New Zealand Mathematical Society Colloquium 2017

5 – 7 December 2017

Programme and Abstracts

THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

Table of contents

Cover image: Gannets at Muriwai
Photograph by Bernd Krauskopf

Welcome

Welcome to the 2017 New Zealand Mathematics Colloquium!

There is an excellent programme of lectures and events, and we hope that you will enjoy the conference hosted by the University of Auckland. We are very grateful for sponsorship from:

- Department of Mathematics, University of Auckland
- Te Pūnaha Matatini, University of Auckland
- The New Zealand Mathematical Society
- ANZIAM (Australia and New Zealand Industrial and Applied Mathematics)
- Operations Research Society of New Zealand

Extra special thanks are due to

John Shanks

of the NZMS for running the conference webpage and registration system, and

Tama Ngahere

of the Department of Mathematics for all the administrative help.

Conference Organising Committee

Hinke Osinga (co-chair)	Tom ter Elst (co-chair)
Anna Barry	Golbon Zakeri
Bernd Krauskopf	Igor Kontorovich
Caroline Yoon	Steven Galbraith

Local Information

Please, ask one of the organisers if you have any questions; they have orange name badges.

In case of medical emergencies:

QuayMed Accident and Medical
Ground floor of QuayPark Health
68 Beach Road, Central Auckland
Phone: (09) 919 2555

Internet

Wireless internet access is available in the conference area via `Eduroam`.

Talks and Prizes

All lectures and social events will be held in the Department of Mathematics on Princes Street, with the exception of the conference dinner. The Department of Mathematics is building 303 on the campus map (see next page). The entry from Princes Street is at the basement level and leads directly to the Basement Lobby and rooms B05, B07, B09, and B11. Invited presentations take place in room MLT1, which is on the ground floor of building 303. The NZMS and ANZIAM AGMs will take place in room SLT1, which is also on the ground floor.

Invited talks have been allocated 50 minutes, followed by 5 minutes for questions.

Contributed talks have been allocated 25 minutes and speakers are advised that their **talks should run for 20 minutes**. Session chairs are listed in the conference programme. Session chairs will ensure that sessions run on time by using green, yellow and red cards:

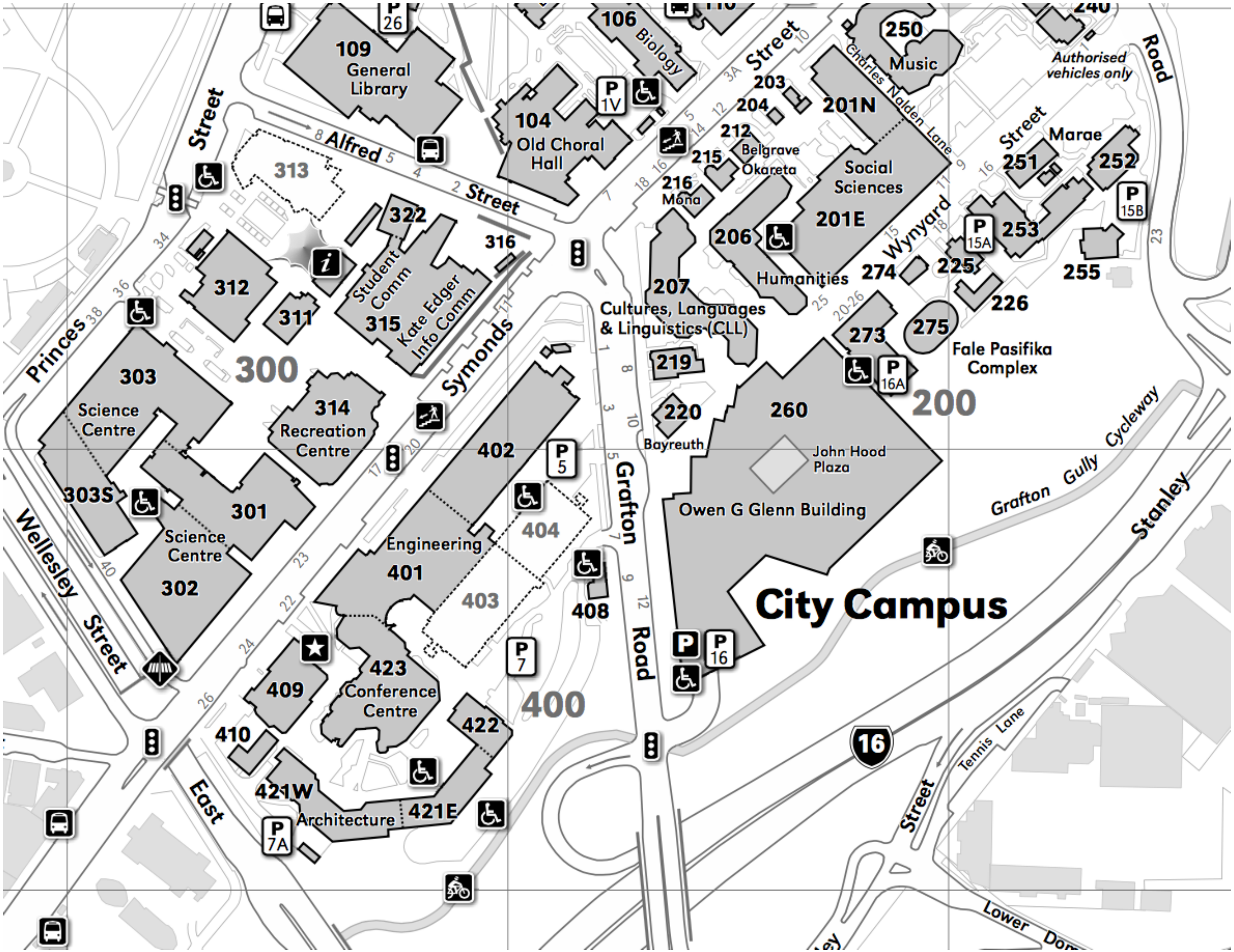
Green card: Five minutes to go. Show after 15 minutes.
Yellow card: Two minutes to go. Show after 18 minutes.
Red card: Time up. Show after 20 minutes if speaker is still talking.
Speaker is permitted one additional minute to conclude talk.

The **Aitken prize** is awarded by the New Zealand Mathematical Society for the best contributed talk by a student at the Colloquium. Entrants must be enrolled (or have been enrolled) for a degree in mathematics at a university or other tertiary institution in New Zealand in 2017. The Aitken Prize honours the New Zealand-born mathematician A. C. Aitken.

The **ANZIAM poster prize** is awarded to the best poster by an early career researcher (i.e., the first author of the paper must be either a student, or within 5 years of the completion of their highest degree).

These prizes as well as the NZMS Research Award, the NZMS Early Career Research Award, and Kalman Prize for Best Paper will be awarded at the Colloquium Dinner.

3


City Campus
109
General Library
106
Biology
104
Old Choral Hall
P 26
P 1V
250
Music
240
Authorised vehicles only
Road
Street
Alfred Street
Princes Street
Symonds Street
Wellesley Street
East Street
Grafton Road
Wynyard Street
Charles Nalden Lane
Stanley Street
Grafton Gully Cycleway
Tennis Lane
Lower Domain
203
204
212
Belgrave
Okareta
215
216
Mona
201N
Social Sciences
201E
206
Humanities
207
Cultures, Languages & Linguistics (CLL)
219
220
Bayreuth
260
John Hood Plaza
Owen G Glenn Building
273
274
275
Fale Pasifika Complex
225
226
251
252
253
255
Marae
P 15A
P 15B
P 16A
P 16
200
313
322
Student Comm
315
Kate Edger Info Comm
316
312
311
300
303
Science Centre
303S
301
Science Centre
302
314
Recreation Centre
402
P 5
404
Engineering
401
403
408
P 7
400
423
Conference Centre
409
410
422
421W
Architecture
421E
P 7A
16

Food and local facilities.

- You can find banks, post-office, bookshop and a mini-supermarket in the Kate Edger Information Commons (315 on the campus map).
- Auckland University student foodcourt (311 and 312 on the campus map): Various take-away food stalls, including Sushi, Chinese, Indian, Kebabs, Pies etc.
- STRATA restaurant, fourth floor of Kate Edger Information Commons (315 on student map): Good quality hot food.
- Relax lounge cafe (312 on the campus map): Nice outside seating facing Albert Park.
- There is also a wide range of restaurants and cafes towards the city (Steven Galbraith recommends High street and Lorne street). For those who want a more "funky" cafe experience then head up Symonds street and turn right onto K-road.

Wednesday Colloquium Dinner

The Colloqium Dinner on Wednesday 6 December will be in the Fale Pasifika (275 on the campus map).

Dinner guests are expected from 6:00pm. The buffet dinner will be served from 7:00pm. Please, try to arrive at the Fale Pasifika between 6:00pm and 6:30pm.

Thursday NZWiM Lunch

The NZ Women in Mathematics Networking event will be a catered lunch 12:30-13:30 on Thursday 7 December, on level 6 of the new Science Building (building 302 on the campus map). This event is made possible with the generous support from Te Pūnaha Matatini and the Department of Mathematics at the University of Auckland.

The purpose of the NZWiM Lunch is to allow for informal networking amongst members of the community who are concerned about the under-representation of women in the NZ mathematics profession.

Mike Plank (University of Canterbury) will be leading small group discussions on how departments can improve their diversity.

Both women and men are strongly encouraged to attend.

Monday 4 December 2017

17:00–19:00	Registration and welcome reception Basement Lobby of the Mathematics Building, 38 Princes Street

Tuesday 5 December 2017

8:45–9:00	2017 NZMS Colloquium opening ceremony by John Hosking, Dean of Science room **MLT1**			
9:00–10:00	NZMS Research Award Winner Plenary Lecture **David Bryant** *The geometry of diversities* (Chair: Vivien Kirk)			
10:00–10:30	Coffee/Tea break, Basement Lobby			
10:30–11:00	**Poster Session**, Basement Lobby			
	Alona Ben-Tal *Nonlinear phenomena in a piece-wise linear model of airflow in birds' lungs* **Anand Rampadarath** *Simulating bronchoconstriction of an airway during asthma* **Ielyaas Cloete** *The straight path to NFAT activation*	**Jesse Hart** *Notions of transfinite diameter on affine algebraic varieties* **Markus Antoni** *Maximal regularity for stochastic Volterra integral equations* **Michael Plank** *Light and shade: modelling growth of Tradescantia fluminensi*	**Samin Aref** *Map of New Zealand scientific collaborations* **Sione Ma'u** *Weighted pluripotential theory and applications* **Valerie Chopovda** *A periodic family of interplay orbits in the Caledonian symmetric four-body problem*	**Vee-Liem Saw** *Helicalised fractals, curved wormholes, time travel, and rotating black hole* **Winston Sweatman** *Four-body orbits close by central configurations* **Xueshan Yang** *Dynamics of a coupled calcium system*

Tuesday 5 December 2017

	room **B07**	room **B09**	room **B05**	room **B11**
	Geometry and topology (Chair: Philip Sharp)	Dynamical systems (Chair: Soizic Terrien)	Mathematical physics (Chair: Rod Gover)	Algebra and number theory (Chair: Yan Bo Ti)
11:00–11:30	**Keegan Flood** *Metrics in projective differential geometry*	**Gemma Mason** *Exploring the neighbourhood of a heterodimensional cycle*	**Vee-Liem Saw** *Helicalised fractals, curved wormholes, time travel, and rotating black hole*	**Shaun Cooper** *The Ramanujan–Mordell theorem for sums of squares, and some extensions*
11:30–12:00	**Michael Lockyer** *Mahavier products: graphs and knots*	**Cris Hasan** *Saddle slow manifolds and canard orbits in the Hodgkin–Huxley model*	**Graham Weir** *Removing classical singularities by using quantum sources*	**Roger Su** *A program algebra with non-halting tests*
12:00–12:30	**Bartek Ewertowski** *Hyperbolic manifolds, group theory, and geometry on the boundary at infinity*	**Elle Musoke** *The geometry of a 3D invariant manifold in a 4D slow-fast system*	**Woei Chet Lim** *Explicit G_1 spike solutions in general relativity*	**Geoff Booth** *On primeness in near-rings of continuous functions*

12:30–13:30	Lunch break
13:30–14:30	Plenary Lecture **Dorit Hochbaum** *Machine learning combinatorial optimization algorithms* (Chair: Golbon Zakeri)

Tuesday 5 December 2017

	room **B07**	room **B09**	room **B05**	room **B11**
	Numerical analysis (Chair: Valerie Chopovda)	Dynamical systems (Chair: Stefanie Hittmeyer)	Mathematical biology (Chair: Anand Rampadarath)	Applied DEs and modelling (Chair: Steve Taylor)
14:30–15:00	**Mohamed Al-Sultani** *Monotone numerical methods for solving nonlinear parabolic problems*	**David Simpson** *The sausage-string structure of mode-locking regions of piecewise-linear maps*	**David Waters** *Calcium signalling in T-cells*	**Fabien Montiel** *Recent advances in modelling ocean waves/sea ice interactions*
15:00–15:30	**Christian Offen** *Bifurcation of solutions to Hamiltonian boundary value problems*	**Andrus Giraldo** *How chaos arises from a homoclinic flip bifurcation*	**Catherine Hassell Sweatman** *Mathematical model of diabetes and lipid metabolism*	**Bruce van Brunt** *On a coagulation-fragmentation equation with exponential growth*
15:30–16:00	Coffee/Tea break, Basement Lobby			
16:00–17:00	Butcher-Kalman Plenary Lecture **Brendan Creutz** *Arithmetic of genus 1 curves* (Chair: Hinke Osinga)			
17:00–18:00	NZMS AGM, room **SLT1**			

Wednesday 6 December 2017

9:00–10:00	ANZIAM Plenary Lecture **Ana Amador** *Low dimensional models to study neural systems for motor control in songbirds* (Chair: Bernd Krauskopf)
10:00–10:30	Coffee/Tea break, Basement Lobby

	room **B07**	room **B09**	room **B05**	room **B11**
	Mathematical education I (Chair: John Griffith Moala)	Mathematical education II (Chair: Igor' Kontorovich)	Mathematical biology (Chair: Catherine Hassell Sweatman)	Analysis (Chair: Markus Antoni)
10:30–11:00	**Peter Radonich** *STEM Online NZ — a free interactive teaching and learning resource for NCEA external standards in STEM subjects*	**Julia Crawford** *School mathematics and NCEA: understanding how it works*	**Faheem Zaidi** *Optimal amplitude and frequency of breathing*	**Jesse Hart** *Transfinite diameter on affine algebraic varieties*
11:00–11:30	**Subash Chandar K** *The journey to loving maths*	**Julia Novak & Cami Sawyer** *First Year in Mathematics (FYiMaths) at University, the challenges faced by lecturers and students in an ever-changing landscape*	**Thomasin Lynch** *Modelling fermentation processes in the rumen*	**Chris Wong** *Elliptic operator with Robin boundary conditions*
11:30–12:00	**Andrew Ricciardi & Damian Campbell** *Growing pains: Implementing innovative ideas in a traditional environment*		**Jack Simpson** *Combining phylogenetic trees into networks: What can be done with just two trees?*	**Gareth Gordon** *The Dirichlet-to-Neumann operator on a Lipschitz hypograph*
12:00–12:30	**Karyn Saunders** *Ako in practice in mathematics teaching and learning*	**Peter Bier & Julia Novak** *Academics anonymous—Bring and share session*	**Nathan Pages** *Anatomically-accurate three-dimensional model of salivary fluid secretion*	

