On computing the subgroup lattice of O'N

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Abstract

Using a series of Magma [2] programs, we determine the full weighted subgroup lattice of the O'Nan sporadic simple group. The same program is applied to compute the subgroup lattices of Fi_{22}, Co_2, Suz and Ru.

Keywords : subgroup lattice, sporadic simple group, computational group theory

1 Introduction

The computation of subgroup lattices of permutation groups has been a subject of interest for many decades. Knowing the subgroup structure of a group, one may then use it to prove theorems, construct geometric objects, better understand the given group.

We are especially interested in the sporadic simple groups. In that framework, the O'Nan group is of particular interest because it is the only sporadic group for which a computer-free existence proof is still missing. In 1973 [10], O'Nan provided strong evidence for the existence of a new sporadic group now called $\mathsf{O'N}$. Later in the seventies, Sims constructed this group with help of a computer (see [6] for a survey of the story of $\mathsf{O'N}$) but his work seems to be unpublished. In 1980, Andrilli published in his PhD Thesis [1], supervised by Sims, an existence and uniqueness proof of $\mathsf{O'N}$. Around 1985, the maximal subgroups of $\mathsf{O'N}$ were determined independently by Yoshiara in his Master's Thesis [16], Wilson [15] and Ivanov, Tsaranov and Shpectorov [7]. The two latter references rely partially on computer algebra.
Some definitions of $\mathsf{O'N}$, its maximal subgroups and its character table are available in the Atlas of Finite Groups [5] as well as a presentation (see also work due to Soïcher [13]). Several people have given computer-based existence and/or uniqueness proofs of this group, including Soïcher [14], Jansen and Wilson [8], and more recently, Michler and Previtali [9].

In 1998, Derek Holt (personal communication) computed one representative of each conjugacy class of subgroups of $\mathsf{O'N}$. However, he did not get the subgroup lattice, i.e. he did not have the inclusions between the classes. In this paper, we give MAGMA programs to compute the subgroup lattice of a permutation group. Then, using these programs, we determine the full weighted subgroup lattice of $\mathsf{O'N}$. We obtain 581 conjugacy classes of subgroups, as Derek Holt did in 1998.

The paper is organised as follows.

In section 2, we give a small MAGMA function which permits to reduce the permutation degree of a group to a certain extend. In section 3, we give MAGMA programs to compute the weighted subgroup lattice of the O'Nan sporadic simple group $\mathsf{O'N}$. These programs may also be applied to the Suzuki sporadic group Suz, the Rudvalis group Ru and the Fischer group Fi_{22}. Finally, in section 4, we give the subgroup lattice of $\mathsf{O'N}$.

2 A program to reduce permutation representation of a permutation group

The program given in Figure 1 is due to Bernd Souvignier. It tries to reduce the permutation representation of a permutation group. As input, it receives a permutation group G and as output, it gives two object : a permutation group H isomorphic to G and an isomorphism ϕ from G to H.

3 Computing the subgroup lattice of O'Nan

Table 2 gives the list of sporadic simple groups together with their order, factored order, smallest permutation representation degree and a reference to the first persons who computed the subgroup lattice of the group (or just the conjugacy classes of subgroups for $\mathsf{O'N}$).

Table 1 gives the time it takes to get the full subgroup lattice of a given sporadic group using MAGMA V2.13. A computer with Dual Core Xeon processors running at 3.2Ghz and 16Gb of Ram was used to get these timings. In that table, we also give the number of conjugacy classes of subgroups ($cc(G)$) and the number of subgroups ($n(G)$). Three groups are a bit special in that table, namely $\mathsf{O'N}$, Ru and Suz. Indeed, it is currently not possible to compute the subgroup lattice of these groups using MAGMA V2.13 but with the programs we present in section 3, it is possible to get them. So the timings given for these latter groups are the ones obtained with the procedure given in this section.

We first recall how MAGMA does nowadays to compute subgroup lattices of groups.

Start with a set `classes` which is empty and a set `sgr` containing just one element, namely the group G for which we want to compute the subgroup lattice.

While `sgr` is nonempty, pick one element out of `sgr` and put it in `classes`. Obvi-

```
RedPerm := function(G)
local phi, H, red, O, orb, I, rho, P, part;
phi := IdentityHomomorphism(G);
H := G @ phi;
red := false;
while not red do
  if not IsTransitive(H) then
    red := true;
    O := Orbits(H);
    for orb in O do
      if #OrbitKernel(H, orb) eq 1 then
        rho, I := OrbitAction(H, orb);
        H := I;
        phi := phi * rho;
        red := false;
        break orb;
      end if;
    end for;
  elif not IsPrimitive(H) then
    red := true;
    part := MaximalPartition(H);
    if #BlocksKernel(H, part) eq 1 then
      rho, I := BlocksAction(H, part);
      H := I;
      phi := phi * rho;
      red := false;
    else
      P := MinimalPartitions(H);
      Sort(~P, func< x,y | #y - #x >);
      for part in P do
        if #BlocksKernel(H, part) eq 1 then
          rho, I := BlocksAction(H, part);
          H := I;
          phi := phi * rho;
          red := false;
          break part;
        end if;
      end for;
    end if;
  else
    red := true;
  end if;
end while;
return H, phi;
```

Figure 1: A function to reduce the permutation degree of a group

G	Order(G)	Deg(G)	$cc(G)$	$n(G)$	CPU Time
M_{11}	7,920	11	39	8,651	0.1s
M_{12}	95,040	12	147	214,871	0.41s
J_1	175,560	266	40	158,485	0.15s
M_{22}	443,520	22	156	941,627	0.47s
J_2	604,800	100	146	1,104,344	0.63s
M_{23}	10,200,960	23	204	17,318,406	0.8s
HS	44,352,000	100	589	149,985,646	5.09s
J_3	50,232,960	6156	137	71,564,248	17.46s
M_{24}	244,823,040	24	1529	1,363,957,253	73.94s
McL	898,128,000	275	373	1,719,739,392	4.51s
He	4,030,387,200	2058	1698	22,303,017,686	177.06s
Ru	145,926,144,000	4060	6035	963,226,363,401	**20117.720s**
Suz	448,345,497,600	1782	6381	4,057,939,316,149	**16130.870s**
O'N	460,815,505,920	122760	581	1,169,254,703,685	**7600s**
Co_3	495,766,656,000	276	2483	2,547,911,497,738	67.92s
Fi_{22}	64,561,751,654,400	3510	111004		**7.3 days**

Table 1: Computing times of subgroup lattices

ously, it is G the first time. Compute its maximal subgroups and for each maximal M, add it to `sgr` provided there is no subgroup in `sgr` conjugate to M in G. During that process, keep track of inclusions of respective subgroups considered.

At the end of this process, in `classes` there is one representative of each conjugacy class of subgroups of G. Moreover, we also have the maximal inclusions between classes. So the subgroup lattice is determined. The weighted subgroup lattice can be determined in the process by computing weighted inclusions at each step.

In Figure 2, a sample of MAGMA-code for computing the subgroup lattice is given. Observe that we use the `RedPerm` function given in the previous section to reduce the representation of each subgroup before computing its maximal subgroups. That is not the case in the SubgroupLattice function of MAGMA. This improvement saves a lot of time and memory. For instance, if we want to compute the maximal subgroups of $L_3(7) : 2$, it will take 20 seconds and 200Mb of memory with their representation on 122760. Using the `RedPerm` function, one may reduce the representation to 456 points for instance and get the maximal subgroups in less than 4 seconds with 35Mb of memory.

The procedure in Figure 2 receives a group `G` for which the subgroup lattice must be computed, a sequence `classes` which is either empty or contains already some representatives of classes of subgroups, a sequence `len` which is either empty or contains the lengths of the conjugacy classes corresponding to the representatives that are in the sequence `classes`, a sequence `sgr` containing maximal subgroups of the subgroups that are in `classes`, a sequence `sgrincl` telling which subgroup of `sgr` is a maximal subgroup of which subgroup in `classes`, a sequence `wlat` which contains 4-tuples

$$a, b, c, d$$

```
load "../RedPerm";
procedure SLat(G, ~classes, ~len, sgr, sgrincl, incls, ~wlat);
  incls2:=[]; weight := [];
  if classes eq [] then
    classes := [G]; incls := [{0}];
    sgr := [x`subgroup : x in  MaximalSubgroups(G) ];
    sgrincl := [[1,Truncate(#G/#Normalizer(G,x`subgroup))]: x in MaximalSubgroups(G)];
  end if;
  while not(sgr eq []) do
    max, nmax := Max([#sgr[i] : i in [1..#sgr]]);
    h := sgr[nmax]; nh := #h; test := true;
    for i := #classes to 1 by -1 do
      if #classes[i] eq nh and IsConjugate(G,classes[i],h) then
        test := false; test2 := true;
        incls[i]:= incls[i] join {sgrincl[nmax][1]};
      for j := 1 to #incls2 do
        if [sgrincl[nmax][1],i] eq incls2[j] then
            weight[j] := weight[j]+sgrincl[nmax][2]; test2 := false; break;
        end if;
      end for;
        if test2 then
          Append(~incls2,[sgrincl[nmax][1],i]);
          Append(~weight,sgrincl[nmax][2]);
        end if;
        break;
      else
        if #classes[i] gt nh then break i;end if;
      end if;
    end for;
    if test then
      Append(~classes,sgr[nmax]);
      Append(~incls,{sgrincl[nmax][1]});
      Append(~incls2,[sgrincl[nmax][1],#classes]);
      Append(~weight,sgrincl[nmax][2]);
      hh,phi := RedPerm(h);
      s := MaximalSubgroups(hh);
      sgr cat:= [x`subgroup@@phi : x in s];
      sgrincl cat:= [[#classes,Truncate(#hh/#Normalizer(hh,x`subgroup))] : x in s];
    end if;
    Remove(~sgr,nmax);
    Remove(~sgrincl,nmax);
  end while;
  len := [[i,#classes[i],Truncate(#G/#Normalizer(G,classes[i]))] : i in [1..#classes] ];
  wlat := [];
  for i := 1 to #incls2 do
    a := incls2[i];
    Append(~a,weight[i]);
    Append(~a,Truncate((len[a[1]][3]*a[3])/len[a[2]][3]));
    Append(~wlat,a);
  end for;
end procedure;
```


Figure 2: A procedure to compute the subgroup lattice of a group

telling that subgroups of class number a in `classes` contains c maximal subgroups in class number b of `classes` and that each subgroup of class number b is contained in d subgroups of class number a.

