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SODO 2012

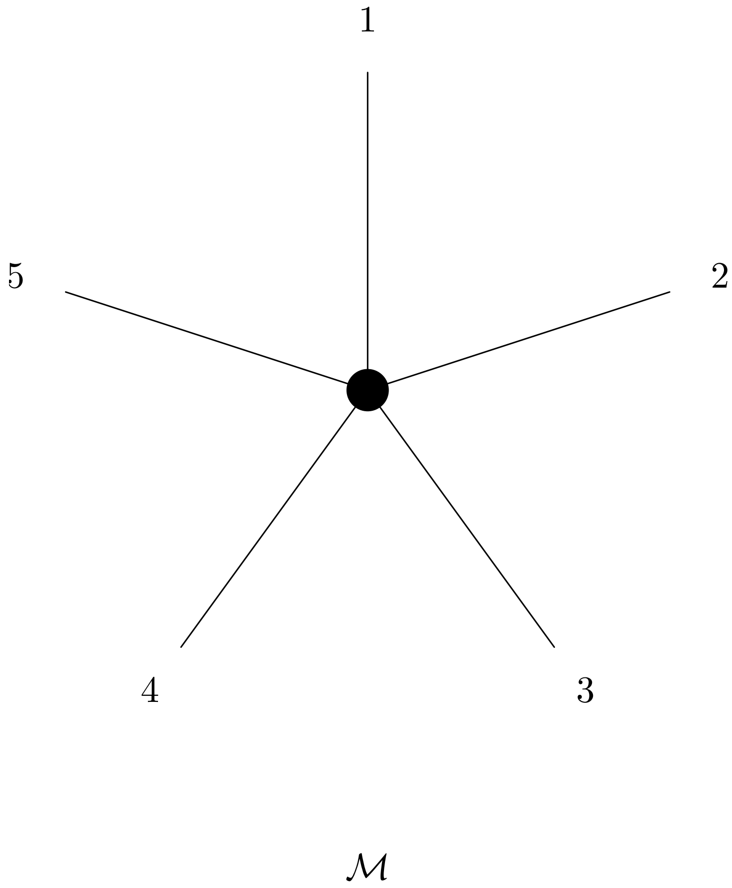
Hole operators

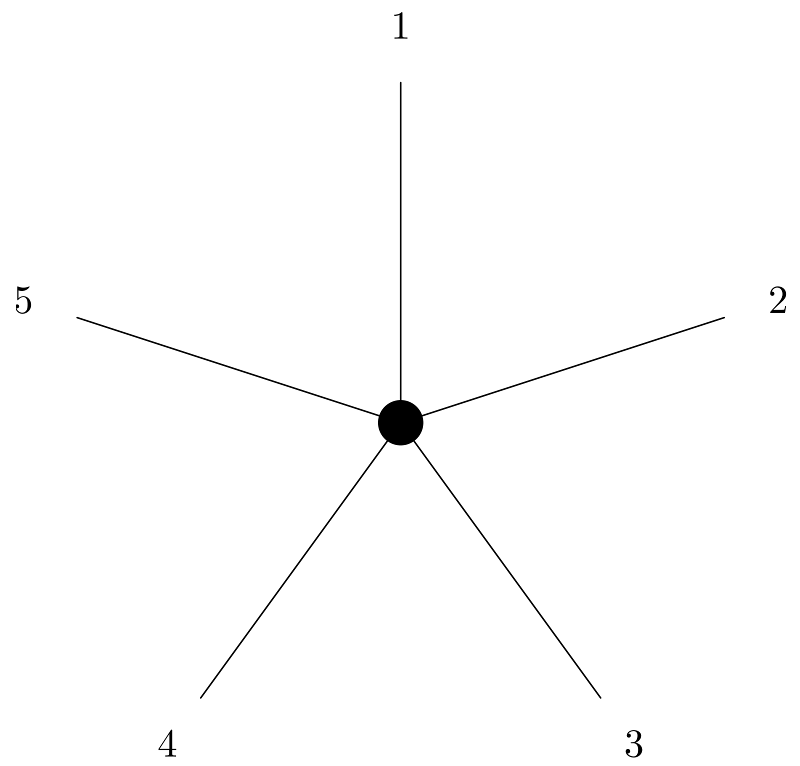
The j-th hole operator H_j creates a new map $\mathcal{M}^{H_j}$ from an oriented map $\mathcal{M}$ such that the faces of $\mathcal{M}^{H_j}$ are the j-th holes of $\mathcal{M}$, that is, cyclic sequences of edges, each two consecutive sharing a vertex, so that at each vertex, the adjacent edges subtend j faces on the right. The map $\mathcal{M}^{H_j}$ is well defined whenever the valency of the map $\mathcal{M}$ is coprime to j. These were introduced by Steve Wilson in 1979.