Wednesday 6 December 2017

12:30–13:30	Lunch break		ANZIAM AGM, room **SLT1** NZMS Education Group AGM, room **B07**	
13:30–14:30	Plenary Lecture **Masina Po'e-Tofaeono** *Bridging the gap between culture and mathematics* (Chair: Caroline Yoon)			
	room **B07**	room **B09**	room **B05**	room **B11**
	Mathematical education I (Chair: Jamie Sneddon)	Mathematical education II (Chair: Jo Knox)	Topology / DS & Combinatorics (Chair: Michael Lockyer)	Applied DEs and modelling (Chair: Andrew Keane)
14:30–15:00	**Graeme Wake** *The graveyard of geometry (a brief sample of)*	**Sze Looi Chin** *Characteristics of identifying narratives about struggling in mathematics*	**Sina Greenwood** *2-manifolds and inverse limits of set-valued functions on intervals*	**Andrea Babylon** *Modelling leptospirosis in livestock*
15:00–15:30	**Chris John** *Case study: developing an engaging, discursive, active and reflective classroom*	**Iresha Ratnayake** *Teaching with digital technology: Factors influencing secondary mathematics teachers' task development and implementation*	**Christopher Tuffley** *Characterisation and classification of signatures of spanning trees of the n-cube*	**Valerie Chopovda** *Families of planar periodic orbits generated from the Schubart-like orbit in the Caledonian four-body problem*
15:30–16:00	Coffee/Tea break, Basement Lobby			

Wednesday 6 December 2017

	room **B07**	room **B09**	room **B05**	room **B11**
	Mathematical education I (Chair: Subash Chandar K)	Mathematical education II (Chair: Iresha Ratnayake)	Mathematical biology (Chair: Nathan Pages)	Algebra and number theory (Chair: Tim Stokes)
16:00–16:30	**James Sneyd, Peter Bier & Peter Radonich** *Modelling in the mathematics curriculum... an impossible ideal?*	**John Griffith Moala** *How do students justify the optimality of their solutions in a contextualized discrete optimization task?*	**Mick Roberts** *Nobody expects a flu epidemic*	**Yan Bo Ti** *Fault attack on supersingular isogeny cryptosystem*
16:30–17:00	**Cami Sawyer & NZMS Education Group** *Working towards a shared vision of school mathematics*	**Tanya Evans** *Improving graduate attributes of students: researching the impact of regular use of non-routine problem solving on creativity and general thinking skills of STEM students*	**Anand Rampadarath** *A distribution-moment approximation for coupled airway dynamics of the airway wall and airway smooth muscle*	**Kevin Broughan** *Small gaps between primes*
17:00–17:30		**Igor' Kontorovich & Sina Greenwood** *Inspiring future researchers: A partnership between a mathematician and a math educator*		

19:00–22:30	**Conference Dinner** in the Fale Pasifika Plan to arrive 18:00-18:30

Thursday 7 December 2017

9:00–10:00	Plenary Lecture **Nick Trefethen** *Cubature, approximation, and isotropy in the hypercube* (Chair: John Butcher)			
10:00–10:30	Coffee/Tea break, Basement Lobby			
10:30–11:30	Plenary Lecture **Julie Clutterbuck** *Shape and resonance* (Chair: Tom ter Elst)			
	room **B07**	room **B09**	room **B05**	room **B11**
	Numerical analysis (Chair: Christian Offen)	Optimisation (Chair: Dion O'Neale)	Applied DEs and modelling (Chair: Andrus Giraldo)	Analysis (Chair: Chris Wong)
11:30–12:00	**Dimitrios Mitsotakis** *Numerical solution of the Serre equations for fully-nonlinear surface water waves*	**Rachel Tappenden** *Flexible alternating direction method of multipliers*	**Soizic Terrien** *Pulsing dynamics in an excitable laser with feedback*	**Melissa Tacy** *Flat models for Laplacian eigenfunction asymptotics*
12:00–12:30	**John Butcher** *Tree stumps and applications*	**Samin Aref** *Balance and frustration in signed graphs under different contexts*	**Phil Wilson** *A spring-dashpot system for modelling clinical lung tumour motion data in radiotherapy*	**Ilija Tolich** *HNN extensions of quasi-lattice ordered groups*
12:30–13:30	Lunch break		NZWiM Lunch, Level 6 of building 302	
13:30–14:30	Plenary Lecture **Bernd Krauskopf** *Why be afraid of state-dependent delays?* (Chair: Jiling Cao)			

Thursday 7 December 2017

	room **B07**	room **B09**	room **B05**	room **B11**
	Industrial mathematics (Chair: Luke Fallard)	Dynamical systems (Chair: Gemma Mason)	Applied DEs and modelling (Chair: Fabien Montiel)	Analysis (Chair: Melissa Tacy)
14:30–15:00	**Graeme Wake & Hyuck Chung** *An initiative for mathematics-in-industry in NZ (MINZ)*	**Stefanie Hittmeyer** *The geometry of blenders in a three-dimensional Hénon-like family*	**Winston Sweatman** *Studies of few-body orbits*	**Robin Havea** *A constructive look at the boundary of the numerical range of a bounded operator*
15:00–15:30	**Mark McGuinness** *Mmm, microwaves measure moisture*	**Andrew Keane** *State-dependent delays in the El Niño Southern Oscillation system*	**Alna van der Merwe** *A locally linear Timoshenko beam model*	**Markus Antoni** *Pathwise regularity for stochastic evolution equations in L^p spaces*

15:30–16:00	Coffee/Tea break, Basement Lobby

	room **B07**	room **B09**	room **B05**	room **B11**
	Industrial mathematics (Chair: Anna Barry)	Mathematical biology (Chair: David Waters)	Applied DEs and modelling (Chair: Cris Hasan)	Algebra and number theory (Chair: Geoff Booth)
16:00–16:30	**Wenjun Zhang** *Changing probability measures in GARCH option pricing models*	**Tertius Ralph** *Hard-core interactions in one-dimensional velocity jump models*	**Philip Sharp** *The fate of Apollo asteroids*	**Tim Stokes** *Constellations: Arrows without targets*
16:30–17:00	**Steve Taylor** *Keeping bags of milk powder nice as they get shipped around the world*	**Shawn Means** *A multi-scaled spatial model of hepatitis-B*	**Liam McMahon** *Hall, viscous and axial modifications of linear reconnection*	**Heung Yeung (Frederick) Lam** *Factorization of theta function identities*

Invited presentations

David Bryant

(University of Otago, david.bryant@otago.ac.nz)

The geometry of diversities

The field of metric geometry was wrenched from mathematical obscurity 20 years ago by the discovery that some really hard optimisation problems in graph theory could be solved, at least approximately, by converting them into metric embedding problems. Recently, we have found that many celebrated results for metric embeddings can be generalised, in a satisfying way, to diversities. A diversity is like a metric except that it defines values for all finite subsets, not just all pairs. The mathematics of diversities may still be obscure, but we've found it to be rich and multifaceted, with connections and applications in all sorts of places.

Dorit S. Hochbaum

(University of California Berkeley, dhochbaum@berkeley.edu)

Machine learning combinatorial optimization algorithms

We present a model for clustering which combines two criteria: Given a collection of objects with pairwise similarity measure, the problem is to find a cluster that is as dissimilar as possible from the complement, while having as much similarity as possible within the cluster. The two objectives are combined either as a ratio or with linear weights. The ratio problem, and its linear weighted version, are solved by a combinatorial algorithm within the complexity of a single minimum s,t-cut algorithm. This problem (HNC) is closely related to the NP-hard problem of normalized cut that is often used in image segmentation and for which heuristic solutions are generated with the eigenvector technique (spectral method).

We demonstrate the use of HNC as a supervised or unsupervised machine learning technique. In an extensive comparative study HNC was compared to leading machine learning techniques on benchmark datasets. The study indicated that HNC and other similarity-based algorithms provide robust performance and improved accuracy as compared to other techniques. Furthermore, the technique of ”sparse-computation” enables the scalability of HNC and similarity-based algorithms to large scale data sets.

Brendan Creutz

(University of Canterbury, brendan.creutz@canterbury.ac.nz)

Arithmetic of genus 1 curves

A polynomial equation like $3x^3+4y^3 = 5$ obviously has infinitely many solutions in real numbers x and y. But what if we instead require that x and y are rational numbers? This question's innocuous appearance belies its difficulty. Indeed, the pursuit of rational solutions to such equations has led to the discovery of deep mathematical structures and intriguing connections between them, as well as suggesting a range of tantalising possibilities and conjectures at the forefront of mathematics. In this talk I will give an overview of what is known, what is expected to be true, and how computational techniques have helped guide our understanding along the way.

Ana Amador

(University of Buenos Aires, ana.amador@gmail.com)

Low dimensional models to study neural systems for motor control in songbirds

Birdsong is a complex motor activity that emerges from the interaction between the peripheral system (PS), the central nervous system (CNS) and the environment. The similarities to human speech, both in production and learning, have positioned songbirds as unique animal models for studying this learned motor skill.

In this work, we developed a low dimensional dynamical system model of the vocal apparatus in which inputs could be related to physiological variables, being the output a synthetic song (SYN) that could be compared with the recorded birdsong (BOS). To go beyond sound comparison, we measured neural activity highly tuned to BOS and found that the patterns of response to BOS and SYN were remarkable similar. This work allowed to relate motor gestures and neural activity, making specific predictions on the timing. To study the dynamical emergence of this feature, we developed a neural model in which the variables were the average activities of different neural populations within the nuclei of the song system. This model can reproduce the measured respiratory patterns and matched the specific predictions on the timing of the neural activity during their production. In this talk, I will present experimental data in accordance with the dynamical model. This interdisciplinary work shows how low dimensional models for the PS and CNS can be a valuable tool for studying the neuroscience of generation and control of complex motor tasks.

Masina Po'e-Tofaeono

(University of Auckland and Manurewa High School, m.poetofaeono@auckland.ac.nz)

Bridging the gap between culture and mathematics

I will be talking about my experiences growing up learning mathematics in Decile 1 schools in NZ and struggling with my culture. I will also cover how I used these experiences to inform my teaching practice, and I will pass on some ideas that I have found successful for you to use in teaching others.

Nick Trefethen

(University of Oxford, trefethen@maths.ox.ac.uk)

Cubature, approximation, and isotropy in the hypercube

The hypercube is the standard domain for computation in higher dimensions. We describe two respects in which the anisotropy of this domain has practical consequences. The first is a matter well known to experts: the importance of axis-alignment in low-rank compression of multivariate functions. Rotating a function by a few degrees in two or more dimensions may change its numerical rank completely. The second is new. The standard notion of degree of a multivariate polynomial, total degree, is isotropic — invariant under rotation. The hypercube, however, is highly anisotropic. We present a theorem showing that as a consequence, the convergence rate of multivariate polynomial approximations in a hypercube is determined not by the total degree but by the *Euclidean degree*, defined in terms of not the 1-norm but the 2-norm of the exponent vector $\mathbf{k}$ of a monomial $x_1^{k_1} \cdots x_s^{k_s}$. The talk will include numerical demonstrations.

Julie Clutterbuck

(Monash University, julie.clutterbuck@monash.edu)

Shape and resonance

The shape of a geometric object determines its resonant frequencies, or spectrum. I will talk about some new and old results describing the kinds of shapes that optimise various spectral quantities.

Bernd Krauskopf

(University of Auckland, b.krauskopf@auckland.ac.nz)

Why be afraid of state-dependent delays?

Delay differential equations (DDEs) are the mathematical models of choice in applications where delays arise naturally, for example, due to the time it takes different subsystems to communicate, process information and finally react. It is a good approximation in many cases to model such delay as constant. On the other hand, communication/processing times may well depend on the state of the system in a significant way. What does this mean for the dynamics of the governing DDE?

After a brief introduction to systems with delays, we present here a case study of a scalar DDE with two delayed feedback terms that depend linearly on the state. The associated constant-delay DDE, obtained by freezing the state dependence, is linear and without recurrent dynamics. With state dependent delay terms, on the other hand, the DDE shows very complicated dynamics. A bifurcation analysis reveals interacting Hopf bifurcations, two-frequency dynamics on invariant tori and associated resonance tongues.

These results may serve as a health warning: state dependence alone is actually capable of generating a wealth of dynamical phenomena. Hence, it must be taken seriously in applications. On the other hand, as this talk will also demonstrate, tools from bifurcation theory and associated numerical methods are now available to deal effectively with state-dependent delays. This means that there is no need to avoid/disregard state dependence in DDE models.

This is joint work with Renato Calleja (UNAM) and Tony Humphries (McGill)

Poster presentations

Alona Ben-Tal

(Massey University, a.ben-tal@massey.ac.nz)

Nonlinear phenomena in a piecewise linear model of airflow in birds' lungs

Avian lungs are remarkably different from mammalian lungs in that air flows unidirectionally through rigid tubes where gas exchange takes place. This unidirectional flow occurs across a wide range of breathing frequencies, amplitudes, and specific species anatomies. It has been hypothesized that the unidirectional flow is due to aerodynamic valving, resulting from the complex anatomical structure of the avian lung and the fluid dynamics involved. To test this hypothesis, we have constructed a novel mathematical model that, unlike previous models, can produce unidirectional flow which is robust to changes in model parameters, breathing frequency and breathing amplitude. The model consists of two piecewise linear ordinary differential equations with lumped parameters and discontinuous, flow-dependent resistances that mimic the experimental observations. The model provides several new physiological insights into the lung mechanics of birds and also serves as a new example of a piecewise linear model that exhibits nonlinear phenomena (such as symmetry breaking).

This is joint work with Emily Harvey (Market Economics).

Anand Rampadarath

(University of Auckland, anandrampadarath@gmail.com)

Simulating bronchoconstriction of an airway during asthma

Asthma is fundamentally a disease of airway constriction. Due to a variety of experimental challenges, the dynamics of airways are poorly understood. Of specific interest is the narrowing of the airway due to forces produced by the airway smooth muscle wrapped around each airway. The interaction between the muscle and the airway wall is crucial for the airway constriction which occurs during an asthma attack. While crossbridge theory is a well-studied representation of complex smooth muscle dynamics, and these dynamics can be coupled to the airway wall, this comes at significant computational cost – even for isolated airways. Because many phenomena of interest in pulmonary physiology cannot be adequately understood by studying isolated airways, this presents a significant limitation. We present results associated with an alternative method which provides a viable option for an orders of magnitude reduction in computational cost, whilst retaining qualitative and quantitative behaviour.

Ielyaas Cloete (University of Auckland)

(i.cloete@auckland.ac.nz)

The straight path to NFAT activation

Calcium in T cells is crucial for cellular signalling. Increases in intracellular calcium regulates a wide variety of cellular functions, arguably the most important such function being the activation of the nuclear factor of activated T cell (NFAT) transcription factors.