Most of the time, when a user wants to compute the subgroup lattice of a group, he just calls `SLat(G, classes, len, [], [], [], wlat)` where `classes`, `len`, `wlat` are empty sequences which are filled in during the execution of `SLat`. If the user wants to compute the subgroup lattice of a group for which MAGMA cannot compute the maximal subgroups, then he may pass this information to the procedure so that the procedure can finish the computation for him. That is what we did for the computation of the subgroup lattice of O′N. We had to compute first one representative of each conjugacy class of maximal subgroups of O′N and then pass this information to the procedure.

Fortunately, for the O'Nan group, permutation generators may be found on the Atlas of Finite Groups website (see `http://for.mat.bham.ac.uk/atlas/`) and the maximal subgroups may be easily generated in MAGMA using the Sporadic Conglomerator of Eamonn O'Brien
(see `http://www.math.auckland.ac.nz/ obrien/Sporadics/conglomerator.php`).
One just then has to provide information in `classes`, `len`, etc. to the procedure to make it able to compute the full subgroup lattice of O′N. The result is given in Section 4.

4 The subgroup lattice of O'Nan

In this section, we give as a table, the weighted subgroup lattice of O'N. The table is read as follows. Column "Nr." gives a number to each conjugacy class. A class will be referred to as "C_x" in this paper where x is the number of the class. Suppose H is a subgroup belonging to class C_x. Column "Struct.", when filled in, gives a structure of H. Column "Order" (resp. "Length") gives the order of H (resp. the number of subgroups conjugated to H). Column "Maximal Subgroups" gives the number of the conjugacy classes of subgroups where the maximal subgroups of H lie. A number between marentheses in that column tells precisely how many such subgroups there are . No number means there is exactly one. Column "Minimal Overgroups" gives the number of the conjugacy classes of subgroups where some representative of these class contain H as a maximal subgroup. A number between marentheses in that column tells precisely how many such subgroups there are . No number means there is exactly one.

Nr.	Struct.	Order	Length	Maximal Subgroups	Min. Over.
1	O'N	460815505920	1	2(122760),3(122760),6(2624832), 7(2857239),11(17778376),12(17778376), 15(30968784),16(30968784),21(42858585), 23(58183776),24(58183776),43(182863296), 44(182863296)	
2	$L_3(7):2$	3753792	122760	4,22(456),65(2793),110(5586),250(26068), 251(26068),285(32928)	1
3	$L_3(7):2$	3753792	122760	5,22(456),66(2793),111(5586),252(26068), 253(26068),285(32928)	1
4	$L_3(7)$	1876896	122760	9(114),171(11172),172(5586),313(26068), 314(26068),372(32928)	2
5	$L_3(7)$	1876896	122760	10(114),173(11172),174(5586),315(26068), 316(26068),372(32928)	3
6	J_1	175560	2624832	119(266),237(1045),277(1463),285(1540), 286(1596),366(2926),397(4180)	1
7	$4 \cdot L_3(4):2$	161280	2857239	8,32(56),33(56),34(56),55(105),67(120), 65(120),66(120),140(280),143(336)	1
8	$4 \cdot L_3(4)$	80640	2857239	30(42),57(56),58(56),59(56),112(120), 113(120),114(120),194(280)	7
9	$7^2:2 \cdot L_2(7):2$	32928	13994640	13,27(8),53(21),82(28),115(49)	4
10	$7^2:2 \cdot L_2(7):2$	32928	13994640	14,28(8),54(21),83(28),116(49)	5
11	$(3^2:4 \times A_6) \cdot 2$	25920	17778376	17,18,19,33(9),36(10),101(36),140(45)	1
12	$3^4:2^{1+4}D_{10}$	25920	17778376	20,26(5),52(16),184(81)	1
13	$7^2:2 \cdot L_2(7)$	16464	13994640	45(14),49(8),175(49)	9
14	$7^2:2 \cdot L_2(7)$	16464	13994640	46(14),50(8),176(49)	10
15	$L_2(31)$	14880	30968784	144(32),367(248),368(248),426(465), 463(496),470(620),471(620)	1
16	$L_2(31)$	14880	30968784	144(32),369(248),368(248),427(465), 463(496),472(620),473(620)	1
17	$3^2:4 \times A_6$	12960	17778376	25,60(9),69(10),161(36),195(45)	11
18	$3^2:4 \times A_6$	12960	17778376	25,61(9),70(10),162(36),196(45)	11
19	$(3^2:2 \times A_6) \cdot 2$	12960	17778376	25,47(12),58(9),71(10),89(30)	11
20	$3^4:2^{1+4}5$	12960	17778376	37,90(16),245(81)	12
21	$4^3L_3(2)$	10752	42858585	55(7),56(7),68(8)	1
22	$7^{1+2}:(3 \times D_8)$	8232	55978560	27,28,29,35,238(49)	2,3
23	M_{11}	7920	58183776	102(11),119(12),252(55),278(66),382(165)	1
24	M_{11}	7920	58183776	103(11),119(12),250(55),279(66),383(165)	1
25	$3^2:2 \times A_6$	6480	17778376	31,48(12),86(12),120(10),146(30)	17,18,19
26	$3^4:2^{1+4}2$	5184	88891880	37,38,39,40,41,42,36,341(81)	12
27	$7^{1+2}:(3 \times 2^2)$	4116	55978560	51,49(2),62,134(14)	9(2),22
28	$7^{1+2}:(3 \times 2^2)$	4116	55978560	50(2),51,63,135(14)	10(2),22
29	$7^{1+2}:12$	4116	55978560	51,64,308(49)	22
30	$4 \cdot 2^4:A_5$	3840	120004038	97(5),133(6),156(10),208(16),209(16), 210(16),211(16)	8
31	$3^2:A_6$	3240	17778376	87(4),141(12),178(10),214(30)	25

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
32	$4 \cdot M_{10}$	2880	160005384	58,197(10),312(36),342(45)	7
33	$A_6 : Q_8$	2880	160005384	60,61,59,198(10),312(36),343(45)	7,11
34	$4 \cdot M_{10}$	2880	160005384	57,199(10),312(36),344(45)	7
35	$7^{1+2} : D_8$	2744	55978560	64,62,63,373(49)	22
36	$(3^2 : 4 \times 3^2 : 4) \cdot 2$	2592	88891880	72,69(2),73,70(2),71,198(18)	11(2),26
37	$3^4 : 2^{1+4}$	2592	17778376	74(5),72(5),75(5),428(81)	20,26(5)
38	$3^4 : 2^{1+3} \cdot 2$	2592	88891880	71,76,75,429(81)	26
39	$3^4 : 8 \cdot 2^2$	2592	88891880	77(2),75,78,73,79(2),430(81)	26
40	$3^4 : 2^{1+3} \cdot 2$	2592	88891880	71,78,74,431(81)	26
41	$3^4 : 8 \cdot 2^2$	2592	88891880	73,76,80(2),74,81(2),432(81)	26
42	$3^4 : 2^{1+3} \cdot 2$	2592	88891880	76,72,78,433(81)	26
43	A_7	2520	182863296	163(7),239(15),240(15),280(21),317(35)	1
44	A_7	2520	182863296	163(7),241(15),239(15),281(21),318(35)	1
45	$7^2 : 2 \cdot S_4$	2352	195924960	84,91(3),136(4),384(49)	13
46	$7^2 : 2 \cdot S_4$	2352	195924960	85,92(3),137(4),385(49)	14
47	$(3^2 : 2 \times A_5) \cdot 2$	2160	213340512	86,147(5),164(6),209(9),215(10)	19
48	$A_6 \times S_3$	2160	213340512	87,104(3),165(6),166(6),215(10),251(15),253(15)	25
49	$7^2 : 2 \cdot 7 : 3$	2058	111957120	88,107,187(7),188(7)	13,27
50	$7^2 : 2 \cdot 7 : 3$	2058	111957120	88,108,189(7),190(7)	14,28
51	$7^{1+2} : 6$	2058	55978560	88,109,187(14),190(14),191(28)	27,28,29
52	$3^2 : 2 \cdot D_{10}$	1620	284454016	90,179(5),490(81)	12
53	$7^2 : 16 : 2$	1568	293887440	91,93,94,434(49)	9
54	$7^2 : 16 : 2$	1568	293887440	95,92,96,435(49)	10
55	$4^3 \cdot S_4$	1536	300010095	97,142(3),157(4),158(4),159(4)	7,21
56	$4^3 \cdot S_4$	1536	300010095	98,99,100,142(3),157(4)	21
57	$4 \cdot A_6$	1440	160005384	105,211(12),254(10),296(30)	8,34
58	$4 \cdot A_6$	1440	160005384	106,208(12),255(10),297(30)	8,32
59	$A_6 : 4$	1440	160005384	104,209(12),256(10),298(30)	8,19,33
60	$4 \times A_6$	1440	160005384	104,257(10),401(36),436(45)	17,33
61	$4 \times A_6$	1440	160005384	104,257(10),402(36),437(45)	18,33
62	$7^{1+2} : 2^2$	1372	55978560	107(2),109,218(14)	27,35
63	$7^{1+2} : 2^2$	1372	55978560	108(2),109,219(14)	28,35
64	$7^{1+2} : 4$	1372	55978560	109,466(49)	29,35
65	$4 \cdot L_3(2) : 2$	1344	342868680	115,117,113,238(8),345(21),386(28)	2,7
66	$4 \cdot L_3(2) : 2$	1344	342868680	116,118,112,238(8),346(21),386(28)	3,7
67	$4 \cdot L_3(2) : 2$	1344	342868680	110,111,114,238(8),347(21),386(28)	7
68	$4^3 \cdot (7 : 3)$	1344	342868680	145,222(7),237(8)	21

