While *Masson and LeBlond* [1989] include extra terms such as non-linear wave coupling and wind generation they were considering the temporal developement of the wave spectra while we are considering the spatial developement for which such terms are not as significant. Of course they could be included straightforwardly in our formulation following the method of *Masson and LeBlond* [1989]. A major advantage of our approach over *Masson and LeBlond* [1989] is that the scattering equation is derived in a far simpler and transparent way and the derivation of *Masson and LeBlond* [1989] included some terms which we believe to be erroneous (they make the scattering equation mathematically inconsistent) which do not appear in our derivation. Further *Masson and LeBlond* [1989] used a different model for the single floe, allowing for its draft but neglecting flexure. This means that there model applies to a MIZ of floes of small aspect ratio which does not include the bulk of the MIZ which consists of floes of hundereds of metres diameter and only a few metres thickness.

Circular Floe Model

A crucial feature of the solitary floe solution used in this paper is that the ice floe is assumed to behave as a flexible thin plate of negligible draft, rather than as a rigid body. The circular geometry chosen simplifies the solution, as it is only for a circular disk that the free modes of vibration can be expressed in an analytic form. Solving for a more complex floe geometry would serve little purpose since what we require is an estimate of the average floe scattering from floes with a distribution of geometries such as make up the MIZ. The problem considered is linear, and time dependence is removed by taking only a single frequency ω in the usual manner. In nondimensional form the boundary value problem to be solved is then

$$\nabla^2\phi = 0 \qquad -\infty < z < 0 \tag{1a}$$

$$\frac{\partial\phi}{\partial z} = 0 \qquad z \to -\infty \tag{1b}$$

$$\frac{\partial\phi}{\partial z} - \alpha\phi = 0 \qquad z = 0 \qquad 1 < r < \infty \tag{1c}$$

$$\left(\beta\nabla^4 + 1 - \alpha\gamma\right)\frac{\partial\phi}{\partial z} - \alpha\phi = 0 \tag{1d}$$

$$z = 0 \qquad 0 \le r \le 1$$

$$\frac{\partial^2}{\partial r^2}\frac{\partial\phi}{\partial z} + \nu\left(\frac{\partial}{\partial r}\frac{\partial\phi}{\partial z} + \frac{\partial^2}{\partial\theta^2}\frac{\partial\phi}{\partial z}\right) = 0 \tag{1e}$$

$$r = 1$$

$$\frac{\partial}{\partial r}\left(\nabla^2\frac{\partial\phi}{\partial z}\right) + (1-\nu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2}\frac{\partial\phi}{\partial z}\right) = 0 \tag{1f}$$

$$r = 1$$

where (r, θ, z) is a cylindrical-polar coordinate frame with its origin at the center of the floe, ϕ is the velocity potential in the water, ν is Poisson's ratio, and α, β, and γ are nondimensional variables defined by MS. The Sommerfield radiation condition [*Sarpkaya and Isaacson,* 1981] makes the problem well-posed.

Using the Green's function for linear water waves, the boundary value problem (1) is solved by converting it to an integral equation, expanding ϕ and its normal derivative in terms of the free modes of vibration of a circular thin plate [*Itao and Crandall,* 1979], and then truncating to achieve the desired accuracy. Defining the (dimensional) Kochin function H [*Wehausen and Laitone,* 1960] as

$$H(\theta) = \int\limits_{\Delta} (k\phi - \frac{\partial \phi}{\partial z}) e^{ik(x\cos\theta + y\sin\theta)} dxdy \tag{2}$$

where k is the wave number, the energy radiated in unit time per unit angle is given by

$$E(\theta) = \frac{\rho\omega^3}{8\pi g} |H(\pi + \theta)|^2 \tag{3}$$

where g is the acceleration due to gravity and ρ is the density of the sea water.