A major question in this study is how spatiotemporal calcium patterns organise to efficiently regulate NFAT proteins. In this project we studied this question by developing a minimal model of calcium dynamics, as well as the downstream regulation of NFAT.

Previous studies have shown that NFAT is most efficiently regulated by an oscillatory calcium signal, particularly at low intracellular concentrations. However, our model predicts that a constant calcium signal is more efficient at regulating NFAT than an oscillatory signal. Furthermore, our model predicts NFAT responses to changes in oscillation period, where NFAT is efficiently regulated at low oscillation periods and become frequency-insensitive at large oscillation period.

Jesse Hart (University of Auckland)

(drjessehart@outlook.com)

Notions of transfinite diameter on affine algebraic varieties

The transfinite diameter is an intrinsic quantity associated to a compact set K that quantifies its spread. In pluripotential theory it has been a subject of interest because it connects to many seemingly unrelated notions which in turn allows pluripotential theory to have applications in the study of asymptotics and approximation. This research was concerned with reconciling two approaches to defining the transfinite diameter on an algebraic variety. Essential to the definition of the transfinite diameter is the use of a polynomial basis. The approach of Cox-Ma'u was to use the monomials from the coordinate ring of the variety $\mathbb{C}[\mathcal{V}] = \mathbb{C}[z]/I(\mathcal{V})$ where $I(\mathcal{V})$ is the ideal associated to the variety, while the approach of Berman-Boucksom was to use an orthonormal basis with respect to some probability measure μ. We showed that when μ is chosen to be the equilibrium measure $(dd^c, V_{T_{\mathcal{V}}})^M$ that these two approaches are equivalent up to a scale factor only depending on dimension.

Markus Antoni

(University of Otago, mantoni@maths.otago.ac.nz)

Maximal regularity for stochastic Volterra integral equations

We present an approach to obtain maximal regularity estimates for solutions of stochastic Volterra integral equations driven by a multiplicative Gaussian noise. The main part consists of the proof of suitable estimates for deterministic and stochastic convolution operators. Starting with the scalar-valued case, we use functional calculi results to obtain the corresponding estimates for the operator-valued setting. Once maximal regularity estimates for convolutions are obtained, appropriate Lipschitz and linear growth assumptions on the nonlinearities will lead to unique mild solutions with Hölder continuous trajectories.

This is joint work with Boris Baeumer (University of Otago).

Michael J Plank

(University of Canterbury, michael.plank@canterbury.ac.nz)

Light and shade: modelling growth of Tradescantia fluminensis

Tradescantia fluminensis is an invasive plant species in New Zealand, Australia and parts of the United States. We develop a model of the vertical variation in plant density and light intensity within a T. fluminensis mat. We parameterise the model using experimental data collected from individual plants. We verify that model predictions for the density and light profiles are qualitatively consistent with the data. The mat eventually reaches steady state due to a balance between energy gained from photosynthesis and energy lost to maintenance and decay. The model will be used to estimate the effects of control strategies, e.g. shading, tip removal.

This is joint work with Alex James, Dave Kelly, Shona Lamoureaux, Graeme Bourdôt.

Samin Aref

(University of Auckland, sare618@aucklanduni.ac.nz)

Map of New Zealand scientific collaborations

Scientific collaborations are among the main enablers of development in small national science systems. Although analysing scientific collaborations is a well-established subject in scientometrics, evaluations of collaborative activities of countries remain speculative with studies based on a limited number of fields or data too inadequate to fully represent collaborations at a national level. This study provides a unique view on the collaborative aspect of scientific activities in New Zealand.

I present the results of a quantitative study based on all Scopus publications in all subjects over a period of 6 years. This research is undertaken to gain insight on the collaboration ties of over 1500 New Zealand institutions which are quantified using a measure based on the number of joint publications.

The data containing thousands of Scopus affiliation IDs was categorised into standard institution names and classes to define nodes of a scientific collaboration network where collaboration ties are represented by weighted edges. This network is unique in its representative capability for research collaborations at a national level.

We observe a power-law distribution indicating that a small number of New Zealand institutions account for a large proportion of national collaborations. The network analysis results reveal the level of collaboration between New Zealand institutions and business enterprises, government institutions, higher education providers, and private not for profit organisations in 2010-2015. Network centrality measures are deployed to identify the most influential institutions of the country in terms of scientific collaboration. We also provide comparative results on 15 universities and crown research institutes based on 27 subject classifications.

This study was based on Scopus custom data and supported by the Te Pūnaha Matatini internship program at Ministry of Business, Innovation & Employment.

This is joint work with Shaun Hendy (University of Auckland) and David Friggens (Ministry of Business Innovation and Employment).

[S. Aref, D. Friggens, and S. Hendy. 2017. Analysing Scientific Collaborations of New Zealand Institutions using Scopus Bibliometric Data. preprint arXiv:1709.02897]

Sione Ma'u

(s.mau@auckland.ac.nz)

Weighted pluripotential theory and applications

A central problem of potential theory is to find the equilibrium distribution of electric charge on a body. The presence of an external electric field modifies the equilibrium distribution on the body. The external field can be accounted for mathematically with an admissible weight, which is a lower semicontinuous nonnegative function on the body. Entering admissible weights into the relevant computations, one obtains weighted potential theory.

Pluripotential theory is a nonlinear generalization of potential theory in the complex plane to higher dimensional spaces; putting in admissible weights then gives weighted pluripotential theory. Admissible weights give extra flexibility; by using appropriate weight functions, one can move between spaces. For example:

1) The pluripotential theory of a circled body in $\mathbb{C}^n$, $n > 1$, is related to a weighted pluripotential theory in $\mathbb{C}^{n-1}$. This has been used in induction arguments.

2) For an admissible weight on a set $K \subset \mathbb{C}^n$ that is algebraic, the weighted pluripotential theory is related to the pluripotential theory of a set contained in an algebraic variety. Computations may be more accessible in one setting compared to the other.

Valerie Chopovda

(Massey University, valerie.chopovda@gmail.com)

A periodic family of interplay orbits in the Caledonian symmetric four-body problem

The general planar four-body problem can be simplified by considering the special case of symmetric motion with collinear initial positions and transverse initial velocities. The simple models that occur may aid our understanding of the general problem. In this study, we generate a family of periodic orbits in which the masses perform an interplay motion similar to Schubart-like systems. The linear stability of the orbits is investigated.

This is joint work with Winston Sweatman and Robert McKibbin (Massey University)

Vee-Liem Saw

(University of Otago, VeeLiem@maths.otago.ac.nz)

Helicalised fractals, curved wormholes, time travel, and rotating black hole

We present an overview on the idea of generating n-manifolds of revolution around a curve. This novel idea originated from the following question: "Given a smooth curve, replace it by another curve that winds around it. After this is done, the resulting curve is replaced by yet another one that winds around it, and so on. What will the ultimate curve be?" Well, this has led us to the formulation of helicalised fractals [1]. Alternatively, if one replaces a given curve by a continuous string of circles S^1 [or in general, $(n-1)$-spheres $S^{(n-1)}$], then one obtains a surface [or in general, an n-manifold] of revolution around the curve. A raft of applications of this method has been carried out in General Relativity, viz. designing "safe" curved traversable wormholes [2, 3], constructing new (linearised) vacuum spacetimes allowing for time travel [4], as well as reproducing the spacetime for a slowly rotating black hole in a geometric way [4].

[1] V.-L. Saw and L.Y. Chew, "Helicalised fractals", Chaos, Solitons and Fractals, 75, 191 (2015).

[2] V.-L. Saw and L.Y. Chew, "A general method for constructing curved traversable wormholes in (2+1)-dimensional spacetime", General Relativity and Gravitation, 44, 2989 (2012).

[3] V.-L. Saw and L.Y. Chew, "Curved traversable wormholes in (3+1)-dimensional spacetime", General Relativity and Gravitation, 46, 1655 (2014).

[4] V.-L. Saw, "Constructing vacuum spacetimes by generating manifolds of revolution around a curve", Classical and Quantum Gravity, 33, 065006 (2016).

Winston Sweatman

(Massey University, w.sweatman@massey.ac.nz)

Four-body orbits close by central configurations

We consider four-body orbits close by the four-body central configurations in the case of equal masses. These orbits can be locally approximated using a perturbation from the homothetic collision/expansion orbit.

Xueshan Yang

(University of Auckland, xyan900@aucklanduni.ac.nz)

Dynamics of a coupled calcium system

Many models have been built to study the behaviour of calcium oscillations in spatially homogeneous cells. In these models, all the variables of interest are assumed to evolve in time only, without spatial dependence. However, in a variety of cell types, intercellular calcium oscillations are observed. Furthermore, it has been observed that the calcium dynamics in one cell can influence the dynamics in neighbouring cells. Motivated by this observation, we are interested in building a model of intercellular calcium oscillations to help understand the mechanisms underlying interactions between cells. This poster presents preliminary results about the dynamics of a coupled pair of calcium oscillators, focussing on the qualitative differences resulting from varying the coupling type and strength.

Contributed presentations

Tuesday 5 December, morning session

Geometry and topology **B07**

Keegan Flood

(University of Auckland, kflood99@gmail.com)

Metrics in projective differential geometry

A projective manifold is a smooth manifold equipped with an equivalence class of affine connections that have the same geodesics as unparametrized curves. It is natural to consider whether there is a Levi-Civita connection corresponding to a metric in that projective class. The existence of such a connection is equivalent to the existence of a maximal rank solution to a particular overdetermined projectively invariant differential equation known as the *metrizability equation*. Dropping the rank assumption we see that solutions to this equation, satisfying relatively mild assumptions, yields a metric on an open dense set. The degeneracy locus of this metric is a in fact a smoothly embedded hypersurface which inherits a projective structure of its own. This is related to projective compactification, which realizes the boundary of a manifold with boundary as the projective infinity of a complete metric on the interior. In particular, we will see that our result gives a curved generalization of the projective compactification of Minkowski space.

Michael Lockyer

(University of Auckland, michael.lockyer@auckland.ac.nz)

Mahavier products: graphs and knots

A Mahavier product is a topological structure that originates from ideas in dynamical systems. They have a rich structure, which is currently not well understood.

In this talk I will introduce Mahavier products, and discuss aspects of their structure with reference to Mahavier products that are knots and topological graphs.

Bartek Ewertowski

(University of Auckland, bartek.ewertowski@gmail.com)

Hyperbolic manifolds, group theory, and geometry on the boundary at infinity

Hyperbolic geometry is an exciting area of mathematics, linking differential geometry with relativity, topology, group theory, representation theory, complex analysis, and number theory. In this talk, we will explore projective compactifications of hyperbolic manifolds, and some of the links with group theory. As a corollary of the Killing-Hopf theorem, every complete connected hyperbolic manifold M is a classifying space $B\Gamma$ for its fundamental group $\pi_1(M) \cong \Gamma$, and hence sheaf cohomology of M with local coefficients is isomorphic to the group cohomology of Γ with coefficients in the corresponding monodromy representation. Since hyperbolic manifolds are projectively flat, the induced projective Cartan geometry of M provides a plentiful supply of geometrically meaningful local systems and natural resolutions by vector bundles and differential operators.

Dynamical systems **B09**

Gemma Mason

(The University of Auckland, g.mason@auckland.ac.nz)

Exploring the neighbourhood of a heterodimensional cycle

We investigate a four-dimensional ordinary differential equation model for intracellular calcium dynamics. This model exhibits a heterodimensional cycle. This is a heteroclinic connection between two saddle periodic orbits whose corresponding stable manifolds are of different dimensions. Heterodimensional cycles are a hallmark of partially hyperbolic dynamics, an exciting frontier in theoretical dynamical systems. Observing it in a model with practical applications opens up new connections between applied mathematics and this emerging theoretical knowledge.

This talk will examine some of the mathematical structures that can be observed near this heterodimensional cycle. We consider the stable and unstable manifolds of the two periodic orbits and find indications as to how the heterodimensional cycle could be structurally stable. We find many periodic orbits in the surrounding phase space. For nearby parameters, these periodic orbits undergo saddle-node bifurcations which form spiralling triangular structures in parameter space. We conjecture that there may be infinitely many such spiralling triangles, and perform a detailed analysis of several of them. We can then seek organising structures by examining data about the behaviour in phase space of the associated orbits. These numerical observations have the potential to guide new theoretical developments.

Cris Hasan

(University of Auckland, rhas033@aucklanduni.ac.nz)

Saddle slow manifolds and canard orbits in the Hodgkin–Huxley model

Many physiological phenomena have the property that some processes evolve much faster than others. For example, neuron models typically involve observable differences in time scales. The Hodgkin–Huxley model is well known for explaining the ionic mechanism that generates the action potential (a pulse of the voltage) in the giant squid axon. Rubin and Wechselberger nondimensionalized this model and obtained a singularly perturbed system an explicit time-scale ratio that separates two fast, two slow variables. The dynamics of this system are very complex and feature periodic orbits with a series of action potentials separated by small-amplitude oscillations. Such periodic orbits are also referred to as mixed-mode oscillations. The system features two-dimensional locally invariant manifolds called slow manifolds, which can be either attracting or of saddle type. We introduce a general method for computing two-dimensional saddle slow manifolds as well as so-called canard orbits. Our method is first tested and implemented for a four-dimensional extended normal form of a folded node. We then illustrate how slow manifolds and associated canard obits organize mixed-mode oscillations and determine the firing rates of action potentials in the Hodgkin–Huxley model.

Elle Musoke

(University of Auckland, elle.musoke@auckland.ac.nz)

The geometry of a three-dimensional invariant manifold in a four-dimensional slow-fast system

Neurons, electric circuits and chemical reactions are examples of systems in which some variables change much more slowly than others. When a system has the defining characteristic of variables evolving on different time scales, we say that it is slow-fast. We consider the four-dimensional Olsen model for peroxidase-oxidase reaction that does not have a clear split into slow and fast variables. Fenichel theory guarantees the existence of so-called slow manifolds and associated stable and unstable manifolds that are locally invariant. Previous studies focused on a reduced three-dimensional version of the Olsen model with a one-dimensional saddle slow manifold having a two-dimensional stable manifold. Building on this work, we use numerical continuation methods in conjunction with appropriately defined boundary-value problems to compute two-dimensional submanifolds of the largely unexplored three-dimensional stable manifold of a slow manifold in the full four-dimensional system.

Mathematical physics **B05**

Vee-Liem Saw

(University of Otago, VeeLiem@maths.otago.ac.nz)

Helicalised fractals, curved wormholes, time travel, and rotating black hole

We present an overview on the idea of generating n-manifolds of revolution around a curve. This novel idea originated from the following question: “Given a smooth curve, replace it by another curve that winds around it. After this is done, the resulting curve is replaced by yet another one that winds around it, and so on. What will the ultimate curve be?” Well, this has led us to the formulation of helicalised fractals. Alternatively, if one replaces a given curve by a continuous string of circles S^1 [or in general, $(n-1)$-spheres $S^{(n-1)}$], then one obtains a surface [or in general, an n-manifold] of revolution around the curve. A raft of applications of this method has been carried out in General Relativity, viz. designing “safe” curved traversable wormholes, constructing new (linearised) vacuum spacetimes allowing for time travel, as well as reproducing the spacetime for a slowly rotating black hole in a geometric way.