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
69	$3^2:4\times 3^2:4$	1296	177783760	121,120,122,257(9),258(9)	17,36
70	$3^2:4\times 3^2:4$	1296	177783760	120,123,122,257(9),259(9)	18,36
71	$3^4:4^2$	1296	88891880	120(2),124,256(18)	19(2),36,38,40
72	$3^4:Q_8:2$	1296	88891880	123,125(2),124,126(2),121,260(18)	36,37,42
73	$3^4:Q_8:2$	1296	88891880	127(2),122(2),128(2),124,260(18)	36,39,41
74	$3^4:D_8:2$	1296	88891880	129,123(2),125,130(2),124,504(81)	37,40,41
75	$3^4:D_8:2$	1296	88891880	121(2),129(2),126,130,124,505(81)	37,38,39
76	$3^4:8:2$	1296	88891880	131(2),124,506(81)	38,41,42
77	$3^2:D_8\cdot 2$	1296	177783760	127,130,132,250(18)	39
78	$3^4:8:2$	1296	88891880	124,132(2),507(81)	39,40,42
79	$3^4:4\cdot 4$	1296	177783760	126,127,132,508(81)	39
80	$3^4:Q_8\cdot 2$	1296	177783760	131,128,125,509(81)	41
81	$3^4:D_8\cdot 2$	1296	177783760	128,129,131,252(18)	41
82	$7^2:2^2:S_3$	1176	391849920	138,134,136,150(3),474(49)	9
83	$7^2:2^2:S_3$	1176	391849920	139,135,137,151(3),475(49)	10
84	$7^2:Q_8:3$	1176	195924960	152,188(4),476(49)	45
85	$7^2:Q_8:3$	1176	195924960	153,189(4),477(49)	46
86	$A_5\times(3^2:2)$	1080	213340512	141,166(6),165(6),216(5),230(6),287(10)	25,47
87	$3:A_6$	1080	71113504	167,231(6),232(6),288(10),319(15),320(15)	31,48(3)
88	$7^{1+2}:3$	1029	55978560	170,247(14),248(28),249(14)	49(2),50(2),51
89	$(3^2:2\times S_4)\cdot 2$	864	533351280	148,147,146,200(3),215(4),298(9)	19
90	$3^4:10$	810	284454016	149,243,549(81)	20,52
91	$7^2:Q_8\cdot 2$	784	293887440	152(2),154,508(49)	45(2),53
92	$7^2:Q_8\cdot 2$	784	293887440	155,153(2),509(49)	46(2),54
93	$7^2:16$	784	293887440	154,510(49)	53
94	$7^2:D_8:2$	784	293887440	150(2),154,511(49)	53
95	$7^2:16$	784	293887440	155,512(49)	54
96	$7^2:D_8:2$	784	293887440	151(2),155,513(49)	54
97	$4^3:A_4$	768	300010095	201,222(4),223(4),224(8),225(4)	30(2),55
98	$4^3\cdot A_4$	768	300010095	160,202,222(4)	56
99	$4^2\times 2\cdot S_4$	768	300010095	160,203(3),226(4)	56
100	$4^2\times 2\cdot S_4$	768	300010095	160,204(3),227(4)	56
101	$3^2:4:5:4$	720	640021536	164,162,161,257(5),312(9)	11
102	M_{10}	720	640021536	168,315(10),491(36),514(45)	23
103	M_{10}	720	640021536	169,313(10),492(36),515(45)	24
104	$2\times A_6$	720	160005384	167,282(12),321(10),387(30)	48(4),59,60,61
105	$2\cdot A_6$	720	160005384	283(12),322(10),385(30)	57
106	$2\cdot A_6$	720	160005384	284(12),323(10),384(30)	58
107	$7^{1+2}:2$	686	111957120	170,291,292(7)	49,62
108	$7^{1+2}:2$	686	111957120	170,293,294(7)	50,63
109	$7^{1+2}:2$	686	55978560	170,292(14),294(14),295(28)	51,62,63,64

Nr.	Struct.	Order	Length	Max. Subs	Minimal Overgroups
110	$L_3(2) : 2^2$	672	342868680	172(2),177,309(8),438(21),478(28)	2(2),67
111	$L_3(2) : 2^2$	672	342868680	173(2),177,310(8),439(21),479(28)	3(2),67
112	$4 \cdot L_3(2)$	672	342868680	176,296(14),308(8)	8,66
113	$4 \cdot L_3(2)$	672	342868680	175,297(14),308(8)	8,65
114	$4 \times L_3(2)$	672	342868680	177,298(14),308(8)	8,67
115	$2 \cdot L_3(2) : 2$	672	342868680	175,310(8),434(21),474(28)	9(2),65
116	$2 \cdot L_3(2) : 2$	672	342868680	176,309(8),435(21),475(28)	10(2),66
117	$2 \cdot L_3(2) : 2$	672	342868680	175,309(8),434(21),475(28)	65
118	$2 \cdot L_3(2) : 2$	672	342868680	176,310(8),435(21),474(28)	66
119	$L_2(11)$	660	698205312	368(11),370(11),375(12),537(55)	6,23,24
120	$3^4 : 4 \times 2$	648	177783760	178,180,181,215(12),324(9)	25,69,70,71
121	$3^4 : 4 \times 2$	648	88891880	182(2),181,325(18)	69(2),72,75(2)
122	$3^4 : 4 \times 2$	648	177783760	179(2),181,326(9),325(9)	69,70,73
123	$3^4 : 4 \times 2$	648	88891880	181,183(2),326(18)	70(2),72,74(2)
124	$3^4 : 4 \times 2$	648	88891880	181,183,182,327(18)	71,72,73,74,75,76,78
125	$3^4 : Q_8$	648	88891880	183(2),182,328(36)	72(2),74,80(2)
126	$3^4 : Q_8$	648	88891880	182(2),183,329(36)	72(2),75,79(2)
127	$3^4 : Q_8$	648	177783760	179(2),183,313(18),329(18)	73,77,79
128	$3^4 : Q_8$	648	177783760	179(2),182,315(18),328(18)	73,80,81
129	$3^4 : D_8$	648	88891880	181(2),182,330(36)	74,75(2),81(2)
130	$3^4 : D_8$	648	88891880	183,181(2),331(36)	74(2),75,77(2)
131	$3^4 : 8$	648	177783760	182,332(18)	76,80,81
132	$3^4 : 8$	648	177783760	183,333(18)	77,78,79
133	$4 \cdot 2^4 D_{10}$	640	720024228	184,185,186,265(5),403(16)	30
134	$7^2 : 3 \times 2^2$	588	391849920	187(2),188,218,310(14)	27(2),82
135	$7^2 : 3 \times 2^2$	588	391849920	189,190(2),219,309(14)	28(2),83
136	$7^2 : 3 : 4$	588	391849920	188,220(3),538(49)	45(2),82
137	$7^2 : 3 : 4$	588	391849920	189,221(3),538(49)	46(2),83
138	$7^2 : D_{12}$	588	391849920	192(2),188,218(3),539(49)	82
139	$7^2 : D_{12}$	588	391849920	193(2),189,219(3),540(49)	83
140	$3^2 : D_8 \cdot 2^3$	576	800026920	195,196,198,200,197,194,199,343(9)	7,11
141	$3^2 \times A_5$	540	213340512	232(2),231(2),289(5),305(6),376(10)	31,86
142	$Syl_2(\mathsf{O'N})$	512	900030285	202,203,205,206,207,204,201	55,56
143	$A_5 : D_8$	480	960032304	212,213,210,299(5),312(6),386(10)	7
144	$31 : 15$	465	991001088	246,304,532(31)	15,16
145	$4^3 : 7$	448	342868680	348,374(8)	68
146	$3^2 : 2 \times S_4$	432	533351280	217,214,216,251(6),253(6),261(3),287(4)	25,89
147	$3^2 : 4 \times A_4$	432	533351280	216,262,288(4),388(9)	47(2),89
148	$3^2 : 2 \times S_4$	432	533351280	216,263(3),290(4),389(9)	89
149	$3^4 : 5$	405	284454016	311,574(81)	90
150	$7^2 : D_8$	392	587774880	220,218(2),556(49)	82(2),94
151	$7^2 : D_8$	392	587774880	221,219(2),556(49)	83(2),96
152	$7^2 : Q_8$	392	195924960	220(3),557(49)	84,91(3)
153	$7^2 : Q_8$	392	195924960	221(3),558(49)	85,92(3)
154	$7^2 : 8$	392	293887440	220,559(49)	91,93,94
155	$7^2 : 8$	392	293887440	221,560(49)	92,95,96