Energy Scattering

In the absence of scatterers it is known that the energy intensity satisfies the following equation [*Phillips,* 1977, p. 26]

$$\frac{1}{c_g}\frac{\partial}{\partial t} I(\boldsymbol{r}, t, \theta) + \theta \cdot \nabla_r I(\boldsymbol{r}, t, \theta) = 0 \tag{4}$$

where $I(\boldsymbol{r}, t, \theta)$ denotes an intensity function that describes the rate of flow of energy traveling in a given direction, across a surface element normal to that direction, per unit of surface, per unit of solid angle of direction, and c_{g} is the velocity of propagation. We have neglected any wave source or dissipation terms in (4) but these could be included straightforwardly as is done in *Masson and LeBlond* [1989]. We prefer to neglect these terms and to concentrate on the effect of the scattering due to the sea ice floes. Because ice floes in the MIZ scatter energy, (4) must be rewritten according to *Howells* [1960] as

$$\begin{aligned}&\frac{1}{c_g}\frac{\partial}{\partial t} I(\boldsymbol{r}, t, \theta) + \theta \cdot \nabla_r I(\boldsymbol{r}, t, \theta) = \\ &-\beta(\boldsymbol{r}, t, \theta) I + \int_0^{2\pi} S(\boldsymbol{r}, t, \theta, \theta') I(\boldsymbol{r}, t, \theta') d\theta'\end{aligned} \tag{5}$$

which is known as the Boltzmann equation. It has been assumed that each floe scatters independently of those around it and that the energy from different scatterers may be added incoherently. This does not mean that model does not include multiple scattering as the Boltzmann equation is precisely an equation for multiple scattering, but simply that coherent interference between adjacent floes has been neglected. Since the scattering model is linear we not allow for an incident wave of one freqency to excite motions at a different freqency, i.e. there is no non-linear coupling of different freqencies. The absorption coefficient $\beta(\boldsymbol{r}, t, \theta)$ represents the fraction of energy lost by scattering and dissipative processes from a pencil of radiation in direction θ, per unit path length traveled in the medium. As such, as well as scattering losses, it includes all irrecoverable dissipation occurring in the MIZ due to hydrodynamical turbulence and wave breaking, collisions and abrasion between ice floes, and hysteresis losses occurring in the brash between floes and through bending of the floes themselves. The scattering function $S(\boldsymbol{r}, t, \theta, \theta')$ specifies the angular distribution of scattered energy in such a way that $SId\Omega dVd\Omega'$ is the rate at which energy is scattered from a pencil of radiation of intensity $I(\boldsymbol{r}, t, \theta')$ in a solid angle $d\Omega'$ in direction θ', by a volume dV of medium at position $\boldsymbol{r}$, into a solid angle $d\Omega$ in direction θ.

To proceed, we omit time dependence, assume that the solution is a function of the x spatial coordinate alone, and restrict our study to uniform MIZs. Then S is a function of θ and θ', β is a constant, and (5) becomes

$$\cos\theta \frac{\partial I}{\partial x} = -\beta I + \int_0^{2\pi} S(\theta, \theta') I(x, \theta') d\theta' \tag{6}$$

At $x = 0$, a directional spectrum I_0 is presumed to be incident on the ice edge, i.e.,

$$I(0, \theta) = I_0(\theta) \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \tag{7}$$

If the MIZ is taken as infinite in extent, as is done in this paper, no net propagation occurs at $x = \infty$, whereas if it is finite the distant ice edge gives rise to a boundary condition that for large times will march back to affect the solution in the interior if no diffusion is applied. In either case the additional boundary condition closes the system to be solved but the latter situation is physically implausible for a time-independent solution.

The scattering function S may be expressed in terms of E as follows [*Meylan and Fox,* 1996] We must now express S in terms of E. The wave energy density is $\frac{1}{2}\rho g a^2$ where a is the wave amplitude. This must be multiplied by the average area occupied by a floe, denoted V_f/ρ_f (where V_{f} is the area of a floe and ρ_f is the floe concentration) and by the wave phase speed ω/k (since the solution is for a single frequency) to get the rate of energy flow per floe. This gives us the following expression for S,

$$S(\theta,\theta') = S(\theta-\theta') = \frac{2\rho_f E(\theta-\theta')k}{V_{\mathrm{f}}\rho g a^2\omega}. \tag{8}$$

Accordingly, the equation for $I(x,\theta)$ is

$$\frac{\partial I}{\partial x} = -\frac{\beta}{\cos\theta}I + \frac{1}{\cos\theta}\int_0^{2\pi} S(\theta-\theta')I(\theta')d\theta' \tag{9}$$

where from conservation of energy

$$\beta = \bar{\beta} + \beta' = \int_0^{2\pi} S(0,\theta')d\theta' + \beta' \tag{10}$$