Graham Weir

(Massey University, grahamweir@xtra.co.nz)

Removing classical singularities by using quantum sources

The classical solutions corresponding to the point sources of an electrical charge and a magnetic dipole in Electromagnetism, and to an isolated mass source in General Relativity, all contain singularities. We investigate the hypothesis that these singularities are unphysical, and can be avoided if the sources are described quantum mechanically by probability functions. By using the quantum mechanical source for a bounded electron, we obtain to leading order, bounded solutions for isolated sources of electrical charge and a magnetic dipole in EM, and for an isolated mass source in GR. In the 'natural GR frame', all non-zero Ricci and Riemann tensor components have the same bounded magnitude at the location of the classical singularity, while the Ricci tensor is exponentially small, far from the isolated mass source. The Equivalence Principle is verified, with the gravitational and inertial masses being equal, and angular momentum of $\hbar$ is encoded from the electron into the gravitational field.

Woei Chet Lim

(University of Waikato, wclim@waikato.ac.nz)

Explicit G_1 spike solutions in general relativity

We present a family of new explicit solutions of Einstein's field equations with stiff fluid. They admit one spacelike Killing vector field, and are generated by applying Stephani's transformation (i.e. stiff fluid version of Geroch's transformation) on Bianchi type V spatially homogeneous solutions. The most interesting phenomenon exhibited is the intersecting spikes near the initial singularity.

Algebra and number theory B11

Shaun Cooper

(Massey University, s.cooper@massey.ac.nz)

The Ramanujan–Mordell theorem for sums of squares, and some extensions

Jacobi's sum of four squares theorem states that for any positive integer n, the number of solutions in integers of the equation

$$x^2 + y^2 + z^2 + w^2 = n$$

is equal to eight times the sum of the divisors of n that are not multiples of 4. It implies Lagrange's theorem that every positive integer is a sum of four squares. Jacobi's theorem can be stated in the analytic form

$$\left(\sum_{j=-\infty}^{\infty} q^{j^2}\right)^4 = 1 + 8\sum_{j=1}^{\infty}\left(\frac{jq^j}{1-q^j} - \frac{4jq^{4j}}{1-q^{4j}}\right)$$

where q is a complex variable with $|q| < 1$. There are analogous results for sums of 2, 6 and 8 squares, all due to Jacobi, and there are also results for 10, 12, 14, 16 and 18 squares due to Eisenstein, Liouville and Glaisher. A general result for sums of an even number of squares was stated by Ramanujan and proved by Mordell.

This talk will survey these results and present some extensions which are new.

Roger Su

(University of Waikato, rcs14@students.waikato.ac.nz)

A program algebra with non-halting tests

An algebraic theory of computer programs with the if-then-else and while-do constructs is not only of great mathematical interest in its own right, but it also enables us to reason about programs in an rigorous equational manner.

Among various schools, Jackson and Stokes have devised a theory where programs are viewed as elements of a semigroup, and the logical tests in the conditional statements are assumed to always halt and hence form a Boolean algebra.

In order to consider non-halting tests, the simple Boolean 2-valued logic has been generalised into a non-commutative 3-valued logic. This 3-valued logic corresponds to what is called a C-algebra, and it models the sequential valuation strategy in most programming languages, where 'non-halting or true' evaluates to 'non-halting', for example.

My research looks at an alternative generalisation into a commutative 3-valued logic, where 'non-halting or true' evaluates to 'true'. This talk will present a historic account of various 3-valued logical systems, and then show how an existing semigroup representation is extended to capture the algebraic properties of the if-then-else operation with these 3-valued tests.

Geoffrey Booth

(Nelson Mandela University, geoff.booth@nmmu.ac.za)

On primeness in near-rings of continuous functions

The set $M(G)$ of mappings of a (not necessarily) abelian group $(G, +)$ into itself is a near-ring with respect to pointwise addition and composition of functions, and in fact is the prototype for near-rings. Likewise the subset $M_0(G)$ of zero preserving mappings gives the prototypical zero-symmetric near ring. These near-rings and certain subnear-rings of them have been extensively studied since near-rings were first introduced over a century ago.

Since the 1970's studies have been made of near-rings of continuous functions of a topological group. If $(G, +)$ is a topologial group, then $N(G)$ and $N_0(G)$ are the subnear-rings of $M(G)$ and $M_0(G)$, respectively which consist of the continuous functions. Unsurprisingly, these near-rings exhibit substantial differences from $M(G)$ and $M_0(G)$, and their structure depends strongly on the topology on G.

Primeness has been extensively studied for near-rings, and several generalisations exist in the literature of the concept for associative rings. In this talk we will outline some results obtained for primeness in $N(G)$ and $N_0(G)$, and for the associated radicals.

Tuesday 5 December, afternoon sessions

Numerical analysis **B07**

Mohamed Al-Sultani

(Massey University, m.al-sultani@massey.ac.nz)

Monotone numerical methods for solving nonlinear parabolic problems

Partial differential equations (PDEs) are used to characterize a wide family of problems in chemistry, physics and egnineering sciences. In general, exact solutions of PDEs are not easily to be computed or not computable, efficient numerical methods are required to find approximate solutions. The method which is considered in the talk is the method of upper and lower solutions. Theorems on the existence and uniqueness of solutions to nonlinear parabolic problems and their discrete approximations are presented. Convergence properties of monotone iterative methods based on the approach of upper and lower solutions are discussed. Numerical experiments, which confirm the theoretical results, are presented.

Christian Offen

(Massey University, c.offen@massey.ac.nz)

Bifurcation of solutions to Hamiltonian boundary value problems

Singularity theory or catastrophe theory has been attracting researcher for more than half a century. Nowadays, a well-developed framework is available to analyse which bifurcations occur in gradient-zero-problems for families of smooth scalar valued map germs. In this talk I will introduce a geometric picture involving the intersection of Lagrangian submanifolds to analyse how and to which extend boundary value problems in Hamiltonian systems fit into the framework of singularity theory. Moreover, I will analyse how conserved quantities and symmetries lead to new bifurcation phenomena.

Dynamical systems **B09**

David Simpson

(Massey University, d.j.w.simpson@massey.ac.nz)

The sausage-string structure of mode-locking regions of piecewise-linear maps

Mode-locking regions are subsets of parameter space where a dynamical system is entrained to a fixed frequency or rotation number. In two-parameter bifurcation diagrams they appear as narrow regions ordered by rotation number. For piecewise-linear maps they have pinch points, called shrinking points, and an overall structure that resembles a string of sausages. This has been identified in models of diverse systems, including power converters, neurons, and economics, and remains incompletely understood. In this talk I will explain how each shrinking point organises the bifurcation structure locally. A handful of key scalar quantities assigned to a shrinking point govern the relative size, properties, and arrangement of nearby mode-locking regions. In sectors radiating from a shrinking point, periodic, quasiperiodic, and chaotic dynamics are accurately captured by a one-dimensional skew sawtooth map

Andrus Giraldo

(the University of Auckland, agir284@aucklanduni.ac.nz)

How chaos arises from a homoclinic flip bifurcation

Studying the qualitative dynamics of a system of differential equations is an important tool for comprehending the behaviour of natural phenomena. Of particular interest are mechanisms that cause a system to exhibit chaotic behaviour. It is well know that homoclinic bifurcations are organizing centres for the creation of such chaotic dynamics. This talk is about a specific homoclinic bifurcation called homoclinic flip bifurcation of case C, where the associated two-dimensional invariant manifold of a saddle equilibrium with real eigenvalues changes from being orientable to nonorientable. This bifurcation is of codimension two, meaning that it can be found as a bifurcation point on a curve of homoclinic bifurcations in a suitable two-parameter plane. The homoclinic flip bifurcation of case C is, in fact, the homoclinic bifurcation of a real saddle of the lowest codimension that generates chaotic behaviour in the form of (suspended) Smale horseshoes and strange attractors.

We present in this talk a detailed study of how global stable and unstable manifolds of the saddle equilibrium and of bifurcating periodic orbits interact in the unfolding of the homoclinic flip bifurcation of case C. This represents the next and final step in understanding the generic cases of homoclinic flip bifurcations, which started with the study of the simpler cases A and B. Our computations confirm what is known from theory but also show the existence of different phenomena that were not considered before. Specifically, we identify the boundaries of the Smale–horseshoe region in the parameter plane, one of which creates a strange attractor of the Rossler type. A computation of a winding number reveals a complicated overall bifurcation structure that involves infinitely many further homoclinic flip bifurcations associated with so-called homoclinic bubbles.

This is joint work with Professors Bernd Krauskopf and Hinke M. Osinga.

Mathematical biology **B05**

David Waters

(University of Auckland, d.waters@auckland.ac.nz)

Calcium signalling in T-cells

Calcium oscillations are ubiquitous in many cell types and control a wide range of cellular activities. Particularly, in T-cells, calcium oscillations are known to control cell differentiation, proliferation, and activation. Very puzzling experimental results from the 1990s showed that blocking certain calcium pumps initiated calcium oscillations. This behaviour is odd, and fairly unique to T-cells, yet remains without theoretical explanation. We construct and study theoretical models of T-cell calcium signalling in an attempt to provide an explanation. We study the bifurcation structure of our models, which allows us to describe a theoretical mechanism for this behaviour.

Catherine Hassell Sweatman

(Auckland University of Technology, catherine.sweatman@aut.ac.nz)

Mathematical model of diabetes and lipid metabolism

Currently there is great debate on the relative merits and dangers of fats and sugars in our diet. How can we avoid insulin resistance, gaining weight and Type II diabetes? How does changing the balance between dietary carbohydrates and lipids aect health markers such as plasma glucose and triglyceride levels? In order to investigate such questions a mathematical model of glucose, insulin, glucagon, β-cell, leptin and fat dynamics and hepatic, peripheral and adipose insulin sensitivity is presented and the steady state behaviour is investigated. The current model includes plasma non-esterified fatty acids, muscle lipids, hepatic lipids and very low density lipoprotein triglycerides, allowing comparisons with recent health research.

Applied differential equations and modelling **B11**

Fabien Montiel

(University of Otago, fmontiel@maths.otago.ac.nz)

Recent advances in modelling ocean waves/sea ice interactions

Sea ice plays an important role in the global climate system, being a key driver of oceanic and atmospheric circulations and controlling a large portion of Earth's reflectivity to sunlight radiations. Yet much of its dynamics is poorly understood and its response to global warming is not captured appropriately by climate models. It is hypothesized that unresolved feedback mechanisms are responsible for this discrepancy. Under the impulse of the Deep South National Science Challenge, NZ is currently developing a new Earth system modelling capability that will resolve the Antarctic sea ice system with unprecedented fidelity. Particular foci are to better represent sea ice interactions with (i) ice shelves at its inner margin and (ii) the open ocean at its outer margin, the so-called marginal ice zone (MIZ). Process-informed modelling of these interactions, supplemented by field observations, are currently being undertaken as part of a 5-year Deep South-funded programme, which is led by the University of Otago.

In this talk, I will discuss recent work on modelling MIZ dynamics. I propose a new theoretical model describing the two-way coupling between ocean waves and the MIZ. I will show that the model is able to simulate a positive wave/ice feedback, though which waves gradually march forward in the ice-covered ocean by breaking up the sea ice cover.

Bruce van Brunt

(Massey University, b.vanbrunt@massey.ac.nz)

On a coagulation-fragmentation equation with exponential growth

In this talk we discuss a certain coagulation-fragmentation partial differential equation that provides a simple model for size structured cell division when the growth is exponential. For linear growth it can be shown that the large time solutions converge to the separable solution, called the "steady size distribution". For exponential growth, however, the long time behaviour of solutions changes markedly. We show that the solutions are asymptotic to a solution that contains a factor which oscillates in time and is dependent on the initial data.

Wednesday 6 December, morning session

Mathematical education I **B07**

Peter Radonich

(University of Auckland (STEM online)/Northcote College, peterr@northcote.school.nz)

STEM Online NZ — a free interactive teaching and learning resource for NCEA external standards in STEM subjects

STEM Online NZ is a free interactive teaching and learning resource that currently covers Mathematics level 1 Tables, Equations and Graphs and Algebra as well as Physics Mechanics level 1 and 2. The aim is to increase the number of secondary school students successfully completing NCEA external standards in STEM (Science, Technology, Engineering and Mathematics) subjects, starting with Mathematics and Physics.

During this presentation you will be introduced to the online learning materials your students would use if you choose to use the resource in your school. The University of Auckland is responding to a New Zealand-wide specialist teacher shortage by harnessing technology in an innovative new way to compensate for STEM teaching shortages. These highly interactive online teaching and learning resources will help teachers, particularly those who are not subject specialists, teach STEM subjects. They do not replace teachers, but are designed to support teachers and engage students with content that is relevant, contextual and exciting.

Subash Chandar K

(Ormiston Senior College, schandark@ormiston.school.nz)

The journey to loving maths

In 2017 the Mathematics & Statistics department at Ormiston Senior College made some major changes with the delivery of the curriculum. There was personalized pathways for NCEA Level 1 students, a shift from traditional forms of assessment where students were assessed when they felt they were ready with the use of Sphero robots, Desmos, Lego & Knex. Desmos played a key part in the development of learning tasks and assessment for Level 2 students. Level 3 students shifted away from reports to presentations in two statistics standards this year. The full time teaching staff in the department were encouraged and enabled to have their YouTube channels to provide personalized resources for students. Some of the ideas on how we shifted will be discussed in this session.

Andrew Ricciardi and Damian Campbell

(Waimea College, andrew.ricciardi@waimea.school.nz and damian.campbell@waimea.school.nz)

Growing pains: Implementing innovative ideas in a traditional environment

For the last two years we have been working on a project through the Teacher Led Innovation Fund. We are three full-time secondary mathematics teachers that have been working with the inquiry of, "Does a blended e- Learning environment foster a growth mindset among students?" Our talk will look at the positives and pitfalls that we have encountered along the way. We can speak from experience about the fallacies and fundamentals of flipped learning. We have learned that the "growth mindset" approach is one of the most misused and overstated concepts in education, yet a powerful approach if done correctly. How has allowing student agency to learn at their own pace changed the way that we teach? How has focusing on effective feedback allowed students to gain confidence? We will speak about implementing change within a department, our school and a Community of Learning. We have done the hard yards and would like to share with you our success and mistakes (that we have learned from) along the way.

Karyn Saunders

(Victoria University of Wellington, karyn.saunders@vuw.ac.nz)

Ako in practice in mathematics teaching and learning

The stubborn disparity between Māori and non-Māori achievement in the New Zealand English-medium secondary education system sees a disproportionate number of Māori students underachieving in this context. Cultural misalignment between teachers and students can contribute to this issue. Ka Hikitia is a strategy document, which guides teacher actions to make a positive difference for Māori students in the New Zealand education system. The principles within this strategy underpin cultural competency indicators as expressed in Tātaiako, a guide for teachers of Māori learners. Both research and policy advocate the inclusion of culturally responsive competencies (e.g., ako) in teacher practice to increase Māori student achievement. Research shows students are able to provide insight and workable solutions to issues within their own education. However, there has been little investigation into student perspectives of the types of teacher behaviour indicators that promote ako in teacher practice, particularly in secondary and tertiary mathematics, a subject crucial for optimising career and study options.