The operator H_{-1} has always been regarded as to produce the mirror image and when restricted to the class of n-valent maps, H_j has been identified with the operator $H_{j \pmod n}$.

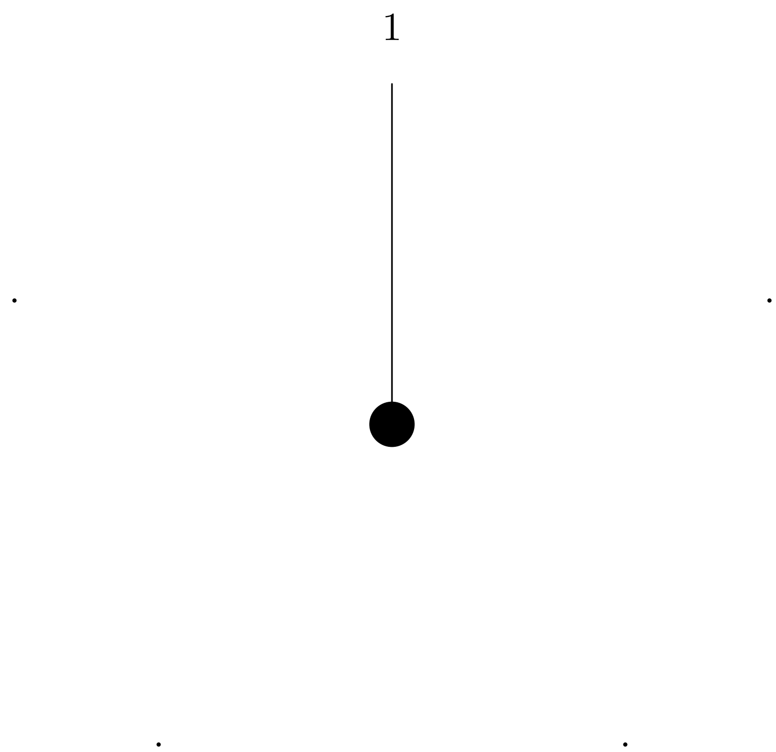
But ... is this really the case?