The decomposition into absorption due to scattering ($\bar{\beta}$) and absorption due to irrecoverable processes (β') is made to allow us to study the effect of different degrees of (constant) dissipation. Equation (8) and Equation (5) depend on the assumption of incoherent scattering from adjacent floes as so will become less valid as the floe concentration is increased. Even at high concentrations the scattering term will be a reasonable estimate scattering provided that the floe size is of the order of the wavelength. Of course the scattering term is only an estimate and the local variation in floe size and geometry are other reasons the scattering will not be absolutely correct.

Following *Ishimaru* [1978] we convert (9) into matrix form by writing

$$\frac{\partial}{\partial x}\mathbf{I} = \mathbf{S}\mathbf{I} \tag{11}$$

with matrix coefficients

$$S_{ij} = \begin{cases} \dfrac{S(0)w_i - \beta}{\cos\theta_i} & i = j \\ \dfrac{S(\theta_i - \theta_j)}{\cos\theta_i} w_j & i \neq j \end{cases} \tag{12}$$

and w_i as weight function. For $\beta' = 0$, (11) has solution

$$\mathbf{I}(\theta_i, x) = c_0\mathbf{f}_0(\theta_i) + c_1\big(x\mathbf{f}_0(\theta_i) + \mathbf{f}_1(\theta_i)\big) + \sum_{n\geq 2} c_n \mathbf{f}_n(\theta_i) e^{\lambda_n x} \tag{13}$$

where $\mathbf{f}_n$ are the eigenvectors of the matrix $\mathbf{S}$ and $\{0, \lambda_n\}$ is its set of eigenvalues. The eigenvector $\mathbf{f}_0$ is associated with the repeated eigenvalue 0 corresponding to the isotropic solution, and because there is only one such independent eigenvector a generalized eigenvector $\mathbf{f}_1$ must be used to obtain a second independent solution $(x\mathbf{f}_0 + \mathbf{f}_1)$ [*Boyce and DiPrima,* 1986]. The values of the coefficients c_n are determined by the boundary conditions. For an infinite MIZ $c_n = 0$ if $\lambda_n > 0$ and $c_1 = 0$, as there is no flow as $x \to \infty$. The rest of the c_n are found from the boundary condition at $x = 0$, i.e.,

$$\sum_n c_n \mathbf{f'}_n(\theta_i) = \mathbf{I}_0(\theta_i) \qquad -\frac{\pi}{2} < \theta_i < \frac{\pi}{2} \tag{14}$$

where the set $\mathbf{f'}_n$ comprises $\mathbf{f}_0$ and the eigenvectors with negative eigenvalues.

When $\beta' \neq 0$ the zero eigenvalue vanishes and consequently the solution becomes simply

$$\mathbf{I}(\theta_i, x) = \sum_n c_n \mathbf{f}_n(\theta_i) e^{\lambda_n x} \tag{15}$$

together with boundary conditions (14) with set $\mathbf{f'}_n$ now comprising just the negative eigenvalues.

Meylan and Fox [1996] provide equivalent equations when the MIZ is of finite extent. Because of the mathematics properties of the Boltzmann equation it does not follow that the results for a semi-infinite MIZ will follow from consideration of a suitably large but finite MIZ. We consider that a semi-infinite MIZ is a more physically realistic model for wave scattering because most MIZ are sufficiently large as to be modelled as semi-infinite.

Results

Throughout this section we consider the development of an open water directional spectrum $E(T, \theta)$ as it proceeds into and through an MIZ composed of 50 m ice floes. In common with other work we use [*Longuet-Higgins et al.,* 1963; *Phillips,* 1977, p. 139]

$$E(T, \theta) \propto T^5 \exp\left(-\frac{5}{4}\left(\frac{T}{T_\mathrm{m}}\right)^4 \right) \cos^{2s}\left(\frac{\theta - \bar{\theta}}{2}\right) \tag{16}$$

where T denotes period, T_m is the peak period, and the principal direction $\bar{\theta} = 0°$. The exponent s is set at 10, after *Ewing and Laing* [1987]. Because we are only interested in relative change, the constant of proportionality is unimportant and we simply set $\max\{E\} = 1$.