This talk describes a large study examining how a non-Māori teacher-as-researcher increased cultural alignment with their students in an English-medium secondary school mathematics classroom, increasing student engagement and achievement. A set of behavioural indicators of ako in teacher practice were synthesised from literature, and used as the basis of discussions with students and whānau in cogenerative dialogue sessions. Data collected through surveys, self-study methods with the support of local cultural advisors, PAT and topic testing were used alongside cogenerative dialogue data to understand teacher behaviours which can enact ako, and which take into account the world views and experiences present in their local context.

The presenter will outline initial study findings, and provide practical ideas, such as ways to find out how students really feel about their learning, that educators could use to reflect ako in their mathematics teaching.

Mathematical education II **B09**

Julia Crawford

(University of Auckland, julia.crawford@auckland.ac.nz)

School mathematics and NCEA: understanding how it works

NCEA has provided schools, teachers and students incredible flexibility. How does this flexibility effect the learning and experience of mathematics at school? This information session is for mathematicians who would like to know more about NCEA, how it works, and what impacts this could have on the nature of undergraduate students.

Julia Novak and Cami Sawyer

(The University of Auckland, j.novak@auckland.ac.nz and
Massey University, c.sawyer@massey.ac.nz)

First Year in Mathematics (FYiMaths) at University, the challenges faced by lecturers and students in an ever-changing landscape

(Part 1) First Year in Maths (FYiMaths) is a network of mathematicians teaching in universities in Australia and New Zealand. The network was established in Australia more than five years ago and there is a newly established New Zealand subgroup. The goal of the group is to improve outcomes for students in undergraduate mathematics courses by sharing and developing teaching practices and building connections between secondary schools and universities. In this session we will have discussions exploring the issues faced by students, secondary and tertiary teachers.

Julia Novak and Cami Sawyer have established the NZ group of FYiMaths because they are interested in furthering conversations around teaching and learning. Julia is a professional teaching fellow at the University of Auckland and the Associate Dean (Teaching and Learning) for the Faculty of Science. Cami is a senior tutor at Massey University.

(Part 2) Continuing on from the discussions in Part 1, please join us as we consider key points and concerns raised in Part 1 and how we currently address these and ways we can aspire to attend to them in the future.

Peter Bier and Julia Novak

(Engineering Science, University of Auckland, p.bier@auckland.ac.nz and
Mathematics, University of Auckland, j.novak@auckland.ac.nz)

Academics anonymous—Bring and share session

Come along to a supportive environment, prepared to share for 1–2 minutes about an idea or experience relating to your teaching of tertiary mathematics. This could be something that worked well for you or something that bombed. Participants would love to beg/borrow/steal your good ideas and will also learn a lot from things that didn't pan out how you hoped. Remember we are all in this together, so sharing failures can be just as valuable as sharing success stories and sometimes has more impact!

Mathematical biology **B05**

Faheem Zaidi

(Massey University, f.zaidi@massey.ac.nz)

Optimal amplitude and frequency of breathing

Physiological levels of oxygen and carbon dioxide in the blood are tightly regulated by varying the amplitude and frequency of breathing, but this can be achieved with different combinations of amplitude and frequency. Why a specific combination of amplitude and frequency is observed remains a mystery. The aim of this study is to explore the hypothesis that the particular combination realised is optimal with respect to some objective function. Several objective functions have been suggested in the literature, such as the rate of work during inhalation, the average force exerted by the respiratory muscles, and the weighted sum of volumetric acceleration and work during inhalation; all of these objective functions provide physiologically acceptable minima under normal conditions. Resolving this issue requires optimal solutions of mathematical models that reflect more accurately the complex interaction between lung mechanics and gas exchange, but this in turn requires the development of new computational methodologies. To help achieve this goal, we constructed a simple mathematical model, consisting of two piecewise linear differential equations, that mimics gas exchange in the lungs. By using concepts from optimal control theory, we found the necessary conditions that minimise a given objective function subject to several constraints, such as satisfying the differential equations and maintaining one of the variables at a given average value. We could then solve the optimal control problem both analytically and numerically. Our methodology can be extended to models with higher dimensions.

Thomasin Lynch

(Massey University, T.A.Lynch@massey.ac.nz)

Modelling fermentation processes in the rumen

Methane production from fermentation processes that occur in the rumen of animals such as cattle and sheep is one of the major sources of anthropogenic greenhouse gas emissions. In this talk, we present a mathematical model created to allow for testing of developing knowledge of enteric fermentation. The model depicts the interaction between microbial populations in the rumen (glucose fermenters and methanogens), their feed substrates (glucose and hydrogen), and the end products of their fermentation processes (such as volatile fatty acids). Thermodynamic control of substrate concentration on the rates of substrate metabolism and hydrogen and volatile fatty acid generation is included. The introduction of thermodynamic control allows for co-existence of microbes that use the same substrate but different fermentation pathways. It also allows for selection of microbes and fermentation pathways based on rumen environment.

Jack Simpson

(University of Canterbury, jrs149@uclive.ac.nz)

Combining phylogenetic trees into networks: What can be done with just two trees?

The study of evolution has been typically conceptualized in simple mathematical structures known as trees. However, it has become increasingly clear that not all evolutionary events can be expressed by trees alone. More complicated non-tree-like evolutionary events, such as hybridization and horizontal gene transfer, demand a more general structure that can express the combination of different trees. Networks are the natural mathematical extension of trees but they come at the cost of the 'nice' tree structure. What is wanted is a network flexible enough to describe all observed evolutionary relationships but still confined by the important properties of trees. In 2015 Francis and Steel introduced the important class of phylogenetic networks known as tree-based networks. A network is tree-based if it can be constructed from an underlying tree by adding only edges between the tree edges. It is know that every vertex in a tree-based network is covered by an underlying base tree. In this talk, I will look at how many trees are required to cover every edge in a tree-based network. Then I will explore what can be done with the sub-class of tree-based networks in which every vertex and edge can be covered by precisely two trees.

Nathan Pages

(University of Auckland, natan.pages@gmail.com)

Anatomically-accurate three-dimensional model of salivary fluid secretion

We constructed a anatomically-accurate three-dimensional model of salivary fluid secretion from a cluster of parotid gland acinar cells.

Parotid acinar cells are responsible for the secretion of saliva. Olfactory and gustatory stimuli provoke the release of agonists that bind to the basal membranes of the acinar cells. This triggers a cascade of events that results in the production of IP3, which, in turn, releases calcium ions from intracellular compartments. Upon release of calcium, chloride channels are activated, leading to chloride ions flowing out of the cell, and water following by osmosis.

The major current question in the modelling of saliva production is whether or not the complex spatial structure of the acinus, or the spatial heterogeneities within each acinar cell, have any significant effect on the rate at which saliva is secreted.

We will discuss how we built a computational model to answer this question. Preliminary results suggest that, contrary to current dogma, the spatial structure of the acinus is an important factor for optimising fluid flow.

Analysis **B11**

Jesse Hart

(University of Auckland, drjessehart@outlook.com)

Transfinite diameter on affine algebraic varieties

The diameter of a compact set $K \subset \mathbb{C}$ is well known to be the maximal distance between two points that lie within the set. One can generalise this notion to the n-diameter by maximising the geometric mean of the distances between n points that lie within the set. Fekete introduced and defined the transfinite diameter for a compact set K by taking the limit of n-diameters for the set as $n \to \infty$. The transfinite diameter is a means to quantify the 'spread' of a set. Spread is a notion which naturally arises in nature such as in the study of capacitors – collections of electrons stored in a capacitor will distribute themselves throughout the capacitor in such a way that maximises their mutual distance. Defining the transfinite diameter on an affine algebraic variety poses some challenges. Cox-Ma'u and Berman-Boucksom give two different approaches this problem. This talk will explain the difficulties in defining the transfinite diameter on an algebraic variety, explore the two approaches and show that the approaches are equivalent up to a scale factor depending only on dimension.

Chris Wong

(University of Auckland, mwon535@aucklanduni.ac.nz)

Elliptic operator with Robin boundary conditions

Second-order strongly elliptic operators in divergence form with real measurable bounded coefficients, subject to Dirichlet, Neumann or mixed boundary conditions, are well studied on a bounded Lipschitz domain Ω. The kernel of the associated semigroup satisfies Gaussian kernel bounds. Recently it was proved that the kernel is even Hölder continuous with the appropriate Hölder Gaussian bounds. The situation is different if the operator has Robin boundary conditions $\partial_\nu u + \beta\, u = 0$, where $\beta \in L_\infty(\partial\Omega)$. If $\beta \geq 0$, then Gaussian kernel bounds exist with a differentiability condition. However no Gaussian bounds are known if the condition $\beta \geq 0$ is dropped and no Hölder Gaussian bounds are known if $\beta \neq 0$.

In this talk, we show that the kernel has both Gaussian and Hölder bounds for any $\beta \in L_\infty(\partial\Omega)$, even for complex valued lower coefficients.

Gareth Gordon

(University of Auckland, g.gordon@auckland.ac.nz)

The Dirichlet-to-Neumann operator on a Lipschitz hypograph

The Dirichlet-to-Neumann operator is an operator on the boundary of some domain $\Omega \subseteq \mathbb{R}^d$ that maps boundary value (Dirichlet) data to the normal derivative (Neumann) data of solutions to an elliptic PDE. Usually, the domain Ω considered is a bounded Lipschitz domain.

In this talk we study the Dirichlet-to-Neumann operator on a Lipschitz hypograph. In particular, we present convergence properties of the Dirichlet-to-Neumann operator resolvents and harmonic lifting operators on a sequence of approximating domains.

Wednesday 6 December, afternoon sessions

Mathematical Education I **B07**

Graeme Wake

(Massey University, g.c.wake@massey.ac.nz)

The graveyard of geometry (a brief sample of)

A largely forgotten piece of classical, but very useful, geometry will be described: that of Steiner symmetrisation. This is a very elegant piece of quite simple mathematics. Recently it has been very successfully applied to several important optimisation problems for things like bubbles, drums and bombs. It is used by myself and others in industry to predict shape factors as a measure of the likelihood of the occurrence of fires, see Wake (1973). This prompted us to develop an extension of Jacob Steiners work, which will be explained. I have also used these ideas as a basis for talks in Schools, which I do frequently. A short puzzle competition will be run as part of the talk with an attractive prize for the first correct answer.

[Wake GC, "Estimation of critical parameters in thermal ignition", Combustion and Flame J. Combustion Institute, Vol 21, (1973) p19-21]

Chris John

(drcmjohn@gmail.com)

Case study: developing an engaging, discursive, active and reflective classroom

Learning mathematics is largely viewed as a sedentary activity and the public image of a mathematician is often that of a introverted figure lost in thought, working alone. This stereotype may in some way contribute to the general model of teaching and learning of Mathematics in classrooms today. In this case study I reflect on five years of developing a traditional classroom by simple and familiar methods requiring only cheap and readily-available materials in an effort to maximize student engagement, enjoyment and ultimately academic progress. Unlike some talks on pedagogy or learning resources, attainment data from the period in question is shared, as an accompaniment to the methods deployed.

James Sneyd, Peter Bier and Peter Radonich

(Maths and Eng Science, University of Auckland and UoA STEM online/Northcote College
sneyd@math.auckland.ac.nz, p.bier@auckland.ac.nz, and peterr@northcote.school.nz)

Modelling in the mathematics curriculum... an impossible ideal?

Three teachers/lecturers of mathematics will share and discuss their experiences of incorporating real world contexts and modelling in their mathematics classes:

- Peter Bier, an engineering scientist, on running annual full-day modelling competitions for secondary school students, nationwide.
- Peter Radonich, a mathematics teacher, HOD and designer, on using modelling contexts for NCEA level 1, Tables Equations and Graphs in the STEM online NZ resource and in his classroom.
- James Sneyd, a mathematical biologist, on developing a new first year mathematics course for scientists, teaching algebra and calculus without x's and y's

This session will discuss the place of modelling and real world contexts in the New Zealand curriculum, which currently states: "In a range of meaningful contexts, students will ... solve problems and model situations that require them to ... "

Cami Sawyer and the NZMS Education Group

(Massey University, c.sawyer@massey.ac.nz)

Working towards a shared vision of school mathematics

The NZMS Education Group aims to advocate for the enhancement of mathematics education in New Zealand. In this session we will discuss the future landscape and work towards developing a shared vision of school mathematics. Some questions we will consider:

- What is the purposes of school mathematics education?
- How can we promote an equitable common mathematics pathway?
- What are the essential mathematical habits of mind or thought processes that all students should practice?
- Can we identify essential concepts that all high school students should learn and understand at a deep level?
- How should changes in technology be reflected in high school mathematics?

Sze Looi Chin

(University of Auckland, schi642@aucklanduni.ac.nz)

Characteristics of identifying narratives about struggling in mathematics

When students talk about their experiences of studying mathematics, they usually talk about three topics: (i) about mathematics, (ii) about themselves and (ii) about other people in the classroom. These three topics are often intertwined into a complex series of utterances which become narratives we tell about ourselves. For students who frequently struggle in mathematics, utterances such as "I'll never be a maths person" and "I just so sick of algebra!" become routine when mathematics is mentioned. These are examples of what Sfard (2008) describes to be utterances which form an identity; a collection of reifying, significant and endorsable narratives about a person.

In this study, I utilize Sfard's (2007) commognitive framework to examine the identifying narratives that three secondary school students create while reflecting on their mathematics classrooms and during two mathematics workshops. In these two workshops, Sara, Ace and Jade, are asked to describe their participation in their classrooms, work collaboratively on two mathematics task, and then reflect on their mathematical thinking. From these narratives that the students create, I examine how they talk about their routines in engaging with the mathematics and the mechanisms that they use to create narratives about themselves. Using fine-grained discourse analysis, I determine a collection of characteristics from these narratives which then reveal the students routines for identifying themselves.

In this talk, I will present excerpts from one students collection of narratives about her repeated experiences of struggle and her attempts to resolve this struggle. I will discuss the characteristics of her identifying narratives and the implications of the mechanisms she uses to tell them.

Iresha Ratnayake

(The University of Auckland, iresha.ratnayake@auckland.ac.nz)

Teaching with digital technology: Factors influencing secondary mathematics teachers' task development and implementation

A crucial step towards improving the conceptual use of digital technology (DT) in the mathematics classroom is to increase teacher involvement in the development of tasks. Hence, my PhD research considers some teacher factors that might influence DT algebra task development and implementation in secondary schools. In order to consider the relationship between teacher resources and task development I employ the theoretical framework of Pedagogical Technology Knowledge (PTK) (Thomas & Hong, 2005). The focus of PTK is to place the enhancement of mathematical thinking at the centre of the processes of design and implementation of tasks, and hence consider mathematics principles, conventions, and techniques required to teach mathematics through the technology. The research was designed with a professional development (PD) intervention and observed and assisted four groups of three teachers as they designed and implemented their own DT tasks. The PD intervention carried out in this research was based on theoretical features of rich DT mathematical task, lesson planning for implementation of the tasks and a student-centred three point framework focussing on students potential confusions and preventative courses of action. The results show that this PD helped to improve significantly the richness of the tasks designed by each group and they produced more student-centred tasks. Further, it identified some factors contributing to this improvement, including the importance of a confident attitude to teaching with DT and the nature of the collaborative communities of inquiry (Jaworski, 2003, 2006) that they constructed.