Proof by pictures

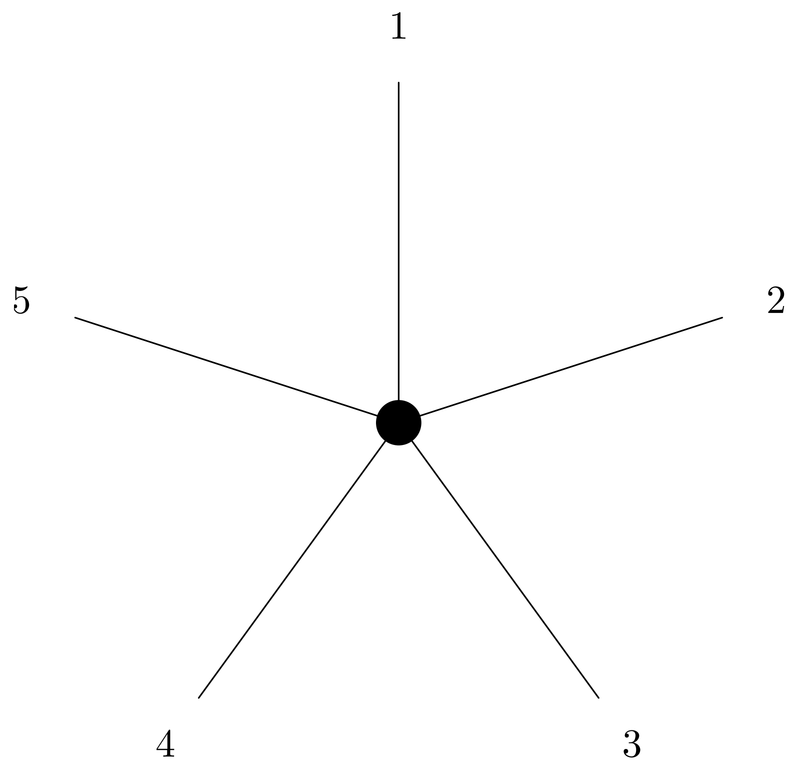




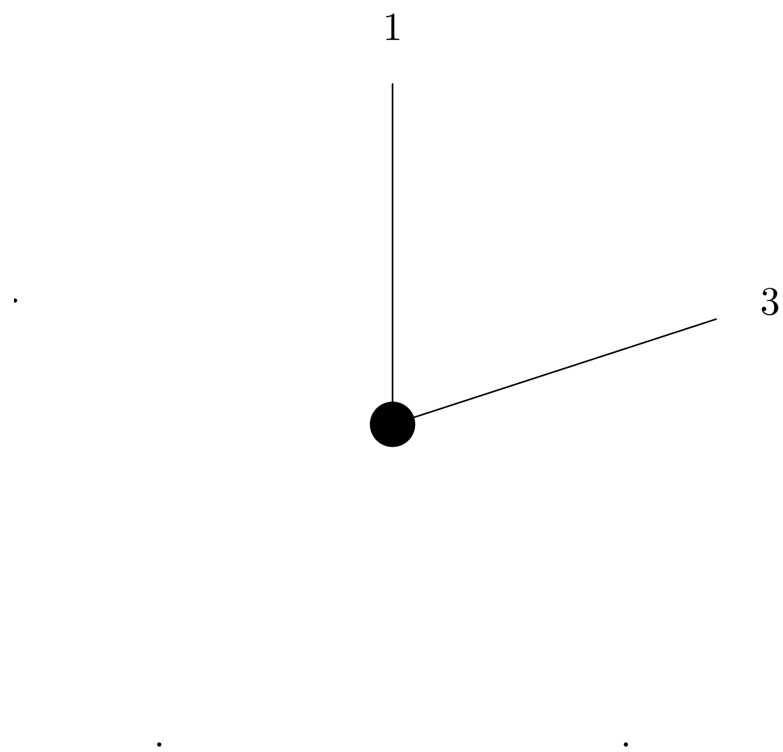
$\mathcal{M}$



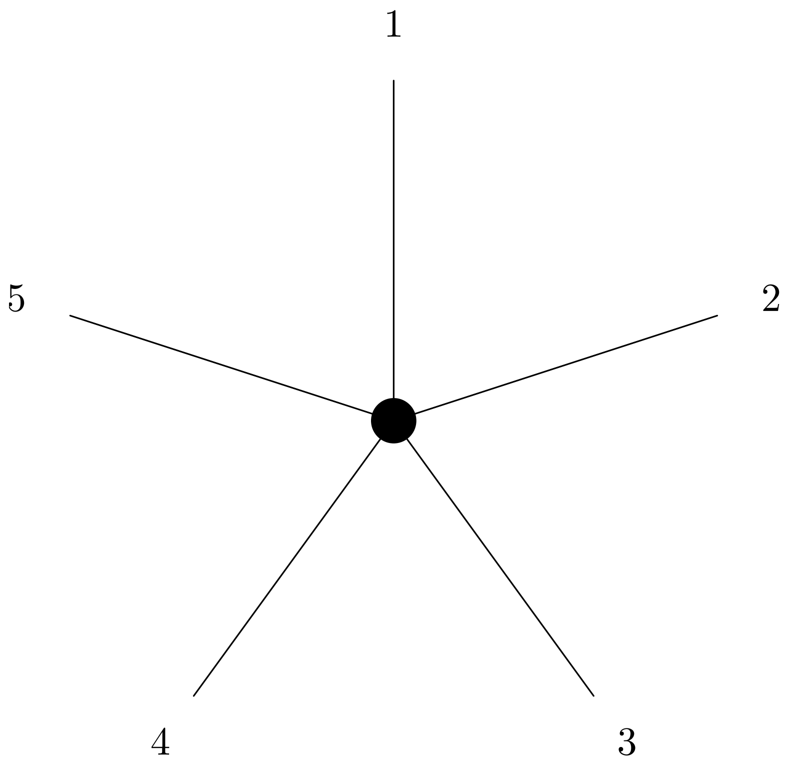