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
156	$4 \cdot 2^4 : S_3$	384	1200040380	224,228,229,265(3),296(4),298(4),297(4)	30
157	$4^3 : S_3$	384	1200040380	226,227,222,266(3),299(4)	55,56
158	$4 \times 2^2 \cdot S_4$	384	1200040380	225,267(3),299(4)	55
159	$4 \times 2^2 \cdot S_4$	384	1200040380	223,268(3),299(4)	55
160	$4^2 \cdot Q_8 : 3$	384	300010095	269,300(4)	98,99,100
161	$3^2 : 5 : (4 \times 2)$	360	640021536	233(2),230,325(5),401(9)	17,101
162	$3^2 : 5 : (4 \times 2)$	360	640021536	234(2),230,326(5),402(9)	18,101
163	A_6	360	1280043072	369(6),367(6),405(10),473(15),471(15)	43,44
164	$3^2 : 5 : (4 \times 2)$	360	640021536	235,230,236,321(5),403(9)	47(2),101
165	$A_5 \times S_3$	360	1280043072	231,282(3),334(5),366(6),406(10)	48,86
166	$A_5 \times S_3$	360	1280043072	232,282(3),335(5),366(6),407(10)	48,86
167	A_6	360	17778376	371(12),408(10),480(30)	87(4),104(9)
168	A_6	360	640021536	368(12),409(10),472(30)	102
169	A_6	360	640021536	368(12),410(10),470(30)	103
170	7^{1+2}	343	55978560	379(2),380(4),381(2)	88,107(2),108(2),109
171	$L_3(2) : 2$	336	1371474720	241,397(8),511(21),539(28)	4
172	$L_3(2) : 2$	336	685737360	242,398(8),511(21),539(28)	4,110
173	$L_3(2) : 2$	336	685737360	242,399(8),513(21),540(28)	5,111
174	$L_3(2) : 2$	336	1371474720	240,397(8),513(21),540(28)	5
175	$2 \cdot L_3(2)$	336	342868680	384(14),400(8)	13(2),113,115,117
176	$2 \cdot L_3(2)$	336	342868680	385(14),400(8)	14(2),112,116,118
177	$2 \times L_3(2)$	336	342868680	242,387(14),400(8)	110,111,114
178	$3^4 : 4$	324	177783760	244,288(4),411(9)	31,120
179	$3^4 : 4$	324	355567520	243,412(9),405(36),409(18),410(18),413(9)	52(4),122,127,128
180	$3^4 : 4$	324	177783760	244,290(12),414(9)	120
181	$3^4 : 2^2$	324	88891880	244(2),243,287(24)	120(2),121,122(2),123, 124,129(2),130(2)
182	$3^4 : 4$	324	88891880	243,412(36),415(18),416(36)	121(2),124,125,126(2), 128(2),129,131(2)
183	$3^4 : 4$	324	88891880	243,415(18),413(36),417(36)	123(2),124,125(2),126, 127(2),130,132(2)
184	$2^{1+4}D_{10}$	320	720024228	245,341(5),490(16)	12(2),133
185	$2^{1+4}D_{10}$	320	720024228	245,349,493(16)	133
186	$2^{1+4} : D_{10}$	320	720024228	245,350(5),494(16)	133
187	$7^2 : 6$	294	783699840	247,292,400(7),399(7)	49,51,134
188	$7^2 : 6$	294	391849920	247,291,399(14)	49(2),84(2),134,136,138
189	$7^2 : 6$	294	391849920	249,293,398(14)	50(2),85(2),135,137,139
190	$7^2 : 6$	294	783699840	249,294,398(7),400(7)	50,51,135
191	$7^2 : 6$	294	1567399680	248,295,400(7),397(7)	51
192	$7^2 : S_3$	294	783699840	247,292(3),571(49)	138
193	$7^2 : S_3$	294	783699840	249,294(3),571(49)	139
194	$3^2 : 4^2 : 2$	288	800026920	254,255,256,440(9)	8,140
195	$3^2 : (8 \times 2) : 2$	288	800026920	255,258,261,436(9)	17,140
196	$3^2 : (8 \times 2) : 2$	288	800026920	254,259,261,437(9)	18,140
197	$3^2 : (8 \times 2) : 2$	288	800026920	259,264,255,441(9)	32(2),140
198	$(4 \times 3^2 : 4) \cdot 2$	288	800026920	259,257(2),258,260(2),256,442(9)	33(2),36(2),140
199	$3^2 : (8 \times 2) \cdot 2$	288	800026920	264,258,254,443(9)	34(2),140
200	$3^2 : 2^3 \cdot 2^2$	288	800026920	261,262(2),264,263(2),256,444(9)	89(2),140

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
201	$4^3 : 2^2$	256	300010095	265(6),270(3),271(3),266(3)	97,142(3)
202	$4^3 \cdot 2^2$	256	300010095	266(3),272(3),269	98,142(3)
203	$4^2 \cdot (8 : 2)$	256	900030285	271,273,269	99,142
204	$4^2 \cdot (8 : 2)$	256	900030285	270,274,269	100,142
205	$4^2 \cdot (D_8 : 2)$	256	900030285	272,275(2),274,271,267(2)	142
206	$4^3 \cdot 4$	256	900030285	266,274,273	142
207	$4^2 \cdot (D_8 : 2)$	256	900030285	272,276(2),273,270,268(2)	142
208	$4 \cdot A_5$	240	1920064608	284,390(5),403(6),481(10)	30,58
209	$4 \times A_5$	240	1920064608	282,388(5),403(6),481(10)	30,47,59
210	$4 \times A_5$	240	960032304	277,391(5),403(6),481(10)	30(2),143
211	$4 \cdot A_5$	240	1920064608	283,392(5),403(6),481(10)	30,57
212	$2 \times S_5$	240	960032304	278(2),277,393(5),402(6),478(10)	143
213	$2 \times S_5$	240	960032304	279(2),277,394(5),401(6),479(10)	143
214	$3^2 : S_4$	216	533351280	289,320(2),319(2),336(3),376(4)	31,146
215	$3^3 : 4 \times 2$	216	2133405120	288,287,290,321(3),481(9)	47,48,89,120
216	$3^2 : (2^3 : 3)$	216	533351280	289,337,334(6),335(6),377(4)	86(2),146,147,148
217	$3^2 : S_4$	216	533351280	289,317(12),318(12),314(6),316(6), 338(3),378(4)	146
218	$7^2 : 2^2$	196	391849920	292(2),291,467(14)	62(2),134,138(3),150(3)
219	$7^2 : 2^2$	196	391849920	294(2),293,468(14)	63(2),135,139(3),151(3)
220	$7^2 : 4$	196	293887440	291,575(49)	136(4),150(2),152(2),154
221	$7^2 : 4$	196	293887440	293,575(49)	137(4),151(2),153(2),155
222	$4^2 : 12$	192	1200040380	300,348,391(4)	68(2),97,98,157
223	$2^3 cdot(Q_8 : 3)$	192	1200040380	351,391(4)	97,159
224	$2^3 cdot(2^3 : 3)$	192	1200040380	301,349,390(4),388(4),392(4),391(8)	97(2),156
225	$2^3 cdot(Q_8 : 3)$	192	1200040380	352,391(4)	97,158
226	$4^2 : D_{12}$	192	1200040380	302(2),300,353(3),394(4)	99,157
227	$4^2 : D_{12}$	192	1200040380	303(2),300,354(3),393(4)	100,157
228	$2^3 \cdot S_4$	192	1200040380	301,355(3),389(4),385(4),384(4)	156
229	$2^3 \cdot S_4$	192	1200040380	301,356(3),387(4),383(4),382(4)	156
230	$D_{10} \times 3^2 : 2$	180	640021536	306,305,307,366(12),418(5)	86(2),161,162,164
231	$3 \times A_5$	180	426681024	371,419(5),464(6),496(10)	87,141,165(3)
232	$3 \times A_5$	180	426681024	371,420(5),464(6),497(10)	87,141,166(3)
233	$3^2 : 5 : 4$	180	1280043072	306,412(5),492(9)	161
234	$3^2 : 5 : 4$	180	1280043072	306,413(5),491(9)	162
235	$3^2 : 5 : 4$	180	640021536	307,421(5),490(9)	164
236	$3^2 : 20$	180	640021536	307,408,493(9)	164
237	$2^3 : 7 : 3$	168	2742949440	374,482(7),488(8)	6,68
238	$21 : 6$	168	2742949440	309,310,308,373,483(7)	22,65,66,67
239	$L_3(2)$	168	2742949440	473(7),471(7),488(8)	43,44
240	$L_3(2)$	168	1371474720	471(14),488(8)	43(2),174
241	$L_3(2)$	168	1371474720	473(14),488(8)	44(2),171
242	$L_3(2)$	168	342868680	484(14),489(8)	172(2),173(2),177
243	$3^4 : 2$	162	17778376	311,378(120)	90(16),179(20),181(5), 182(5),183(5)
244	$3^4 : 2$	162	177783760	311,377(4),376(12)	178,180,181
245	$2^{1+4}5$	160	720024228	428,549(16)	20(2),184,185,186
246	$31 : 5$	155	991001088	462,574(31)	144