Monochromatic Seas

In all the results that follow, unless otherwise stated, we assume that the Poisson's ratio for the sea ice is 0.3 and the Young's modulus is $6 times 10^9$Pa. Figures 1–3, drawn respectively for 0, 5%, and 10% irrecoverable damping, illustrate the way in which the $\cos^{2s}\theta/2$ open sea spread alters as 10-s-period ocean waves travel into an open MIZ. In each case a homogeneous distribution of 1-m-thick circular ice floes is used, and polar plots are given at the edge (0 km) and at penetrations of 1, 10, and 50 km. Each set of four polar plots is qualitatively similar. For example, Figure 1a is made up of an incident part lying to the right of the origin that is described by $\cos^{2s}\theta/2$, and a reflected isotropic part lying to the left of the origin. At 1 km penetration (Figure 1b) the appearance is similar, though a slight broadening of the forward-going lobe and a smearing out of the backscattered energy is evident. This effect is more noticeable 10 km from the ice edge (Figure 1c), and is very conspicuous in Figure 1d, which shows that the spread at 50 km is becoming quite isotropic. Figures 2 and 3 are rather similar to Figure 1 except that the nonzero β' diminishes the scattered and forward-going lobes as would be anticipated. Scattered waves are particularly affected by the damping.

It is of interest to consider shorter and longer period seas. This is done for 5- and 20-s-period waves respectively in Figures 4 and 5, where the irrecoverable damping is set at 5%. In the 5-s-case the spread becomes isotropic within a kilometer or so, but the waves are also strongly attenuated so that by 10 km their intensity is negligible. Figure 5, on the other hand, shows that 20-s-waves can travel far into the interior of the MIZ with little attenuation and next to no change in directional spread.

It is also of interest to consider what effect the inclusion of the floe flexure in the calculation of the scattering operator has on our results. Figure 6 is a plot of the directional spectrum for 10-s period waves with 5% damping at the ice edge (a1 and b1) and 500 m from the ice edge (a2 and b2). The floe thickness is 0.5 m and the floe diameter is 100 m. In a1 and a2 the standard values of the the sea ice parameters are used while the stiffness is set to ∞ in b1 and b2 (i.e. no floe flexure is allowed as in the model of *Masson and LeBlond* [1989]). While we have choosen parameters which will show clearly how different the results from the two models can be they are in no way unrepresentative of actual parameters found in the MIZ. The results at the ice edge are not substantitally different but the estimates of the wave energy 500 m from the ice edge are completely different.

Summarizing, we find that the amount of directional spreading caused by ice floes in an MIZ is strongly dependent on the period of the incoming wave train. Short period seas entering an ice field will quickly become isotropic, whereas long period swells will retain their directionality to far greater penetrations. This is in agreement with field observations, which will be discussed later.

Evolution of Directional Spectra

Figures 7 and 8 invoke expression (16) to track the progress of a directional energy spectrum as it moves into and through an ice field composed of 50-m-floes. For Figure 7 ice thickness is 1 m, for Figure 8 it is 5 m. Note the considerable distortion of the spectral shape that occurs, both in terms of directional spread and the distribution of energy between periods. The gradual broadening of the spread seen in the earlier polar plots is evident for 1-m-thick ice floes, along with a selective filtering effect that favours the damping of shorter waves in preference to longer ones. By 50 km (Figure 7d) the spectrum is close to being isotropic for all but the longest waves, and the spectral energy is biased towards large periods. Because the scattering and consequently the damping is greater for thicker ice, Figure 8 is a more extreme version of Figure 7. Here an isotropic spread is achieved within a kilometer or so of the ice edge, and by 50 km most of the energy has been removed from the spectrum; only a perfectly isotropic ring of long period energy remains with no suggestion of the original directional structure. These theoretical results are supported by field data, as we shall now show.

Field Observations

Data reported by *Wadhams et al.* [1986] are, to the authors' knowledge, the only published field observations that follow the progress of a directional sea through pack ice. Experiments conducted in the Greenland Sea, as part of the MIZEX-84 programme, allow qualitative comparison with the theoretical predictions of the present paper. There, a pitch-roll buoy was deployed in the waters off the ice edge, and an equivalent unit was placed on suitable ice floes within the ice cover. In each case vertical acceleration and two orthogonal tilts referenced to north were measured to allow the directional wave spectrum at each location to be found using the method of *Cartright* [1963].