Supervisors: Emeritus Professor Michael O. J. Thomas
Dr. Barbara Kensington-Miller

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: Towards a theoretical framework based on co-learning partnership. *Educational Studies in Mathematics, 54*(2), 249-282. doi: `10.1023/B:EDUC.0000006160.91028.f0`

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching *Journal of Mathematics Teacher Education, 9*(2), 187-211. doi: `10.1007/s10857-005-1223-z`

Thomas, M. O. J., & Hong, Y. Y. (2005). Teacher factors in integration of graphic calculators into mathematics learning. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 257-264). Australia: University of Melbourne.

John Griffith Moala

(The University of Auckland, john.moala@auckland.ac.nz)

How do students justify the optimality of their solutions in a contextualized discrete optimization task?

In their perfectly named paper, "The Complement of Research in Undergraduate Mathematics Education", Speer and Kung (2016) identified optimization as one of the topics that needs more research attention. Students understanding of optimization problems is currently a not-very-well understood phenomenon. The majority of the little research that has been conducted focuses on continuous optimization (e.g., Bremigan, 2005; Larue, 2015; White & Mitchelmore, 1996; Whiteley & Mamolo, 2012). These researchers have shed some light on important issues such as: the aspects of learners concept images that influence their understanding of optimization; the role of intuition and rigor in the solving of optimization problems; the difficulties that students have with formulating the optimizing function (i.e., the function to be optimized) and the processes by which students justify the optimality of a result. However, the ever-growing importance of discrete mathematics—"the math of our time" according to Dossey (1991)—suggests the need to also look at how students learn and understand discrete optimization problems. It is unreasonable to assume that the findings from the research on continuous optimization will apply to the discrete setting. In this talk I will give a brief overview of the existing literature on optimization in mathematics education, but focusing more on discrete optimization. I will discuss some preliminary findings that have started to emerge from the analyses of data gathered from the work of several groups of students on a contextualized discrete optimization task. I will highlight some of the difficulties that the students faced during their work, particularly when they attempted to examine whether or not a feasible solution was optimal.

Tanya Evans

(University of Auckland, t.evans@auckland.ac.nz,)

Improving graduate attributes of students: researching the impact of regular use of non-routine problem solving on creativity and general thinking skills of STEM students

In this session we will give an overview of a project aimed at incorporating non-routine problem solving into university courses by introducing collaborative small team problem solving activities during traditional lectures. The non-routine problems that were used included puzzles, paradoxes and sophisms (PPS). The impact of the regular use of non-routine problems was evaluated as a pedagogical strategy to enhance generic problem-solving and thinking skills of tertiary STEM (Science, Technology, Engineering and Mathematics) students. A significant number of tertiary STEM students drop out from their study during the first-year not because the courses are too difficult but because, in their words, they are too dry and boring. Terms such as emotional disengagement and academic disinterest have even been associated with STEM courses. The intention of using PPS in teaching/learning is to engage students' emotions, creativity and curiosity and also enhance their critical thinking skills and lateral thinking outside the box. The theoretical considerations of the project were based on the Puzzle-Based Learning concept that has become increasingly popular worldwide. The impact of this pedagogical strategy was evaluated via comprehensive questionnaires, interviews and class observations involving 137 STEM students from four groups at AUT and the University of Auckland. The vast majority of the participants reported that the regular use of PPS helped them to enhance their problem-solving skills (91%) and generic thinking skills (92%). Moreover, 82% of the participants commented on other benefits of this pedagogical strategy. After analysing the observed overwhelmingly positive students attitudes we suggest that there is a need for further and more rigorous investigation of the suggested pedagogical strategy.

This is joint work with Mike Thomas and Sergiy Klymchuk (PI of project), University of Auckland.

Igor' Kontorovich and Sina Greenwood

(University of Auckland, i.kontorovich@auckland.ac.nz and, sina@math.auckland.ac.nz)

Inspiring future researchers: A partnership between a mathematician and a math educator

Cross-disciplinary collaborations may not be easy to establish and maintain, but the many points of tangency between the mathematics and the mathematics education communities promise otherwise. Having observed that students prefer discussing mathematical ideas to being told them, Sina set out to require her graduate students to collaborate in developing theory for her courses, and present their findings. She partnered with Igor to examine her approach with a view to enhancing students' engagement with topology. In the talk, we will reflect on different stages that our year and a half-long partnership went through, and the gains and potentials of the partnership in terms of research and teaching practice.

Topology/DS and Combinatorics **B05**

Sina Greenwood

(University of Auckland, sina@math.auckland.ac.nz)

2-manifolds and inverse limits of set-valued functions on intervals

Suppose for each $n \in \mathbb{N}$, $f_n\colon [0,1] \to 2^{[0,1]}$ is a function whose graph

$$\Gamma(f_n) = \big\{(x,y) \in [0,1]^2 \colon y \in f_n(x)\big\}$$

is closed in $[0,1]^2$ (here $2^{[0,1]}$ is the space of non-empty closed subsets of $[0,1]$). We show that the generalized inverse limit

$$\varprojlim(f_n) = \big\{(x_n) \in [0,1]^{\mathbb{N}} \colon \forall n \in \mathbb{N},\ x_n \in f_n(x_{n+1})\big\}$$

of such a sequence of functions cannot be an arbitrary continuum, answering a long-standing open problem in the study of generalized inverse limits. In particular we show that if such an inverse limit is a 2-manifold then it is a torus and hence it is impossible to obtain a sphere.

Christopher Tuffley

(Massey University Manawat, c.tuffley@massey.ac.nz)

Characterisation and classification of signatures of spanning trees of the n-cube

The signature of a spanning tree of the n-dimensional cube Q_n is the n-tuple recording the number of edges of the tree in each of the n directions of the cube. We use Hall's Theorem to characterise the n-tuples that can occur as a signature, and classify signatures as reducible and irreducible, with the reducible signatures being further divided into strictly reducible and quasi-irreducible signatures. The classification of a signature has structural implications for the set of spanning trees with that signature.

Joint work with my (now finished) PhD student Howida al Fran and co-supervisor David Simpson.

Applied differential equations and modelling **B11**

Andrea Babylon

(Massey University (Albany), A.Babylon@massey.ac.nz)

Modelling leptospirosis in livestock

Leptospirosis is a zoonotic infectious disease resulting from a bacterial infection. Humans can become infected via contact with material contaminated by infectious animals, or via contact with the infectious animals themselves. Symptoms are usually flu-like and result in an average of six weeks absence from work. It is the highest occurring occupational disease in New Zealand, with between 80 and 180 cases per year, 60% of which result in hospitalisation. Studying the dynamics of infection and possible control measures for the disease in animals is an important area of research in terms of minimising exposure to humans.

Mathematical models of leptospirosis in livestock will be presented. The models show the dynamics of the spreading of infection in sheep, and the interaction between lambs and ewes. The models are cyclical in nature, with the system reset to an initial condition at the beginning of each time period. The limit cycles and bifurcation diagrams of the systems will be presented and conditions under which the disease will persist in the population are predicted.

Valerie Chopovda

(Massey University, valerie.chopovda@gmail.com)

Families of planar periodic orbits generated from the Schubart-like orbit in the Caledonian four-body problem

We consider the special case of the general planar gravitational four-body problem with motion symmetrical about the centre of mass and in time. The four bodies of the system are in a mirror configuration.

In my talk, I will review the planar equal-mass family of periodic orbits generated from the Schubart-like orbit and discuss the effect of different mass ratios on the family. The linear stability of the orbits is studied by applying various perturbations to the orbits.

Mick Roberts

(Massey University, m.g.roberts@massey.ac.nz)

Nobody expects a flu epidemic

We propose and analyse a model for the dynamics of a single strain of an influenza-like infection. The model incorporates a waning acquired immunity to infection and punctuated antigenic drift of the virus, employing a set of differential equations within a season and a discrete map between seasons. We show that the dynamics of the between-season map depend on parameter values, with attractors being fixed points, periodic solutions, or showing complicated dynamics. For some example parameters we demonstrate the existence of two distinct basins of attraction, with the long term dynamics depending on the initial conditions.

Collaborative work with Roslyn Hickson, James McCaw and Laure Talarmain.

Anand Rampadarath

(The University of Auckland, anandrampadarath@gmail.com)

A distribution-moment approximation for coupled airway dynamics of the airway wall and airway smooth muscle

Asthma is fundamentally a disease of airway constriction. Due to a variety of experimental challenges, the dynamics of airways are poorly understood. Of specific interest is the narrowing of the airway due to forces produced by the airway smooth muscle wrapped around each airway. The interaction between the muscle and the airway wall is crucial for the airway constriction which occurs during an asthma attack. While crossbridge theory is a well-studied representation of complex smooth muscle dynamics, and these dynamics can be coupled to the airway wall, this comes at significant computational cost – even for isolated airways. Because many phenomena of interest in pulmonary physiology cannot be adequately understood by studying isolated airways, this presents a significant limitation. We present a distribution moment approximation of this coupled system as well as comparative results between this approximation and the full PDE based model. These results show that in many situations the distribution moment approximation is a viable option which provides an orders of magnitude reduction in computational complexity; this is valuable not only for isolated airway studies, but moreover offers the prospect that rich ASM dynamics might be incorporated into interacting airway models where previously this was precluded by computational cost.

Yan Bo Ti

(University of Auckland, yan.ti@auckland.ac.nz)

Fault attack on supersingular isogeny cryptosystem

The advent of post-quantum computers have necessitated the adoption of cryptosystems resistant to quantum algorithms. A candidate for such a cryptosystem is one based on supersingular isogenies. The security of communications rely on the ability of cryptosystems to withstand all sort of attacks. Traditional attacks can exploit mathematical properties of a cryptosystem. Fault injection, on the other hand, is a physical attack on cryptosystems implemented on hardware. The goal of the fault injection is for the implementation to compute unintended values in the hope that the output would leak sensitive data.

The supersingular isogeny cryptosystem uses isogenies as its secret and is founded on the difficulty of finding isogenies between arbitrary elliptic curves. Inherent in the system is the computation of auxiliary points under the secret isogeny. There have been attempts to exploit this computation in an active attack. In this talk, we present the first fault attack on cryptosystems based on supersingular isogenies. During the computation of the auxiliary points, the attack aims to change the base point to a random point on the curve via a fault injection. We will show that this would reveal the secret isogeny with one successful perturbation with high probability. The result therefore demonstrates the need to incorporate checks in implementations of the cryptosystem.

Kevin Broughan

(University of Waikato, Department of Mathematics and Statistics, kab@waikato.ac.nz)

Small gaps between primes

One of the more spectacular recent advances in analytic number theory has been the proof of the existence of an infinite number of pairs of prime numbers at a constant small distance apart. Work on this goes back many years. An illustrated overview of developments will be given, tracing through the work of Erdos, Bombieri/Davenport, Goldson/Pintz/Yildrim, Zhang, Tao/Polymath8a/b, and Maynard. A recent breakthrough by students of Ken Ono will also be described.

Thursday 7 December, morning session

Numerical Analysis **B07**

Dimitrios Mitsotakis

(Victoria University of Wellington, dimitrios.mitsotakis@vuw.ac.nz)

Numerical solution of the Serre equations for fully-nonlinear surface water waves

We solve numerically the Serre-Green-Naghdi (SGN) system using stable, accurate and efficient fully discrete numerical schemes based on Galerkin/finite element methods. Although the SGN equations contain third-order derivatives, a modified Galerkin/finite element method allows the use of Lagrange finite elements and combined with explicit Runge-Kutta schemes for the discretization in time approximate solutions of the SGN system with variable bottom efficaciously. Compared to other methods, such as finite volume and discontinuous Galerkin methods, that have been applied for the same system, finite element methods appeared to have certain advantages since they are not dissipative and also can approximate the high order nonlinear terms very accurately. After reviewing the convergence properties of the new numerical scheme, a detailed study of the dynamics of the solitary waves of the SGN system over variable bottom topographies is presented. A numerical study of the various collisions of solitary waves with wall boundaries is being performed while in some cases a comparison with experimental data is presented.

John Butcher

(University of Auckland, butcher@math.auckland.ac.nz)

Tree stumps and applications

A tree stump is a rooted tree from which some of the vertices, and their descendants, have been removed but the edges, connecting the deleted vertices to their parents, are left intact and correspond to unfilled valencies. Right multiplication by further stumps is regarded as completing these valencies. This defines a semigroup. A special type of "atomic" stump consists of a root with children, but no grandchildren, and only the root has unfilled valencies.

The classical theory of the order of Runge–Kutta methods, can be written in terms of atomic stumps. Analysing the order in this way clarifies the relationship between scalar test problems and general, multi-dimensional problems. For example, order 5 requires 16 conditions for scalar problems instead of 17 for more general problems. For order 6, the corresponding numbers of conditions become 31 and 37 respectively. The discrepancy is explained by commutation of some of the atoms, which occurs only in one dimension.

Rachael Tappenden

(University of Canterbury, rachael.tappenden@canterbury.ac.nz)

Flexible alternating direction method of multipliers

In this talk we present a flexible Alternating Direction Method of Multipliers (FADMM) algorithm for solving optimization problems involving a strongly convex objective function that is separable into n blocks, subject to linear equality constraints. The F-ADMM algorithm updates the blocks of variables in a Gauss-Seidel fashion, and the subproblems within F-ADMM include a regularization term. The algorithm is globally convergent. We also introduce a hybrid variant called H-ADMM that is partially parallelizable, which is important in a big data setting. Convergence of H-ADMM follows directly from the convergence properties of F-ADMM. We present numerical experiments to demonstrate the practical performance of this algorithm.

This is joint work with Prof. Daniel P. Robinson.

Samin Aref

(Department of Computer Science, University of Auckland, sare618@aucklanduni.ac.nz)

Balance and frustration in signed graphs under different contexts

Structural balance in networks with positive and negative edges has become a focus in network science. In signed networks, the frustration index determines distance of a network from a state of structural balance making it a key measure for analysing positive and negative interactions under different contexts. Originally suggested decades ago, the complexity involved in computing the frustration index has restricted its usage. We develop new discrete optimisation models to compute this measure in decent-sized instances or alternatively approximate it for relatively large networks. Our extended optimisation models include several speed-up techniques involving prioritised branching and valid inequalities that improve the branch and bound algorithm. The speed-up techniques make our models capable of processing graphs with thousands of nodes and edges in seconds on inexpensive hardware.

We also discuss some applications of the proposed models in social networks, physics, chemistry, finance, international relations, and biology. The structural balance of such signed networks has different meanings and interpretations depending on the context. We use several social networks, such as signed networks inferred from students' choice and rejection and a signed network of US senators. The second discipline we investigate includes gene regulatory networks of several organisms as well as biological networks related to blood cells and proteins. We also discuss networks of formal alliances and antagonisms between countries. The fourth application is in financial portfolio networks. Molecular graphs of carbon and Ising spin glass models are among other applications of this NP-hard graph optimisation problem. The findings unify the applications of a graph-theoretical measure in understanding complex structures in several fields of research.