$\mathcal{M}^{H_2}$



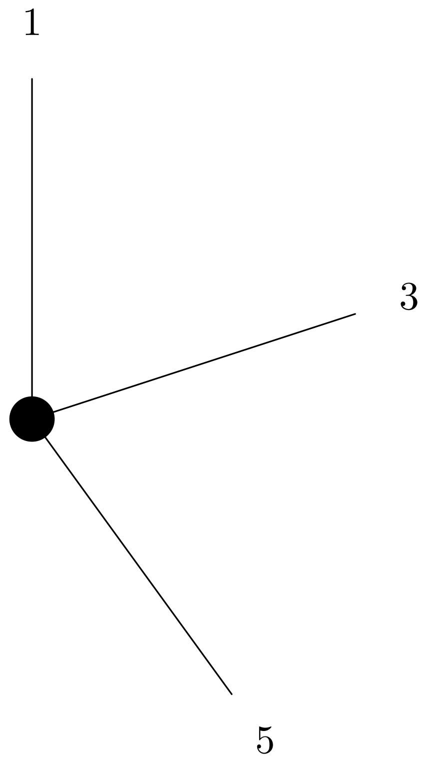
$\mathcal{M}$



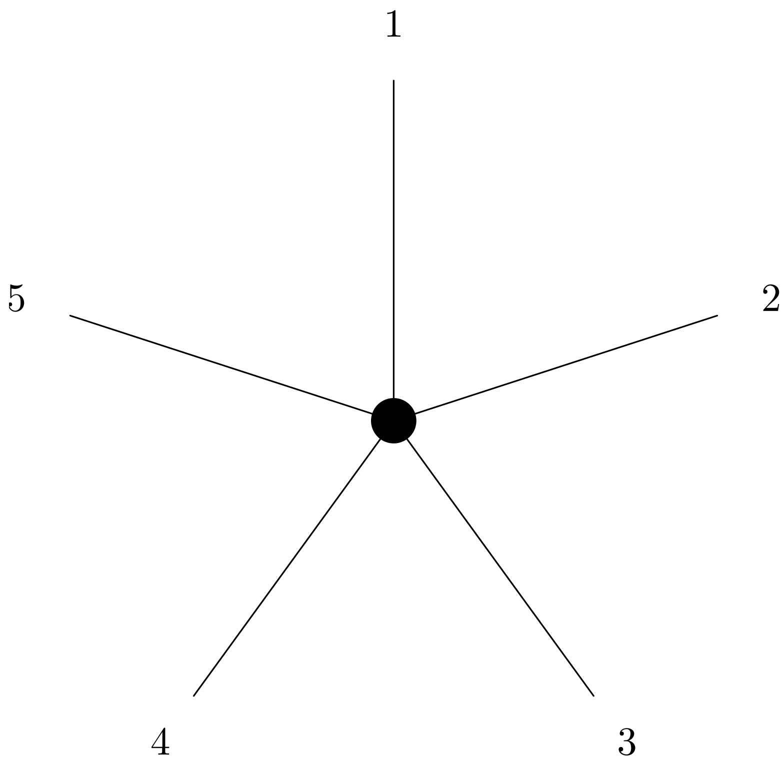
$\mathcal{M}^{H_2}$



