The Hole Problem

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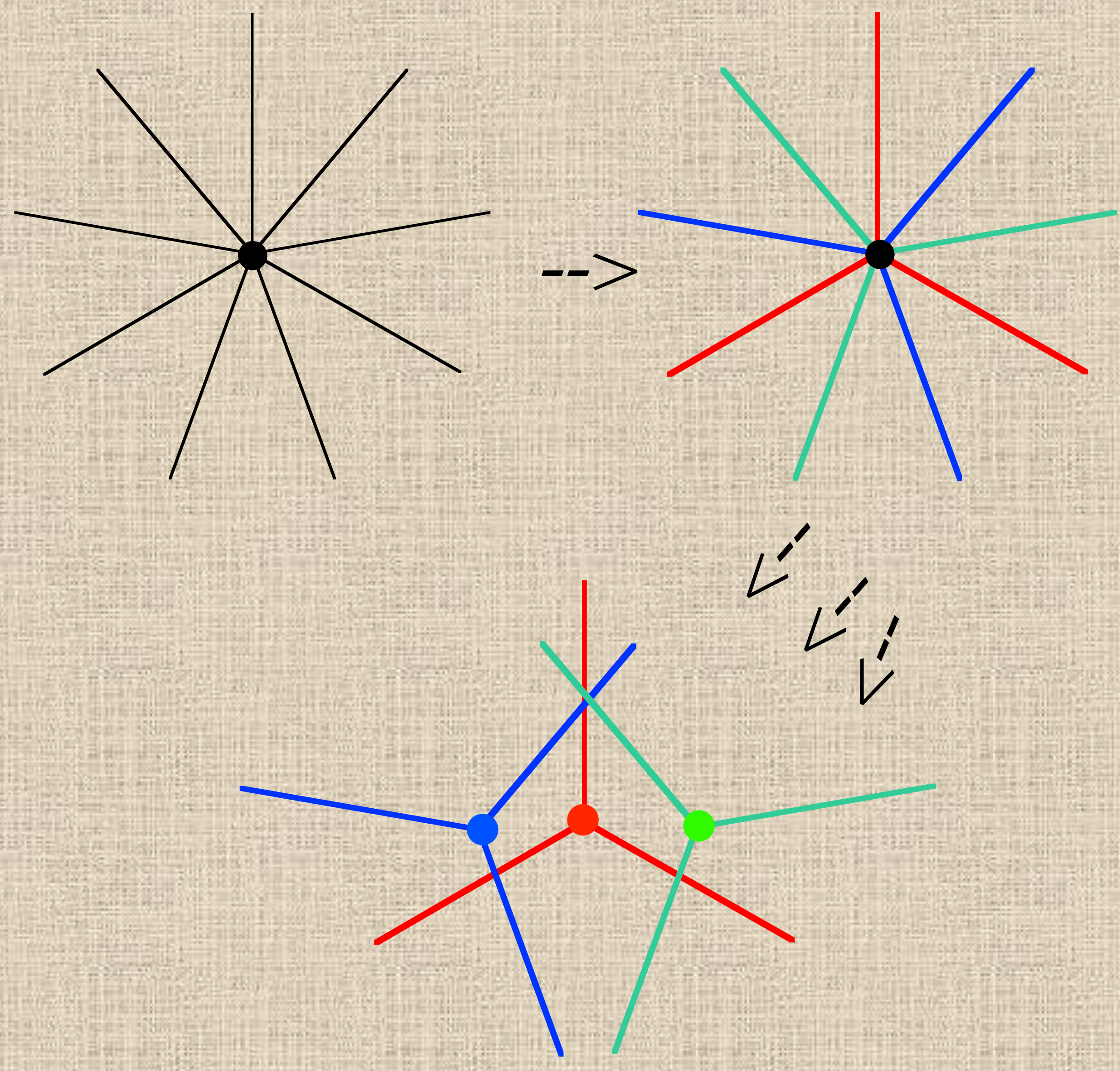
If M is a regular map of type {p, q}, we saw, in talks by Martin Macaj and Juergen Wolfart, that if j and q are relatively prime, then $H_j(M)$ is another (possibly isomorphic) regular map, and its type is {p′, q} for some p′.

But if j and q are *not* relatively prime, then $H_j(M)$ is still defined, though we have to be careful about vertices.

For example, if we consider H_3 of some map of type {p, 9}, the 3rd-order holes at each vertex make, not one complete cycle of length 9, but three separate cycles of length 3

We then separate the vertex into three vertices, each of degree 3, as on the next slide.



This process might or might not disconnect the map.

For example, if M is the torus map $\{3, 6\}_{2, 0}$ of4 vertices and 12 edges, $H_2(M)$ is the torus map $\{6, 3\}_{2, 0}$ of 8 vertices and 12 edges.

Contrast this with $N = \{3, 6\}_{3, 0}$ of 9 vertices and 27 edges. $H_2(N)$ is three copies of the torus map $\{6, 3\}_{1,1}$ of 3 vertices and 9 edges.

In this case, we keep one copy of the duplicated map and say that $H_2(\{3, 6\}_{3, 0}) = \{6, 3\}_{1,1}$.

In this case, H_j acts (sortakinda) like a projection, in that it takes in one map and gives back a smaller related map.

Projections and coverings of regular maps have been studied. I have an algorithm, and Martin Skoviera has one he has been keeping secret :)

I think some related techniques can be used to consider this question: Given a small map N, find all regular maps M such that $H_j(M) = N$.

Perhaps even the less restrictive problem might be more approachable: Given a regular map N, and numbers j and k, find all regular maps M such that $H_j(M)$ results in k copies of N.