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
247	$7^2:3$	147	391849920	379,489(14)	88(2),187(2),188,192(2)
248	$7^2:3$	147	1567399680	380,488(7),489(7)	88,191
249	$7^2:3$	147	391849920	381,489(14)	88(2),189,190(2),193(2)
250	$M_9:2$	144	3200107680	333,313,331,515(9)	2,24,77
251	$S_4\times S_3$	144	3200107680	319,314,334,387(3),386(3),406(4)	2,48,146
252	$M_9:2$	144	3200107680	332,315,330,514(9)	3,23,81
253	$S_4\times S_3$	144	3200107680	320,316,335,387(3),386(3),407(4)	3,48,146
254	$3^2:8\times 2$	144	800026920	322(2),324,516(9)	57(2),194,196,199
255	$3^2:8\times 2$	144	800026920	323(2),324,517(9)	58(2),194,195,197
256	$3^2:4^2$	144	800026920	327,324,321,518(9)	59(2),71(2),194,198,200
257	$4\times 3^2:4$	144	1600053840	326,325,321,519(9)	60,61,69,70,101(2),198
258	$3^2:4\cdot 4$	144	800026920	325(2),324,519(9)	69(2),195,198,199
259	$3^2:4\cdot 4$	144	800026920	326(2),324,519(9)	70(2),196,197,198
260	$3^2:Q_8:2$	144	1600053840	327,329(2),328(2),326,325,520(9)	72,73,198
261	$3^2:2^3:2$	144	800026920	338(2),337(2),339,324,336,386(12)	146(2),195,196,200
262	$3^2:4\times 2^2$	144	533351280	321(3),340(3),337,521(9)	147,200(3)
263	$3^2:2^3\cdot 2$	144	1600053840	340,327,337,522(9)	148,200
264	$3^2:4\cdot 4$	144	800026920	340(2),324,519(9)	197,199,200
265	$2^3\cdot 2^3\cdot 2$	128	1800060570	357,350(2),358,341(2),353,355(2),359, 356(2),354,360,349	133(2),156(2),201
266	$4^3:2$	128	900030285	354(2),361,348,353(2),357	157(4),201,202,206
267	$2^3\cdot 8:2$	128	1800060570	362,363,352	158(2),205
268	$2^3\cdot 8:2$	128	1800060570	364,365,351	159(2),207
269	$4^2\cdot D_8$	128	300010095	361(3)	160,202,203(3),204(3)
270	$4^2\cdot D_8$	128	900030285	361,359(2),354(2),351,358	201,204,207
271	$4^2\cdot D_8$	128	900030285	353(2),352,361,359(2),360	201,203,205
272	$4^2\cdot 2^3$	128	900030285	362(2),343(4),347(4),364(2),357(2),361	202,205,207
273	$4^2\cdot 8$	128	900030285	361,365(2)	203,206,207
274	$4^2\cdot 8$	128	900030285	361,363(2)	204,205,206
275	$(8\times 2)\cdot D_8$	128	1800060570	362,345(2),363,342(2),360	205
276	$(8\times 2)\cdot D_8$	128	1800060570	364,344(2),365,346(2),358	207
277	$2\times A_5$	120	960032304	370,482(5),494(6),537(10)	6(4),210,212,213
278	S_5	120	1920064608	370,472(5),491(6),537(10)	23(2),212
279	S_5	120	1920064608	370,470(5),492(6),537(10)	24(2),213
280	S_5	120	3840129216	369,473(5),492(6),540(10)	43
281	S_5	120	3840129216	367,471(5),491(6),539(10)	44
282	$2\times A_5$	120	1920064608	371,485(5),494(6),537(10)	104,165(2),166(2),209
283	$2\cdot A_5$	120	1920064608	477(5),490(6),538(10)	105,211
284	$2\cdot A_5$	120	1920064608	476(5),490(6),538(10)	106,208
285	$19:6$	114	4042241280	372,404,572(19)	2,3,6
286	$11:10$	110	4189231872	375,487,549(11)	6
287	$3^3:2^2$	108	2133405120	376,377,378,407(6),418(3),406(6)	86,146,181,215
288	$3^3:4$	108	711135040	377,408,541(9)	87,147(3),178,215(3)
289	$3^2\times A_4$	108	533351280	422(4),423(4),419(2),420(2),424,469(4)	141(2),214,216,217
290	$3^3:4$	108	2133405120	377,421(3),538(9)	148,180,215

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
291	$7^2:2$	98	13994640	379,533(56)	107(8),188(28),218(28),220(21)
292	$7^2:2$	98	783699840	379,533,534(7)	107,109,187,192(3),218
293	$7^2:2$	98	13994640	381,535(56)	108(8),189(28),219(28),221(21)
294	$7^2:2$	98	783699840	381,535,534(7)	108,109,190,193(3),219
295	$7^2:2$	98	1567399680	380,536,534(7)	109,191
296	$Q_8 \cdot D_{12}$	96	4800161520	385,382,392,445(3),481(4)	57,112,156
297	$Q_8 \cdot D_{12}$	96	4800161520	384,383,390,446(3),481(4)	58,113,156
298	$2^3 \cdot D_{12}$	96	4800161520	388,387,389,444(3),481(4)	59,89,114,156
299	$2^3 \cdot D_{12}$	96	4800161520	394,393,391,447(3),486(4)	143,157,158,159
300	$4^2:6$	96	1200040380	395,448,482(4)	160,222,226,227
301		96	1200040380	449,476(4),485(4),482(8),477(4)	224,228,229
302	$4^2:S_3$	96	2400080760	395,450(3),470(4)	226
303	$4^2:S_3$	96	2400080760	395,451(3),472(4)	227
304	$31:3$	93	991001088	462,579(31)	144
305	$3^2 \times 5:2$	90	640021536	396,464(4),498(5)	141(2),230
306	$3^2 \times 5:2$	90	640021536	396,463(12),499(5)	230,233(2),234(2)
307	$3^2 \times 5:2$	90	640021536	396,465(12),500	230,235,236
308	$28:3$	84	2742949440	400,466,541(7)	29,112,113,114,238
309	$D_{28}:3$	84	2742949440	400,398(2),468,542(7)	110,116,117,135(2),238
310	$D_{28}:3$	84	2742949440	400,399(2),467,543(7)	111,115,118,134(2),238
311	3^4	81	17778376	469(40)	149(16),243,244(10)
312	$D_{10}:D_8$	80	5760193824	401,403,402,519(5)	32,33,34,101,143
313	$3^2:Q_8$	72	3200107680	410(2),417,557(9)	4,103(2),127,250
314	$3^2:Q_8$	72	3200107680	419,474(3),484(3),473(6),501(4)	4,217,251
315	$3^2:Q_8$	72	3200107680	409(2),416,558(9)	5,102(2),128,252
316	$3^2:Q_8$	72	3200107680	420,475(3),471(6),484(3),502(4)	5,217,253
317	$(A_4 \times 3):2$	72	6400215360	422,473(6),471(3),475(3),503(4)	43,217
318	$(A_4 \times 3):2$	72	6400215360	423,474(3),473(3),471(6),503(4)	44,217
319	$3 \times S_4$	72	1066702560	419,480,483(3),496(4)	87,214,251(3)
320	$3 \times S_4$	72	1066702560	420,480,483(3),497(4)	87,214,253(3)
321	$2 \times 3^2:4$	72	800026920	421,418,408,561(9)	104(2),164(4),215(8),256,257(2),262(2)
322	$3^2:8$	72	1600053840	414,560(9)	105,254
323	$3^2:8$	72	1600053840	414,559(9)	106,255
324	$3^2:4 \times 2$	72	800026920	414,411,418,481(12)	120(2),254,255,256,258,259,261,264
325	$2 \times 3^2:4$	72	1600053840	412(2),418,562(9)	121,122,161(2),257,258,260
326	$2 \times 3^2:4$	72	1600053840	413(2),418,562(9)	122,123,162(2),257,259,260
327	$2 \times 3^2:4$	72	800026920	415(2),418,563(9)	124(2),256,260(2),263(2)
328	$3^2:Q_8$	72	3200107680	412,413,415,558(9)	125,128,260
329	$3^2:Q_8$	72	3200107680	415,412,413,557(9)	126,127,260
330	$3^2:D_8$	72	3200107680	416,407(2),564(9)	129,252
331	$3^2:D_8$	72	3200107680	406(2),417,565(9)	130,250
332	$3^2:8$	72	3200107680	416,560(9)	131,252
333	$3^2:8$	72	3200107680	417,559(9)	132,250
334	$S_3:A_4$	72	3200107680	419,479,485(3),496(4)	165(2),216,251
335	$S_3:A_4$	72	3200107680	420,478,485(3),497(4)	166(2),216,253
336	$12:6$	72	800026920	424(2),411,483(4)	214,261