[Aref S, Mason AJ, Wilson MC. An exact method for computing the frustration index in

signed networks using binary programming. Preprint arXiv:1611.09030 (2016)]
[Aref S, Mason AJ, Wilson MC. Computing the line index of balance using integer programming optimisation. Preprint arXiv:1710.09876 (2017)]

Applied differential equations and modelling **B05**

Soizic Terrien

(The Dodd-Walls Centre, University of Auckland, s.terrien@auckland.ac.nz)

Pulsing dynamics in an excitable laser with feedback

As sources of short, high-amplitude light pulses, self-pulsing lasers are key elements in many applications. We consider here an excitable semiconductor laser subject to delayed optical feedback. Recent experiments demonstrated that a single external optical perturbation can trigger a train of optical pulses, whose repetition rate is determined by the delay time. These pulse trains can be controlled reliably through external optical perturbations: several pulse trains can be switched on and sustained simultaneously in the external cavity with different interpulse timing.

We focus on the theoretical investigation of such pulsing dynamics. We show that the Yamada model with feedback - a system of three delay differential equations for the gain G, absorption Q and intensity I - reproduces pulsing dynamics in very good agreement with the experiment. A bifurcation analysis shows that several stable periodic solutions coexist, which correspond to pulsing regimes with equidistant pulses. Although coexisting pulse trains may seem independent from each other on the timescale of the experiment or in simulation, we demonstrate that they rather correspond to long transients toward one of the stable periodic solutions. The maximum number of pulse trains that can be sustained simultaneously in the external cavity corresponds to the number of periodic solutions that coexist and are stable, and it strongly depends on the delay time and strength of the feedback.

These results provide a better understanding of pulsing dynamics in excitable lasers with feedback, which may be useful for all-optical control of pulse train duration. Because the mechanism for self-pulsations described here relies only on excitability and delayed feedback, our results might be of interest beyond the specific device considered here.

Phillip Wilson
(University of Canterbury, phillip.wilson@canterbury.ac.nz)

A spring-dashpot system for modelling clinical lung tumour motion data in radiotherapy

Lung cancer remains one of the most commonly diagnosed cancers worldwide, and is a leading cause of death from cancer. A common treatment is radiation therapy, the efficacy of which is limited by breath-induced tumour motion. We present a simple three-dimensional system of springs and dashpots to model the movement of a lung tumour during respiration. This system is driven by an external abdominal signal, which is three-dimensional and time-dependent. In the limit of small lateral and transverse tumour motions, asymptotic analysis shows that the dominant motion is given by a single equation driven by the 3D time-dependent signal. A simple numerical scheme is compared to further asymptotic analysis of this 1D-3D model. The model is applied to clinical data obtained from 52 treatments, or beams, of 10 patients. In a two-stage process, we first optimise the model parameters for individual patients and beams, comparing the numerical predictions with clinical data. In the second stage, the numerical predictions based on optimised parameters for individual patients are compared to clinical data obtained from other beams, in order to investigate whether the parameters from a single beam are sufficient to accurately model the lung tumour motion over a course of treatment. We present a statistical analysis of our results, showing that the obtained model output for tumour position is accurate 95% of the time, and that the model deals well with baseline, amplitude, and frequency variations of the input data. This shows that it may be possible to obtain patient-specific model parameters during or prior to the first beam which can be used throughout the course of treatment.

ANALYSIS **B11**

Melissa Tacy
(University of Otago, mtacy@maths.otago.ac.nz)

Flat models for Laplacian eigenfunction asymptotics

Laplacian eigenfunctions arise as stationary states of quantum systems whose classical analogue is free particle motion. The eigenvalue is interpreted as the energy of the system. Therefore where the eigenvalue is large it is expected that the eigenfunctions will display some echoes of the classical dynamics. One way this can happen is concentration of the eigenfunction. This concentration can be measured by studying L^p norms. However, on a general manifold it is quite difficult to determine when any given family of L^p estimates are sharp. In this talk I will introduce a family of flat model examples that can be used to test sharpness. There is a correspondence between this family of examples and spherical harmonics so any estimate that is sharp for the model family is sharp for eigenfunctions on the sphere S^n.

Ilija Tolich

(University of Otago, tolil653@student.otago.ac.nz)

HNN extensions of quasi-lattice ordered groups

Nica introduced a class of partially ordered groups called quasi-lattice ordered groups and studied the C^*-algebras generated by semigroups of isometries satisfying a covariance condition. A quasi-lattice ordered group is amenable if its universal algebra and reduced algebra are isomorphic. The Baumslag-Solitar group is an example of a quasi-lattice ordered HNN extension of $(\mathbb{Z}, \mathbb{N})$. For a quasi-lattice ordered group (G, P) we find conditions for its HNN-extension to be quasi-lattice ordered. In that case, if (G, P) is an amenable quasi-lattice ordered group, then so is its HNN extension.

Thursday 7 December, afternoon sessions

Industrial mathematics **B07**

Graeme Wake and Hyuck Chung

(Massey University, g.c.wake@massey.ac.nz and
Auckland University of Technology, hchung@aut.ac.nz)

An initiative for mathematics-in-industry in NZ (MINZ)

Programs to enable Mathematics to better address Industrial and Community Problems are on the "crest of a wave". The initiative described here is a modification of the tried and tested approach of week-long Mathematics-in-Industry Study Groups, which were introduced at Oxford in the 1960s, which have now spread world-wide. The distinct feature of this approach is that here the work starts and (in many cases) stops with the Industry problem as posed by the Problem Challenge provider, and the week is largely taken up in small groups working with the Industry-based people to address the problem as stated, not as you might want it to be. Ongoing work is encouraged and usually occurs. Typically these involve graduate work and future employment opportunities and/or contracts are common.

For NZ, it is suggested that this will work best if it is seen as a "whole nation" initiative which means that should be run by a national group rather than just one institution, moving usually annually around the country. It may be fostered by a professional grouping, by government, a professional society, or even a nation-wide grouping formed just for this specific purpose. Ideally it could be a blend of all three. We have made a start with this, forming a grouping called **Mathematics-in-Industry for NZ (MINZ)** which is administrated by a **national consortium (KiwiNet) of Industry and Universities** formed largely to bring clever science to address Industry problems.

The NZ initiative was initially in partnership with Australia, which was formalised in 1993 under the then new two-country grouping ANZIAM (=Australian and NZ Industrial and Applied Mathematics) group. During the following two decades this worked well, but the meetings were mostly in Australia, and both Industry and our Mathematical Scientists often were only getting together there, rather than here in our own country. So gradually the national need for this to happen regularly in, and for, NZ became clearer. So in 2015 the MINZ initiative was launched and meetings have been held in Auckland, Massey 2015; Wellington VUW 2016; Palmerston North, Massey 2017; and the next one scheduled is in Auckland, AUT in mid-2018. Attempts have been, and are still being made, to obtain NZ government endorsement, such as usually happens elsewhere.

Here the MINZ initiative will be described, including details of one Challenge from MINZ 2017, and with the help of a short video.

References

- `http://www.minz.org.nz`
- `https://www.youtube.com/channel/UCBdlFHZ1WA4kiy3WQABUGjA`

Mark McGuinness

(Victoria University of Wellington, Mark.McGuinness@vuw.ac.nz)

Mmm, microwaves measure moisture

Measure the moisture content of bauxite in real time as it is offloaded from a ship - this is the challenge that alumina company Rusal Aughinish brought to a recent European Study Group with Industry. This took place during one week in June this year at the University of Limerick in Ireland. Rusal are using a recently installed microwave analyser, and sought our view on how reliable the measurements are. If you come to this talk, you will hear how we tackled the data provided and the physics of microwaves propagating through a field of polarisers, during and shortly after the Study Group. Come and be amazed at the twists and turns we negotiated on the path to enlightenment.

Wenjun Zhang

(Auckland University of Technology, wenjun.zhang@aut.ac.nz)

Changing probability measures in GARCH option pricing models

In this talk, we consider several option pricing models with stochastic volatility in the context of the generalized autoregressive conditional heteroskedastic (GARCH) processes. We propose a globally risk-neutral valuation relationship (GRNVR) to derive the model dynamics under risk-neutral measure and obtain the corresponding closed-form pricing formula for the Chicago Board Options Exchange Volatility Index (CBOE VIX). The parameters of the proposed models are then calibrated using the S&P 500 returns data and the CBOE VIX. Based on the empirical pricing performances, we observe that the proposed GRNVR generally performs better than the locally risk-neutral valuation relationship (LRNVR). We also provide theoretical justification of the proposed GRNVR.

Steve Taylor

(University of Auckland, s.taylor@auckland.ac.nz)

Keeping bags of milk powder nice as they get shipped around the world

Milk powder can undergo a complexity of chemical reactions, all influenced by oxygen, moisture and temperature. Yet, even small changes in milk powder lead to noticeable changes in its flavour. When milk powder is shipped around the world, it is subjected to diverse environmental conditions for prolonged periods of time. We discuss how mathematical modelling sheds light on what can be done to preserve its flavour under these conditions.

This problem was a challenge from Fonterra to the mathematics in industry study group MINZ 2016.

Problem moderators: Luke Fullard, Valerie Chopovda, Steve Taylor.

Stefanie Hittmeyer

(University of Auckland, stefanie.hittmeyer@auckland.ac.nz)

The geometry of blenders in a three-dimensional Hénon-like family

A blender is a geometric object with intricate spacial structure in diffeomorphisms of dimension at least three and vector fields of dimension at least four. Its characterising feature is that its invariant manifolds behave as geometric objects of a dimension that is larger than expected from the dimensions of the manifolds themselves. We consider an explicit family of three-dimensional Hénon-like maps that exhibit blenders in a specific regime in parameter space. With advanced numerical techniques we compute stable and unstable manifolds in this system, enabling us to show one of the first numerical pictures of an actual blender. Moreover, we present numerical evidence for the existence of the blenders over a larger region in parameter space.

This is joint work with Bernd Krauskopf and Hinke Osinga (University of Auckland) and Katsutoshi Shinohara (Hitotsubashi University).

Andrew Keane

(University of Auckland, a.keane@auckland.ac.nz)

State-dependent delays in the El Niño Southern Oscillation system

Delay differential equations (DDEs) have been used successfully in the past to model climate systems at a conceptual level. An important aspect of these models is the existence of feedback loops that feature a delay time, usually associated with the time required to transport energy through the atmosphere and/or oceans across the globe. So far, the values of the delays are generally assumed to be constant. Recent studies have demonstrated that even simple DDEs with nonconstant delay times, which change depending on the state of the system, can produce surprisingly rich dynamical behaviour. Here, we identify physical arguments for the existence of such state-dependent delays in a DDE model for the El Niño Southern Oscillation climate system. We then conduct a bifurcation analysis by means of continuation software to investigate the effects of state-dependent delays on the dynamics of the system.

This is joint work with Bernd Krauskopf (University of Auckland).

Tertius Ralph

(University of Auckland, tral001@aucklanduni.ac.nz)

Hard-core interactions in one-dimensional velocity jump models

Excluded-volume effects can play an important role in determining transport properties in diffusion of particles through crowded environments. Here, the diffusion of finite-sized hard-core inter-acting particles is considered systematically using the method of matched asymptotic expansions. We will use the Langevin approach to diffusion where stochastic increments are applied to the velocity rather than to the space variable. The result is a non-linear PDE for the one-particle probability density function taking into account crowding effects. Stochastic simulations will be used for a comparison with the analytic/numerical solutions derived. The analytic/numerical solutions compare well with stochastic simulations provided the excluded volume fraction is small.

Shawn Means

(Auckland Bioengineering, University of Auckland, s.means@auckland.ac.nz)

A multi-scaled spatial model of hepatitis-B

A complex and heterogeneous network of metabolic activity and blood flow, the spatial landscape of liver structure may influence successful invasion of viruses or propagation of tissue damage. Known spatial distributions of nutrients or metabolic roles across lobular sinuses and proximal hepatocytes may further be key to persistence of infection, toxicity or, say, fatty liver diseases. We are developing a multi-scaled mathematical modeling framework to investigate the influence of micro-scaled sinusoids on the macro, and make our initial foray with hepatitis-B viral (HBV) dynamics. We deploy a mathematical model of intracellular HBV infection across an array of hepatocytes coupled via sinusoidal blood transport. Results indicate that spatially concentrating HBV replication, without coincidentally-distributed immune clearance, may result in persistent infections according to our in silico study.

APPLIED DIFFERENTIAL EQUATIONS AND MODELLING **B05**

Winston Sweatman

(Massey University, w.sweatman@massey.ac.nz)

Studies of few-body orbits

In this talk we will consider some aspects of the few-body problem of celestial mechanics/stellar dynamics.

Alna van der Merwe

(Auckland University of Technology, alna.vandermerwe@aut.ac.nz)

A locally linear Timoshenko beam model

The Timoshenko beam model consists of two linear partial differential equations formulated in terms of the deflection of the beam, and the angle due to rotation of a cross section. Shear and rotary inertia are taken into account in this model. The model may be used only for small vibrations, and it is well-known that in many applications this model compares well with higher dimensional beam models.

In this talk, I consider a locally linear Timoshenko beam model derived for the planar motion of a beam where the material is linearly elastic. As large displacements are allowed, a system of nonlinear partial differential equations is obtained.

Elastic waves in the beam are simulated using a mixed finite element method and the linear and locally linear Timoshenko beam models are compared.

Philip Sharp

(University of Auckland, sharp@math.auckland.ac.nz)

The fate of Apollo asteroids

JPL's Sentry system is a highly automated system that uses observations and orbits provided by the Minor Planet Center to estimate the probability of Near Earth Asteroids hitting Earth over the next century or more. Table 1 below lists information about the six NEAs currently listed by Sentry as having the highest cumulative probability of hitting Earth.

Object	Year range	Number impacts	Impact prob. (cumulative)	Diamtr. (km)	Energy (Mt)
29075 (1950 DA)	2880 - 2880	1	1.2×10^{-4}	1.300	75,000
101955 Bennu (1999 RQ36)	2175 - 2199	78	3.7×10^{-4}	0.490	1,200
410777 (2009 FD)	2185 - 2198	7	1.6×10^{-3}	0.160	140
2017 UR51	2030 - 2115	102	2.5×10^{-5}	0.111	54
2008 UB7	2044 - 2100	33	3.5×10^{-5}	0.058	14
2008 EX5	2061 - 2090	16	4.7×10^{-5}	0.059	7.3

All six objects listed in Table 1 are Apollo asteroids. These are asteroids with an orbital semi-major axis greater than Earth's and a perihelion distance less than Earth's aphelion distance. Eight thousand, one hundred and eighty Apollo asteroids were known as of November, 2016. Of these, 1,133 were classified as potentially hazardous asteroids, meaning they have the potential to come close to Earth and are large enough to cause regional destruction.

Impact probabilities provide insight about the evolution of the Apollo asteroids as a group. Another way to get insight is to perform long N-body simulations of the asteroids and record their fate. I have done this and will present the rests of my investigation.