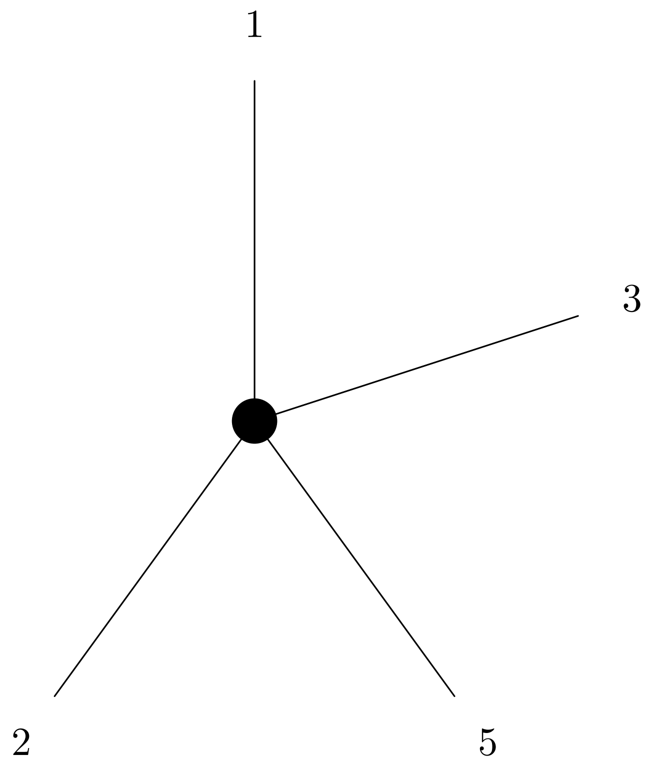
$\mathcal{M}$



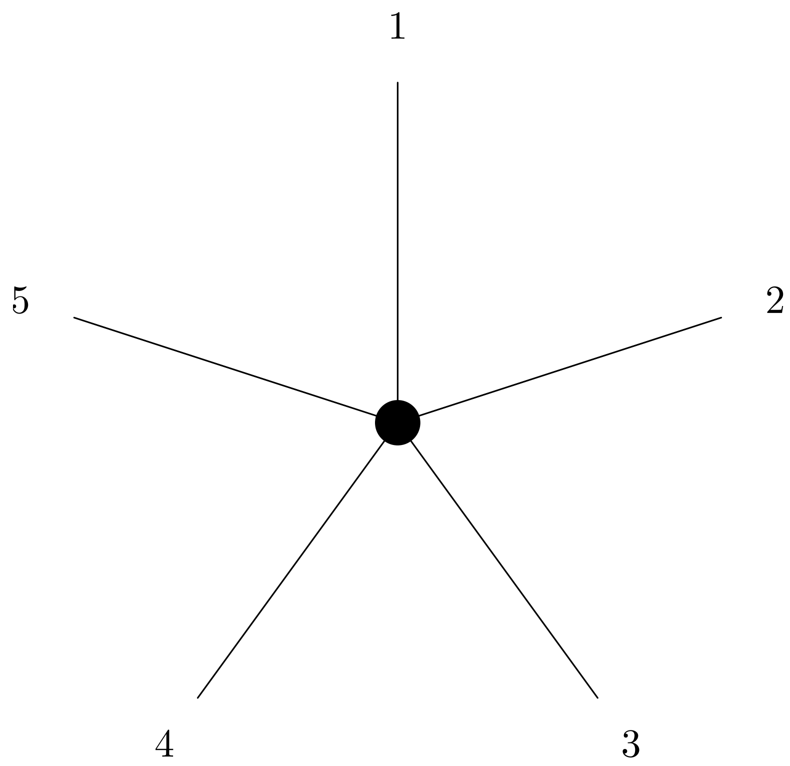
$\mathcal{M}^{H_2}$



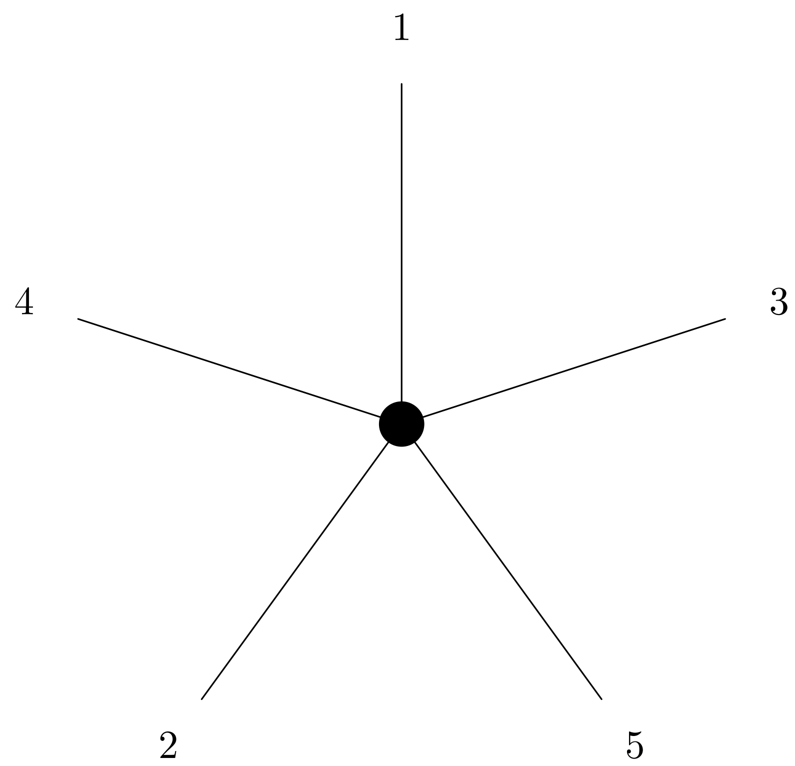