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
337	$3^2 : 2^3$	72	533351280	425(3),418(3),424,479(6),478(6)	216,261(3),262,263(3)
338	$3^2 : D_8$	72	1600053840	424,414,425,474(6),475(6)	217,261
339	$12 : S_3$	72	800026920	411,425(2),486(12)	261
340	$3^2 : 4 \times 2$	72	1600053840	421(2),418,562(9)	262,263,264
341	$4^2 \cdot 2^2$	64	3600121140	429,433,431,428,442,432,430	26(2),184,265
342	$8 \cdot D_8$	64	3600121140	441,452,446	32(2),275
343	$4^2 \cdot 2^2$	64	3600121140	436,441,437,443,444,440,442	33(2),140(2),272
344	$8 \cdot D_8$	64	3600121140	443,453,445	34(2),276
345	$16 : 2^2$	64	3600121140	438,434(2),452,446,426(2)	65(2),275
346	$16 : 2^2$	64	3600121140	439,453,427(2),435(2),445	66(2),276
347	$4^2 \cdot 2^2$	64	3600121140	444,440,438,437,439,454,436	67(2),272
348	4^3	64	42858585	448(7)	145(8),222(28),266(21)
349	$2^3 \cdot 2^3$	64	120004038	449(10),455(15),428(6)	185(6),224(10),265(15)
350	$4^2 \cdot 2^2$	64	3600121140	456,451,457,433,458,450,428	186,265
351	$2^3 \cdot Q_8$	64	300010095	459(3)	223(4),268(6),270(3)
352	$2^3 \cdot Q_8$	64	300010095	460(3)	225(4),267(6),271(3)
353	$4^2 : 2^2$	64	1800060570	448,450(2),431(2),460,455	226(2),265,266,271
354	$4^2 : 2^2$	64	1800060570	448,451(2),429(2),459,455	227(2),265,266,270
355	$4^2 : 2^2$	64	3600121140	449,432,451,456,430,450,461	228,265
356	$4^2 : 2^2$	64	3600121140	454,457,429,461,431,458,449	229,265
357	$4^2 \cdot 2^2$	64	900030285	447(4),444(4),448,456(2),455(2),442,454	265(2),266,272(2)
358	$8 \cdot 2^3$	64	900030285	459,458(4),445(4),455(2),432(4)	265(2),270,276(2)
359	$2^3 \cdot 4 \times 2$	64	1800060570	459,433(2),460,455,461(2)	265,270,271
360	$8 \cdot 2^3$	64	900030285	455(2),446(4),430(4),460,457(4)	265(2),271,275(2)
361	$4^2 \cdot 2^2$	64	900030285	460,440(4),459,448	266,269,270,271,272,273,274
362	$8 : D_8$	64	1800060570	436(2),438,441,447(2),460	267,272,275
363	$8 : D_8$	64	1800060570	460,452(2)	267,274,275
364	$8 : D_8$	64	1800060570	447(2),437(2),459,439,443	268,272,276
365	$8 \cdot D_8$	64	1800060570	459,453(2)	268,273,276
366	$S_3 \times D_{10}$	60	7680258432	464,465,463,494(3),537(5)	6,165,166,230
367	A_5	60	3840129216	544(5),550(6),571(10)	15(2),163(2),281
368	A_5	60	7680258432	545(5),550(6),571(10)	15,16,119,168,169
369	A_5	60	3840129216	546(5),550(6),571(10)	16(2),163(2),280
370	A_5	60	960032304	545(5),550(6),573(10)	119(8),277,278(2),279(2)
371	A_5	60	213340512	547(5),551(6),573(10)	167,231(2),232(2),282(9)
372	$19 : 3$	57	4042241280	495,579(19)	4,5,285
373	D_{56}	56	2742949440	466,468,467,566(7)	35,238
374	$4^2 : 7$	56	2742949440	567,569(8)	145,237
375	$11 : 5$	55	4189231872	548,574(11)	119(2),286
376	$S_3 \times 3^2$	54	2133405120	469,497(2),496(2),498(3)	141,214,244,287
377	$3^3 : 2$	54	711135040	469,500,496(6),497(6)	216(3),244,287(3),288,290(3)
378	$3^3 : 2$	54	2133405120	469,501(6),502(6),503(24),499(3)	217,243,287
379	7^2	49	13994640	570(8)	170(8),247(28),291,292(56)
380	7^2	49	223914240	569(7),570	170,248(7),295(7)
381	7^2	49	13994640	570(8)	170(8),249(28),293,294(56)

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
382	$GL_2(3)$	48	4800161520	477,514(3),537(4)	23(2),229,296
383	$GL_2(3)$	48	4800161520	476,515(3),537(4)	24(2),229,297
384	$Q_8 : 3$	48	4800161520	476,508(3),538(4)	45(2),106,175,228,297
385	$Q_8 : 3$	48	4800161520	477,509(3),538(4)	46(2),105,176,228,296
386	$12 : 2^2$	48	9600323040	475,483,474,478,479, 486,481,523(3)	65,66,67,143,251,253,261
387	$2 \times S_4$	48	4800161520	484,485,480,523(3),537(4)	104,177,229,251(2),253(2),298
388	$2^3 \cdot 6$	48	4800161520	485,521,541(4)	147,209(2),224,298
389	$2^3 \cdot S_3$	48	4800161520	485,522(3),538(4)	148,228,298
390	$Q_8 \cdot 6$	48	4800161520	476,524,541(4)	208(2),224,297
391	$2^3 \cdot 6$	48	4800161520	482,525,541(4)	210,222,223,224(2),225,299
392	$Q_8 \cdot 6$	48	4800161520	477,526,541(4)	211(2),224,296
393	$2 \times S_4$	48	4800161520	472(2),482,527(3),539(4)	212,227,299
394	$2 \times S_4$	48	4800161520	482,470(2),527(3),540(4)	213,226,299
395	$4 \cdot A_4$	48	1200040380	528,545(4)	300,302(2),303(2)
396	15×3	45	640021536	532(4),552	305,306,307
397	$7 : 6$	42	10971797760	488,536,572(7)	6,171,174,191
398	$7 : 6$	42	5485898880	489,535,572(7)	172,189,190,309
399	$7 : 6$	42	5485898880	489,533,572(7)	173,187,188,310
400	$14 : 3$	42	2742949440	489,534,572(7)	175,176,177,187(2),190(2),191(4),308, 309,310
401	$10 : 4$	40	5760193824	492(2),494,562(5)	60,161,213,312
402	$10 : 4$	40	5760193824	494,491(2),562(5)	61,162,212,312
403	$20 : 2$	40	5760193824	490,493,494,561(5)	133(2),164,208(2),209(2),210,211(2),312
404	D_{38}	38	4042241280	495,580(19)	285
405	$3^2 : 4$	36	12800430720	503,575(9)	163,179
406	$S_3 \times S_3$	36	6400215360	496(2),501,537(6)	165(2),251(2),287(2),331
407	$S_3 \times S_3$	36	6400215360	497(2),502,537(6)	166(2),253(2),287(2),330
408	$3^2 : 4$	36	17778376	500,576(9)	167(10),236(36),288(40),321(45)
409	$3^2 : 4$	36	6400215360	502,575(9)	168,179,315
410	$3^2 : 4$	36	6400215360	501,575(9)	169,179,313
411	12×3	36	800026920	498,541(4)	178(2),324,336,339
412	$3^2 : 4$	36	3200107680	499,575(9)	179,182,233(2),325,328,329
413	$3^2 : 4$	36	3200107680	499,575(9)	179,183,234(2),326,328,329
414	$3^2 : 4$	36	800026920	498,538(12)	180(2),322(2),323(2),324,338(2)
415	$3^2 : 4$	36	1600053840	499,575(9)	182,183,327,328(2),329(2)
416	$3^2 : 4$	36	3200107680	502,575(9)	182,315,330,332
417	$3^2 : 4$	36	3200107680	501,575(9)	183,313,331,333
418	$2 \times 3^2 : 2$	36	800026920	499,498,500,537(12)	230(4),287(8),321,324,325(2),326(2), 327,337(2),340(2)
419	$3 \times A_4$	36	1066702560	546(2),543,547,553(4)	231(2),289,314(3),319,334(3)
420	$3 \times A_4$	36	1066702560	542,544(2),547,554(4)	232(2),289,316(3),320,335(3)
421	$3^2 : 4$	36	800026920	500,575(9)	235(4),290(8),321,340(4)
422	$3 \times A_4$	36	2133405120	546(2),542,544,555(4)	289,317(3)
423	$3 \times A_4$	36	2133405120	546,543,544(2),555(4)	289,318(3)
424	$3^2 \times 2^2$	36	533351280	498(3),542(2),543(2)	289,336(3),337,338(3)
425	$3^2 : 2^2$	36	1600053840	498,499(2),540(6),539(6)	337,338,339