On 12–13 July the incident sea was composed of a wind sea and a swell. One open water station and four ice stations are of interest here, at penetrations of -8.2, 5.6, 11.2, 17.8, and 22.5 km. At these locations the significant wave height was found to be 150, 12.9, 3.5, 2.3, and 1.4 mm, respectively, illustrating the substantial damping of the waves brought about by the pack ice. Of importance, the directional spread at the peak frequency changed significantly from about 40° off the ice edge to 67° at 17.8 km and 76° at 22.5 km. Such large values suggest an isotropic wave field in the vicinity of the sites deep in the ice cover, even though the periods here are relatively long (14–15 s).

In a second experiment on 13–14 July, a station at 1.2 km penetration was compared with several off the ice edge. Here the change in directional spread of two distinct peaks in each spectrum could be tracked, the first at about 7 s corresponding to a short swell, and the second at about 3.3 s corresponding to a local wind sea. In both cases the open water angular spread was about 30°. It was found that the wind sea's spread broadened on entry to the ice field, becoming undefined and isotropic by the time it reached the station at 1.2 km, while longer waves were unaffected that close to the ice margin.

Accordingly a picture emerges in the data of a process that acts to broaden the angular spread of a penetrating sea to a degree dependent on period. Short period waves are quickly scattered to become isotropically distributed in direction, whereas longer waves penetrate further into the pack ice before their directional structure eventually becomes isotropic. In the long run all waves are affected, but the penetration at which isotropy is attained depends on the period in relation to the ice field's morphology. This is as predicted by the theoretical model described earlier in the paper.

Conclusion

This paper, along with that of *Masson and LeBlond* [1989], is a serious attempt to describe the behavior of ocean waves in a marginal ice zone in as authentic a manner as possible. Both papers have the same goal, but each tackles the problem in a slightly different way. We believe, supported by arguments in MS, that the natural compliancy of each ice floe significantly distorts the scattering function and, consequently, that it must be included when solitary floe results are synthesized over the MIZ. Our results provide compelling evidence that scattering causes the broadening in the spread of directional seas observed by *Wadhams et al.* [1986], in contrast to earlier theoretical models based on a continuous surface boundary condition that lead to collimation. The ratio of the wave's length to the floe diameter and the ice thickness are both found to be crucial in determining the penetration at which isotropy is first achieved.

The present model certainly has deficiencies. Of importance: (i) the MIZ has been assumed to be uniform; (ii) the absorption coefficient β is constant; (iii) no dependence on time is included; and (iv) the solution depends only on the x-coordinate. M.H.M. is currently working hard to circumvent these assumptions.

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Figure 1. Polar plots showing changes in the spread of a 10 s sea traveling into a typical MIZ at penetrations (a) 0 km, (b) 1 km, (c) 10 km, and (d) 50 km. Concentration is 50%, floe thickness is 1 m, and floe diameter is 50 m. The damping coefficient $\beta' = 0$

Figure 2. As Figure 1 with 5% irrecoverable damping

Figure 3. As Figure 1 with 10% irrecoverable damping

Figure 4. As Figure 2 for a 5 s sea.

Figure 5. As Figure 2 for a 20 s sea.

Figure 6. Polar plots showing changes in the spread of a 10 s sea traveling into a typical MIZ at penetrations (a1 and b1) 0 km and (a2 and b2) 500 m. Standard sea ice parameters are used in a1 and a2 while no floe flexure is permitted in b1 and b2. Concentration is 50%, floe thickness is 0.5 m, and floe diameter is 100 m. The damping coefficient $\beta' = 5\%$

Figure 7. Three-dimensional plots showing changes in the directional spectrum traveling into an MIZ of 1 m thick sea ice at penetrations (a) 0 km, (b) 1 km, (c) 10 km, and (d) 50 km. Concentration is 50%, floe diameter is 50 m, and 5% damping is used.

Figure 8. As Figure 6 for 5-m-thick ice floes.

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