Liam McMahon

(University of Waikato, Liamcm21@Gmail.com)

Hall, viscous and axial modifications of linear reconnection

Magnetic reconnection is a model for energy release that predicts rates too slow to explain the rapid release of energy observed in solar flares. Hence, we turn to generalisations of purely resistive magnetic reconnection such as the incorporation of the Hall effect and viscosity. We describe Hall and viscous linear reconnection, in particular we consider eigenequations, decay and oscillation rate scalings and long-time behaviour. We give further evidence to a recent result that while viscous effects may slow down magnetic reconnection, viscous dissipation during reconnection can result in fast energy release for the order of one Aflv en time that could be consistent with a solar flare. Finally, we compare linear reconnection at two different 3D geometries- a quasi-separatrix layer (QSL) and a 3D null point. We find that fast reconnection persists at a 3D null point, but is stalled at a QSL.

Analysis **B11**

Robin Havea

(University of the South Pacific, robin.havea@usp.ac.fj)

A constructive look at the boundary of the numerical range of a bounded operator

Working within the framework of Bishop–style constructive mathematics, we investigate some well known classical results and see how it can be pushed into and adopted in a constructive setting where we based our mathematics on intuitionistic logic. Computing and locating the boundaries of numerical range can be a straight forward exercise in a classical setting (where it rests on classical logic) but we explore the challenges arise when it is dealt with in a constructive manner. We will also touch on key questions regarding the convexity of the numerical range and use the work of Bridges and Havea to demonstrate such challenges and limitations.

Markus Antoni
(University of Otago, mantoni@maths.otago.ac.nz)

Pathwise regularity for stochastic evolution equations in L^p spaces

In this talk we give an introduction to the theory of stochastic integrals and stochastic evolution equations in L^p spaces. In contrast to the classical semigroup approach we interchange the order of the spacial and temporal regularity leading to new challenges and tasks which we meet by using techniques taken from functional and harmonic analysis. In the end, this approach turns out to be fruitful yielding, e.g., pathwise Hölder estimates for the solutions of a large class of stochastic partial differential equations.

Algebra and number theory **B11**

Tim Stokes
(University of Waikato, tim.stokes@waikato.ac.nz)

Constellations: Arrows without targets

Constellations are partial algebras that are one-sided generalisations of categories. Categories model classes of objects together with suitably defined mappings between them. Each mapping, or arrow, has a domain and codomain (source and target), and composition of mappings $f \cdot g$ is defined precisely when the codomain of f coincides with the domain of g.

An alternative notion of composition arises if one only requires the codomain of f to be a *subset* of the domain of g. When this is done, precise information about codomains is no longer needed, and "arrows" have sources but no targets. This is more natural in many examples, for example all mappings between sets having infinite domain but arbitrary image. The abstract concept corresponding to these concrete examples is that of a constellation, a concept first introduced by Gould and Hollings (where it was shown that the category of so-called inductive constellations is isomorphic to the category of left restriction semigroups).

Here we consider constellations in full generality, giving many examples. We characterise those small constellations that are isomorphic to constellations of partial functions, as well as those constellations that arise as (sub-)reducts of categories, and show that categories are nothing but two-sided constellations. We demonstrate that the naive notion of substructure can be captured within constellations but not within categories. We show that every constellation P gives rise to a category $\mathcal{C}(P)$, its canonical extension, in a simplest possible way, and that P is a quotient of $\mathcal{C}(P)$ obtained by factoring out a so-called canonical congruence, and that many familiar concrete categories may be constructed from simpler quotient constellations in this way. A correspondence between constellations and categories equipped with a canonical congruence is established.

This is joint work with Victoria Gould.

Heung Yeung (Frederick) Lam

(Massey University (Auckland), h.y.lam@massey.ac.nz)

Factorization of theta function identities

In Ramanujan's lost notebook, infinite product formulas are recorded for each of the functions

$$\phi(q)+\phi(q^5),\quad \phi(q)-\phi(q^5),\quad \phi(q)+\sqrt{5}\phi(q^5)\quad \text{and}\quad \phi(q)-\sqrt{5}\phi(q^5)$$

where $\phi(q)=\sum_{n=-\infty}^{\infty} q^{n^2}$ is the generating function for squares. Ramanujan also gave similar results that involve the Rogers–Ramanujan continued fraction. In this talk, I will present some cubic analogues of Ramanujan's results, many of which are new. That is, we provide factorizations for the eight functions

$$\phi(q^3)\pm\phi(q),\quad \phi(q^3)\pm i\,\phi(q),\quad \sqrt{3}\,\phi(q^3)\pm i\,\phi(q)\quad \text{and}\quad \sqrt{3}\phi(q^3)\pm\phi(q)$$

as well as corresponding results for the generating function of the triangular numbers.

List of participants

Mohamed Al-Sultani	Massey U.	m.al-sultani@massey.ac.nz
Ana Amador	U. Buenos Aires, Argentina	ana.amador@gmail.com
Jianbei An	U. Auckland	j.an@auckland.ac.nz
Astrid an Huef	Victoria U.	astrid.anhuef@vuw.ac.nz
Jacek Andrzejczak	Albany Senior High School	jacekand@gmail.com
Markus Antoni	U. Otago	mantoni@maths.otago.ac.nz
Samin Aref	U. Auckland	sare618@aucklanduni.ac.nz
Andrea Babylon	Massey U. (Albany)	a.babylon@massey.ac.nz
Boris Baeumer	U. Otago	bbaeumer@maths.otago.ac.nz
craig baily	waitakere college	bl@waitakerecollege.school.nz
Anna Barry	U. Auckland	anna.barry@auckland.ac.nz
Anton Baykalov	U. Auckland	a.baykalov@auckland.ac.nz
Alona Ben-Tal	Massey U.	a.ben-tal@massey.ac.nz
Peter Bier	U. Auckland	p.bier@auckland.ac.nz
Christian Blasche	Massey U.	c.b@mail.de
Geoffrey Booth	Nelson Mandela U., SA	geoff.booth@nmmu.ac.za
Kevin Broughan	U. Waikato	kab@waikato.ac.nz
David Bryant	U. Otago	david.bryant@otago.ac.nz
John Butcher	U. Auckland	butcher@math.auckland.ac.nz
Damian Campbell	Waimea College	damian.campbell@waimea.school.nz
Jiling Cao	AUT	jiling.cao@aut.ac.nz
Subash Chandar K	Ormiston Senior College	schandark@ormiston.school.nz
Sze Looi Chin	U. Auckland	schi642@aucklanduni.ac.nz
Valerie Chopovda	Massey U.	valerie.chopovda@gmail.com
hyuck chung	AUT	hchung@aut.ac.nz
Ielyaas Cloete	U. Auckland	i.cloete@auckland.ac.nz
Julie Clutterbuck	Monash U., Australia	julie.clutterbuck@monash.edu
Shaun Cooper	Massey U.	s.cooper@massey.ac.nz
Julia Crawford	U. Auckland	julia.crawford@auckland.ac.nz
Brendan Creutz	U. Canterbury	brendan.creutz@canterbury.ac.nz
Giovanni De Franceschi	U. Auckland	giovanni.defranceschi@auckland.ac.nz
Nur Atiqah Dinon	U. Auckland	nur.dinon@auckland.ac.nz
Min-Chih Embleton	Waitakere College	em@waitakerecollege.school.nz
Tanya Evans	U. Auckland	t.evans@auckland.ac.nz
Bartek Ewertowski	U. Auckland	bartek.ewertowski@gmail.com
Hammed Fatoyinbo	Massey U.	h.fatoyinbo@massey.ac.nz
Keegan Flood	U. Auckland	kflood99@gmail.com
Gillian Frankcom	U. Auckland	g.frankcom@auckland.ac.nz
Luke Fullard	Massey U.	l.fullard@Massey.ac.nz
Andrus Giraldo	U. Auckland	agir284@aucklanduni.ac.nz
Gareth Gordon	U. Auckland	g.gordon@auckland.ac.nz
Rod Gover	U. Auckland	r.gover@auckland.ac.nz
Sina Greenwood	U. Auckland	sina@math.auckland.ac.nz
John Griffith Moala	U. Auckland	john.moala@auckland.ac.nz
Jesse Hart	U. Auckland	drjessehart@outlook.com
Cris Hasan	U. Auckland	rhas033@aucklanduni.ac.nz
Catherine Hassell Sweatman	AUT	catherine.sweatman@aut.ac.nz
Robin Havea	USP, Fiji	robin.havea@usp.ac.fj
Stefanie Hittmeyer	U. Auckland	stefanie.hittmeyer@auckland.ac.nz

Dorit S. Hochbaum	UC Berkeley, USA	dhochbaum@berkeley.edu
Audris Hong	St Mary's College	ahong@stmaryak.school.nz
Zan Iqbal		zan_in_nz@yahoo.co.nz
Frederic Jaccard	Kristin School	fjaccard@kristin.school.nz
Chris John		drcmjohn@gmail.com
Andrew Keane	U. Auckland	a.keane@auckland.ac.nz
Jeong-Hoon Kim	Yonsei U., South Korea	jhkim96@yonsei.ac.kr
Steve King	Maths teacher	stevemich@outlook.com
Vivien Kirk	U. Auckland	v.kirk@auckland.ac.nz
Jo Knox	U. Auckland	j.knox@auckland.ac.nz
Igor' Kontorovich	U. Auckland	i.kontorovich@auckland.ac.nz
Bernd Krauskopf	U. Auckland	b.krauskopf@auckland.ac.nz
Anna Kuan	AMA	akuan@eggs.school.nz
Heung Yeung Lam	Massey U.	h.y.lam@massey.ac.nz
Harris Leung	U. Auckland	pleu073@aucklanduni.ac.nz
Wendy Light	WestLake Boys' High School	wlight@westlake.school.nz
Woei Chet Lim	U. Waikato	wclim@waikato.ac.nz
Arthur Liu	Massey U.	sirarthur2479@hotmail.com
Michael Lockyer	U. Auckland	michael.lockyer@auckland.ac.nz
Thomasin Lynch	Massey U.	t.a.lynch@massey.ac.nz
Sione Ma'u	U. Auckland	s.mau@auckland.ac.nz
Gemma Mason	U. Auckland	g.mason@auckland.ac.nz
Stephen McConnachie	Middleton Grange School	s.mcconnachie@middleton.school.nz
Mark McGuinness	Victoria U.	mark.mcGuinness@vuw.ac.nz
Liam McMahon	U. Waikato	liamcm21@gmail.com
Shawn Means	U. Auckland	s.means@auckland.ac.nz
Tomas Michalik	U. Auckland	tomas.michalik@auckland.ac.nz
Dimitrios Mitsotakis	Victoria U.	dimitrios.mitsotakis@vuw.ac.nz
Fabien Montiel	U. Otago	fmontiel@maths.otago.ac.nz
Rua Murray	U. Canterbury	rua.murray@canterbury.ac.nz
Elle Musoke	U. Auckland	elle.musoke@auckland.ac.nz
Shumaila Noreen	Massey U.	s.noreen@massey.ac.nz
Julia Novak	U. Auckland	j.novak@auckland.ac.nz
Christian Offen	Massey U.	c.offen@massey.ac.nz
Dion O'Neale	U. Auckland	d.oneale@auckland.ac.nz
Hinke Osinga	U. Auckland	h.m.osinga@auckland.ac.nz
Nathan Pages	U. Auckland	natan.pages@gmail.com
Michael Plank	U. Canterbury	michael.plank@canterbury.ac.nz
Masina Po'e-Tofaeono	U. Auckland / Manurewa High	m.poetofaeono@auckland.ac.nz
Peter Radonich	U. Auckland / Northcote College	peterr@northcote.school.nz
Iain Raeburn	Victoria U.	iain.raeburn@vuw.ac.nz
Tertius Ralph	U. Auckland	tral001@aucklanduni.ac.nz
Anand Rampadarath	U. Auckland	anandrampadarath@gmail.com
Pinki Rani	U. Auckland	pinkimalik012@gmail.com
Reena Raphael	Epsom Girls Grammar	rraphael@eggs.school.nz
Iresha Ratnayake	U. Auckland	iresha.ratnayake@auckland.ac.nz
Nicolette Rattenbury	U. Auckland	nicolette.rattenbury@auckland.ac.nz
Andrew Ricciardi	Waimea College	andrew.ricciardi@waimea.school.nz
Mick Roberts	Massey U.	m.g.roberts@massey.ac.nz
Karyn Saunders	Victoria U.	karyn.saunders@vuw.ac.nz
Vee-Liem Saw	U. Otago	veeliem@maths.otago.ac.nz
Cami Sawyer	Massey U.	c.sawyer@massey.ac.nz

Nesa Selvarajah	Avondale College	svh@avcol.school.nz
Philip Sharp	U. Auckland	sharp@math.auckland.ac.nz
David Simpson	Massey U.	d.j.w.simpson@massey.ac.nz
Jack Simpson	U. Canterbury	jrs149@uclive.ac.nz
Jamie Sneddon	Saint Kentigern College	jamie.sneddon@saintkentigern.com
James Sneyd	U. Auckland	sneyd@math.auckland.ac.nz
Kerri Spooner	AUT	kspooner@aut.ac.nz
Tim Stokes	U. Waikato	tim.stokes@waikato.ac.nz
Roger Su	U. Waikato	rcs14@students.waikato.ac.nz
Winston Sweatman	Massey U.	w.sweatman@massey.ac.nz
Melissa Tacy	U. Otago	mtacy@maths.otago.ac.nz
Rachael Tappenden	U. Canterbury	rachael.tappenden@canterbury.ac.nz
Steve Taylor	U. Auckland	s.taylor@auckland.ac.nz
Tom ter Elst	U. Auckland	terelst@math.auckland.ac.nz
soizic terrien	U. Auckland	s.terrien@auckland.ac.nz
Yan Bo Ti	U. Auckland	yan.ti@auckland.ac.nz
Ilija Tolich	U. Otago	tolil653@student.otago.ac.nz
Nick Trefethen	Oxford U., UK	trefethen@maths.ox.ac.uk
Christopher Tuffley	Massey U.	c.tuffley@massey.ac.nz
Bruce van Brunt	Massey U.	b.vanbrunt@massey.ac.nz
Alna van der Merwe	AUT	alna.vandermerwe@aut.ac.nz
Matthew van Oudenaaren	St Peter's College	mvanoudenaaren@st-peters.school.nz
Graeme Wake	Massey U.	g.c.wake@massey.ac.nz
David Waters	U. Auckland	d.waters@auckland.ac.nz
Graham weir	Massey U.	grahamweir@xtra.co.nz
Phillip Wilson	U. Canterbury	phillip.wilson@canterbury.ac.nz
Trevor Wilson		trevorawilson@gmail.com
Chris Wong	U. Auckland	mwon535@aucklanduni.ac.nz
Xueshan Yang	U. Auckland	xyan900@aucklanduni.ac.nz
Caroline Yoon	U. Auckland	c.yoon@auckland.ac.nz
Faheem Zaidi	Massey U.	f.zaidi@massey.ac.nz
Golbon Zakeri	U. Auckland	g.zakeri@auckland.ac.nz
Wenjun Zhang	AUT	wenjun.zhang@aut.ac.nz