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
426	D_{32}	32	7200242280	529,510,511	15(2),345
427	D_{32}	32	7200242280	530,512,513	16(2),346
428	2^{1+4}	32	720024228	504(5),520(5),505(5)	37(2),245,341(5),349,350(5)
429	$2^{1+3} \cdot 2$	32	3600121140	518,506,505	38(2),341,354,356
430	$8 \cdot 2^3$	32	3600121140	515(2),508(2),505,507,520	39(2),341,355,360
431	$2^{1+3} \cdot 2$	32	3600121140	518,507,504	40(2),341,353,356
432	$8 \cdot 2^3$	32	3600121140	504,509(2),514(2),506,520	41(2),341,355,358
433	$2^{1+3} \cdot 2$	32	3600121140	507,506,520	42(2),341,350,359
434	$16 : 2$	32	7200242280	511,508,510	53(2),115,117,345
435	$16 : 2$	32	7200242280	513,512,509	54(2),116,118,346
436	$(8 \times 2) : 2$	32	3600121140	523,517,519	60(2),195(2),343,347,362
437	$(8 \times 2) : 2$	32	3600121140	523,516,519	61(2),196(2),343,347,364
438	$D_8 : 2^2$	32	1800060570	511(4),523(2),517	110(4),345(2),347(2),362
439	$D_8 : 2^2$	32	1800060570	513(4),523(2),516	111(4),346(2),347(2),364
440	$4^2 \cdot 2$	32	3600121140	518,516,517	194(2),343,347,361
441	$(8 \times 2) \cdot 2$	32	1800060570	517,519(2)	197(4),342(2),343(2),362
442	$2^2 \cdot 2^3$	32	900030285	520(2),518,519(4)	198(8),341(4),343(4),357
443	$(8 \times 2) \cdot 2$	32	1800060570	516,519(2)	199(4),343(2),344(2),364
444	$2^3 \cdot 2^2$	32	3600121140	523,518,522(2),521(2),519	200(2),298(4),343,347,357
445	$Q_8 \cdot 2^2$	32	3600121140	526(2),516,530,514(2),509	296(4),344,346,358
446	$Q_8 \cdot 2^2$	32	3600121140	524(2),517,529,515(2),508	297(4),342,345,360
447	$2^3 \cdot 2^2$	32	3600121140	527(2),525,522(2),519,523	299(4),357,362,364
448	$4^2 \times 2$	32	300010095	525(3),518(3),528	300(4),348,353(6),354(6),357(3),361(3)
449	$2^3 : 2^2$	32	1200040380	531(3),527(6),504(3),505(3)	301,349,355(3),356(3)
450	$4^2 : 2$	32	3600121140	528,504,507	302(2),350,353,355
451	$4^2 : 2$	32	3600121140	528,506,505	303(2),350,354,355
452	$16 : 2$	32	3600121140	517,510(2)	342,345,363
453	$16 : 2$	32	3600121140	516,512(2)	344,346,365
454	$4^2 : 2$	32	900030285	531(2),523(4),518	347(4),356(4),357
455	$D_8 \cdot 2^2$	32	1800060570	524(2),525,504(2),521(2),505(2), 520,526(2),527(2),531	349,353,354,357,358,359,360
456	$4^2 : 2$	32	1800060570	531,528,522(4),520	350(2),355(2),357
457	$8 : 2^2$	32	3600121140	531,529(2),507,505,515(2)	350,356,360
458	$8 : 2^2$	32	3600121140	531,530(2),506,504,514(2)	350,356,358
459	$8 : 2^2$	32	900030285	506(4),525,516(2)	351,354(2),358,359(2),361,364(2),365(2)
460	$8 : 2^2$	32	900030285	507(4),525,517(2)	352,353(2),359(2),360,361,362(2),363(2)
461	$2^3 \cdot 4$	32	3600121140	531,506,507	355,356,359
462	31	31	991001088	580	246,304
463	D_{30}	30	7680258432	532,550(3),571(5)	15(2),16(2),306,366
464	$15 : 2$	30	2560086144	532,551,572(5)	231,232,305,366(3)
465	$S_3 \times 5$	30	7680258432	532,549(3),573	307,366
466	28	28	2742949440	534,576	64,308,373
467	D_{28}	28	2742949440	534,533(2),577(7)	218(2),310,373
468	D_{28}	28	2742949440	534,535(2),577(7)	219(2),309,373
469	3^3	27	711135040	555(8),553(2),554(2),552	289(3),311,376(3),377,378(3)

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
470	S_4	24	9600323040	545,565(3),571(4)	15(2),169(2),279,302,394
471	S_4	24	19200646080	544,556(3),571(4)	15,163,239,240,281,316,317,318(2)
472	S_4	24	9600323040	545,564(3),571(4)	16(2),168(2),278,303,393
473	S_4	24	19200646080	546,556(3),571(4)	16,163,239,241,280,314,317(2),318
474	$3 : D_8$	24	9600323040	538,543,539,556(3)	82(2),115,118,314,318(2),338,386
475	$3 : D_8$	24	9600323040	538,542,540,556(3)	83(2),116,117,316,317(2),338,386
476	$Q_8 : 3$	24	4800161520	557,572(4)	84(2),284(2),301,383,384,390
477	$Q_8 : 3$	24	4800161520	558,572(4)	85(2),283(2),301,382,385,392
478	$3 : 2^3$	24	3200107680	537(3),539(3),542,568(3)	110(3),212(3),335,337,386(3)
479	$3 : 2^3$	24	3200107680	540(3),537(3),543,568(3)	111(3),213(3),334,337,386(3)
480	S_4	24	533351280	547,566(3),573(4)	167,319(2),320(2),387(9)
481	$3 : 4 \times 2$	24	9600323040	538,541,537,561(3)	208(2),209(2),210,211(2),215(2), 296(2),297(2),298(2),324,386
482	$2^3 : 3$	24	4800161520	545,567,572(4)	237(4),277,300,301(2),391,393,394
483	$3 \times D_8$	24	3200107680	541,542,543,566	238(6),319,320,336,386(3)
484	S_4	24	4800161520	547,556(3),571(4)	242,314(2),316(2),387
485	$2^3 : 3$	24	4800161520	547,568,572(4)	282(2),301,334(2),335(2),387,388,389
486	$3 : D_8$	24	9600323040	541,540,539,566(3)	299(2),339,386
487	D_{22}	22	4189231872	548,580(11)	286
488	$7 : 3$	21	10971797760	569,579(7)	237(2),239(2),240,241,248,397
489	$7 : 3$	21	2742949440	570,579(7)	242,247(2),248(4),249(2),398(2),399(2),400
490	$5 : 4$	20	5760193824	549,575(5)	52(4),184(2),235,283(2),284(2),403
491	$5 : 4$	20	11520387648	550,575(5)	102(2),234,278,281(2),402
492	$5 : 4$	20	11520387648	550,575(5)	103(2),233,279,280(2),401
493	20	20	5760193824	549,576	185(2),236,403
494	D_{20}	20	5760193824	551,549,550,578(5)	186(2),277,282(2),366(4),401,402,403
495	19	19	4042241280	580	372,404
496	$3^2 : 2$	18	4266810240	553,573,572(3)	231,319,334(3),376,377,406(3)
497	$3^2 : 2$	18	4266810240	554,573,572(3)	232,320,335(3),376,377,407(3)
498	$3^2 \times 2$	18	800026920	552,572(4)	305(4),376(8),411,414,418,424(2),425(2)
499	$3^2 : 2$	18	800026920	552,571(12)	306(4),378(8),412(4),413(4),415(2), 418,425(4)
500	$3^2 : 2$	18	17778376	552,573(12)	307(36),377(40),408,418(45),421(45)
501	$3^2 : 2$	18	3200107680	553,571(12)	314(4),378(4),406(2),410(2),417
502	$3^2 : 2$	18	3200107680	554,571(12)	316(4),378(4),407(2),409(2),416
503	$3^2 : 2$	18	12800430720	555,571(12)	317(2),318(2),378(4),405
504	$D_8 : 2$	16	3600121140	565(2),563,562(2),558,564	74(2),428,431,432,449,450,455,458
505	$D_8 : 2$	16	3600121140	564(2),562(2),563,557,565	75(2),428,429,430,449,451,455,457
506	$8 : 2$	16	3600121140	563,560(2)	76(2),429,432,433,451,458,459,461
507	$8 : 2$	16	3600121140	563,559(2)	78(2),430,431,433,450,457,460,461
508	$Q_8 \cdot 2$	16	3600121140	559,557(2)	79(4),91(4),384(4),430(2),434(2),446
509	$Q_8 \cdot 2$	16	3600121140	558(2),560	80(4),92(4),385(4),432(2),435(2),445
510	16	16	7200242280	559	93(2),426,434,452
511	$D_8 : 2$	16	7200242280	556(2),559	94(2),171(4),172(2),426,434,438
512	16	16	7200242280	560	95(2),427,435,453
513	$D_8 : 2$	16	7200242280	556(2),560	96(2),173(2),174(4),427,435,439
514	$8 : 2$	16	7200242280	564,560,558	102(4),252(4),382(2),432,445,458
515	$8 : 2$	16	7200242280	565,559,557	103(4),250(4),383(2),430,446,457

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
516	8×2	16	1800060570	561,560(2)	254(4),437(2),439,440(2),443,445(2),453(2),459
517	8×2	16	1800060570	561,559(2)	255(4),436(2),438,440(2),441,446(2),452(2),460
518	4×2^2	16	900030285	561(2),563	256(8),429(4),431(4),440(4),442,444(4),448,454
519	$4 \cdot 4$	16	3600121140	561,562(2)	257(4),258(2),259(2),264(2),312(8),436,437, 441,442,443,444,447
520	$Q_8 : 2$	16	1800060570	557(2),558(2),563,562(2)	260(8),428(2),430(2),432(2),433(2),442,455,456
521	4×2^2	16	1200040380	561(3),562(3),568	262(4),388(4),444(6),455(3)
522	$2^3 \cdot 2$	16	7200242280	568,562,563	263(2),389(2),444,447,456
523	$2^3 : 2$	16	3600121140	568(2),561,556(2),566(2)	386(8),387(4),436,437,438,439,444,447,454
524	$Q_8 : 2$	16	1200040380	561(3),565(3),557	390(4),446(6),455(3)
525	4×2^2	16	300010095	563(3),561(3),567	391(16),447(12),448(3),455(6),459(3),460(3)
526	$Q_8 : 2$	16	1200040380	564(3),561(3),558	392(4),445(6),455(3)
527	$Q_8 : 2$	16	3600121140	567,562,564(2),565(2),568	393(4),394(4),447(2),449(2),455
528	4×4	16	300010095	563(3)	395(4),448,450(12),451(12),456(6)
529	$Q_8 : 2$	16	3600121140	565(2),559	426(2),446,457(2)
530	$Q_8 : 2$	16	3600121140	564(2),560	427(2),445,458(2)
531	$Q_8 : 2$	16	1800060570	564(2),565(2),568(2),563	449(2),454,455,456,457(2),458(2),461(2)
532	15	15	2560086144	574,579	144(12),396,463(3),464,465(3)
533	D_{14}	14	783699840	570,580(7)	291,292,399(7),467(7)
534	14	14	2742949440	570,580	292(2),294(2),295(2),400,466,467,468
535	D_{14}	14	783699840	570,580(7)	293,294,398(7),468(7)
536	D_{14}	14	1567399680	569,580(7)	295,397(7)
537	D_{12}	12	9600323040	573,572,571,578(3)	119(4),277,278(2),279(2),282(2),366(4),382(2), 383(2),387(2),406(4),407(4),418,478,479,481
538	$3 : 4$	12	9600323040	572,575(3)	136(2),137(2),283(2),284(2),290(2),384(2), 385(2),389(2),414,474,475,481
539	D_{12}	12	9600323040	571(2),572,577(3)	138(2),171(4),172(2),281(4),393(2),425,474, 478,486
540	D_{12}	12	9600323040	571(2),572,577(3)	139(2),173(2),174(4),280(4),394(2),425,475, 479,486
541	12	12	3200107680	572,576	288(2),308(6),388(6),390(6),391(6),392(6), 411,481(3),483,486(3)
542	6×2	12	1066702560	572(3),577	309(18),420,422(2),424,475(9),478(3),483(3)
543	6×2	12	1066702560	572(3),577	310(18),419,423(2),424,474(9),479(3),483(3)
544	A_4	12	2133405120	577,579(4)	367(9),420,422,423(2),471(9)
545	A_4	12	4800161520	578,579(4)	368(8),370,395,470(2),472(2),482
546	A_4	12	2133405120	577,579(4)	369(9),419,422(2),423,473(9)
547	A_4	12	533351280	577,579(4)	371(2),419(2),420(2),480,484(9),485(9)
548	11	11	4189231872	580	375,487
549	10	10	5760193824	574,580	90(4),245(2),286(8),465(4),490,493,494
550	D_{10}	10	5760193824	574,580(5)	367(4),368(8),369(4),370,463(4),491(2), 492(2),494
551	D_{10}	10	640021536	574,580(5)	371(2),464(4),494(9)
552	3^2	9	17778376	579(4)	396(36),469(40),498(45),499(45),500
553	3^2	9	355567520	579(4)	419(12),469(4),496(12),501(9)

Nr.	Struct.	Order	Length	Max. Subs	Min. Over.
554	3^2	9	355567520	579(4)	420(12),469(4),497(12),502(9)
555	3^2	9	1422270080	579(4)	422(6),423(6),469(4),503(9)
556	D_8	8	7200242280	577(2),575	150(4),151(4),471(8),473(8),474(4),475(4),484(2), 511(2),513(2),523
557	Q_8	8	1200040380	575(3)	152(8),313(24),329(24),476(4),505(3),508(6),515(6), 520(3),524
558	Q_8	8	1200040380	575(3)	153(8),315(24),328(24),477(4),504(3),509(6),514(6), 520(3),526
559	8	8	3600121140	575	154(4),323(4),333(8),507(2),508,510(2),511(2),515(2), 517,529
560	8	8	3600121140	575	155(4),322(4),332(8),506(2),509,512(2),513(2),514(2), 516,530
561	4×2	8	900030285	576,575,578	321(8),403(32),481(32),516(2),517(2),518(2), 519(4),521(4),523(4),524(4),525,526(4)
562	4×2	8	3600121140	578,575(2)	325(4),326(4),340(4),401(8),402(8),504(2), 505(2),519(2),520,521,522(2),527
563	4×2	8	900030285	578,575(2)	327(8),504(4),505(4),506(4),507(4),518,520(2), 522(8),525,528,531(2)
564	D_8	8	3600121140	578(2),575	330(8),472(8),504,505(2),514(2),526,527(2),530(2),531
565	D_8	8	3600121140	578(2),575	331(8),470(8),504(2),505,515(2),524,527(2),529(2),531
566	D_8	8	800026920	577(2),576	373(24),480(2),483(4),486(36),523(9)
567	2^3	8	42858585	578(7)	374(64),482(112),525(7),527(84)
568	2^3	8	1200040380	578(3),577(4)	478(8),479(8),485(4),521,522(6),523(6),527(3),531(3)
569	7	7	1567399680	580	374(14),380,488(7),536
570	7	7	55978560	580	379(2),380(4),381(2),489(49),533(14),534(49),535(14)
571	S_3	6	9600323040	579,580(3)	192(4),193(4),367(4),368(8),369(4),463(4),470(4), 471(8),472(4),473(8),484(2),499,501(4),502(4),503(16), 537,539(2),540(2)
572	6	6	3200107680	579,580	285(24),397(24),398(12),399(12),400(6),464(4),476(6), 477(6),482(6),485(6),496(4),497(4),498,537(3),538(3), 539(3),540(3),541,542,543
573	S_3	6	213340512	579,580(3)	370(45),371(10),465(36),480(10),496(20),497(20), 500,537(45)
574	5	5	640021536	580	149(36),246(48),375(72),532(4),549(9),550(9),551
575	4	4	900030285	580	220(16),221(16),405(128),409(64),410(64),412(32), 413(32),415(16),416(32),417(32),421(8),490(32),491(64), 492(64),538(32),556(8),557(4),558(4),559(4),560(4),561, 562(8),563(2),564(4),565(4)
576	4	4	2857239	580	408(56),466(960),493(2016),541(1120),561(315),566(280)
577	2^2	4	533351280	580(3)	467(36),468(36),539(54),540(54),542(2),543(2), 544(4),546(4),547,556(27),566(3),568(9)
578	2^2	4	300010095	580(3)	494(96),537(96),545(16),561(3),562(12),563(3), 564(24),565(24),567,568(12)
579	3	3	71113504	580	304(432),372(1080),488(1080),489(270),532(36),544(120), 545(270),546(120),547(30),552,553(20),554(20),555(80), 571(135),572(45),573(3)
580	2	2	2857239	580	404(26880),487(16128),533(1920),534(960),535(1920), 536(3840),549(2016),550(10080),551(1120),571(10080), 572(1120),573(224),575(315),576,577(560),578(315)
581	1	1	1	20	462(991001088),495(4042241280),548(4189231872), 569(1567399680),570(55978560),574(640021536), 579(71113504),580(2857239)

G	Factored order of G	Degree of G	Reference
M_{11}	$2^4 \cdot 3^2 \cdot 5 \cdot 11$	11	Buekenhout [3], 1984
M_{12}	$2^6 \cdot 3^3 \cdot 5 \cdot 11$	12	Buekenhout-Rees [4], 1988
J_1	$2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$	266	Buekenhout [3], 1984
M_{22}	$2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	22	Pfeiffer [12], 1997
J_2	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7$	100	Pahlings [11], 1987
M_{23}	$2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	23	Pfeiffer [12], 1997
HS	$2^9 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 11$	100	
J_3	$2^7 \cdot 3^5 \cdot 5 \cdot 17 \cdot 19$	6156	Pfeiffer, 1991
M_{24}	$2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	24	Pfeiffer [12], 1997
McL	$2^7 \cdot 3^6 \cdot 5^3 \cdot 7 \cdot 11$	275	Pfeiffer [12], 1997
He	$2^{10} \cdot 3^3 \cdot 5^2 \cdot 7^3 \cdot 17$	2058	Merkwitz, 1997
Ru	$2^{14} \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 13 \cdot 29$	4060	
Suz	$2^{13} \cdot 3^7 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1782	
$\mathsf{O'N}$	$2^9 \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 11 \cdot 19 \cdot 31$	122760	Holt, 1998 (subgroups only)
Co_3	$2^{10} \cdot 3^7 \cdot 5^3 \cdot 7 \cdot 11 \cdot 23$	276	Merkwitz, 1997
Co_2	$2^{18} \cdot 3^6 \cdot 5^3 \cdot 7 \cdot 11 \cdot 23$	2300	
Fi_{22}	$2^{17} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	3510	
HN	$2^{14} \cdot 3^6 \cdot 5^6 \cdot 7 \cdot 11 \cdot 19$	1140000	
Ly	$2^8 \cdot 3^7 \cdot 5^6 \cdot 7 \cdot 11 \cdot 31 \cdot 37 \cdot 67$	8835156	
Th	$2^{15} \cdot 3^{10} \cdot 5^3 \cdot 7^2 \cdot 13 \cdot 19 \cdot 31$	143127000	
Fi_{23}	$2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$	31671	
Co_1	$2^{21} \cdot 3^9 \cdot 5^4 \cdot 7^2 \cdot 11 \cdot 13 \cdot 23$	98280	
J_4	$2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11^3 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot 43$	173067389	
Fi'_{24}	$2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$	920808	
BM	$2^{41} \cdot 3^{13} \cdot 5^6 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 31 \cdot 47$	13571955000	
M	$2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$	97239461142009186000	

Table 2: Sporadic groups and what is known about their subgroup lattice

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