$\mathcal{M}$



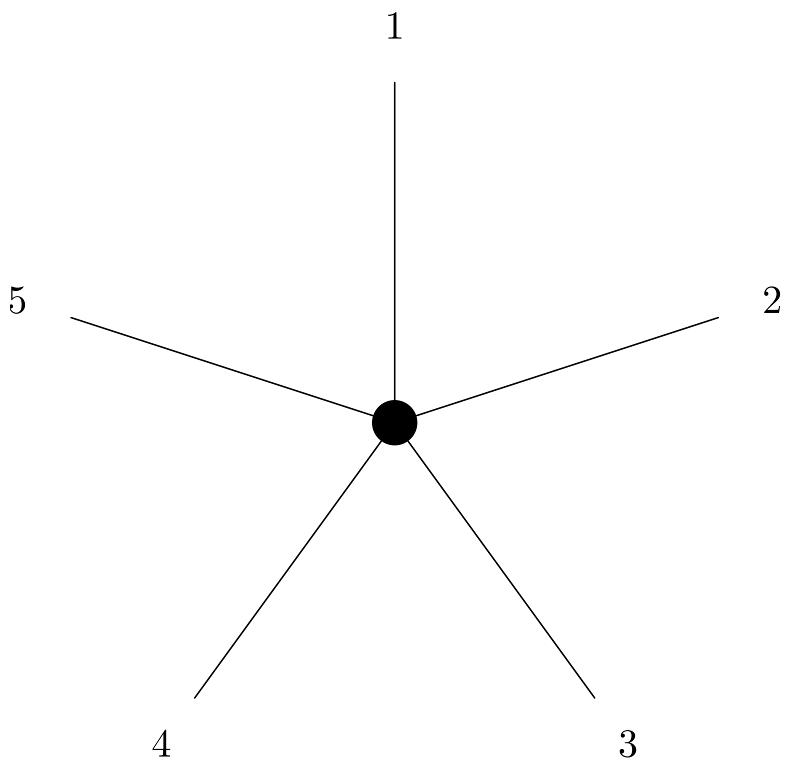
$\mathcal{M}^{H_2}$



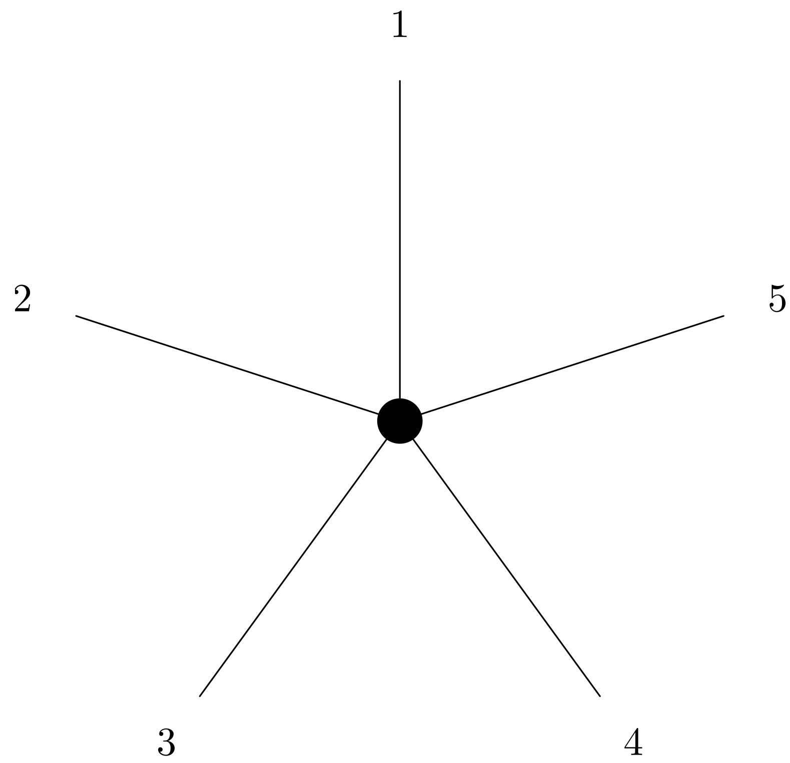
$\mathcal{M}$



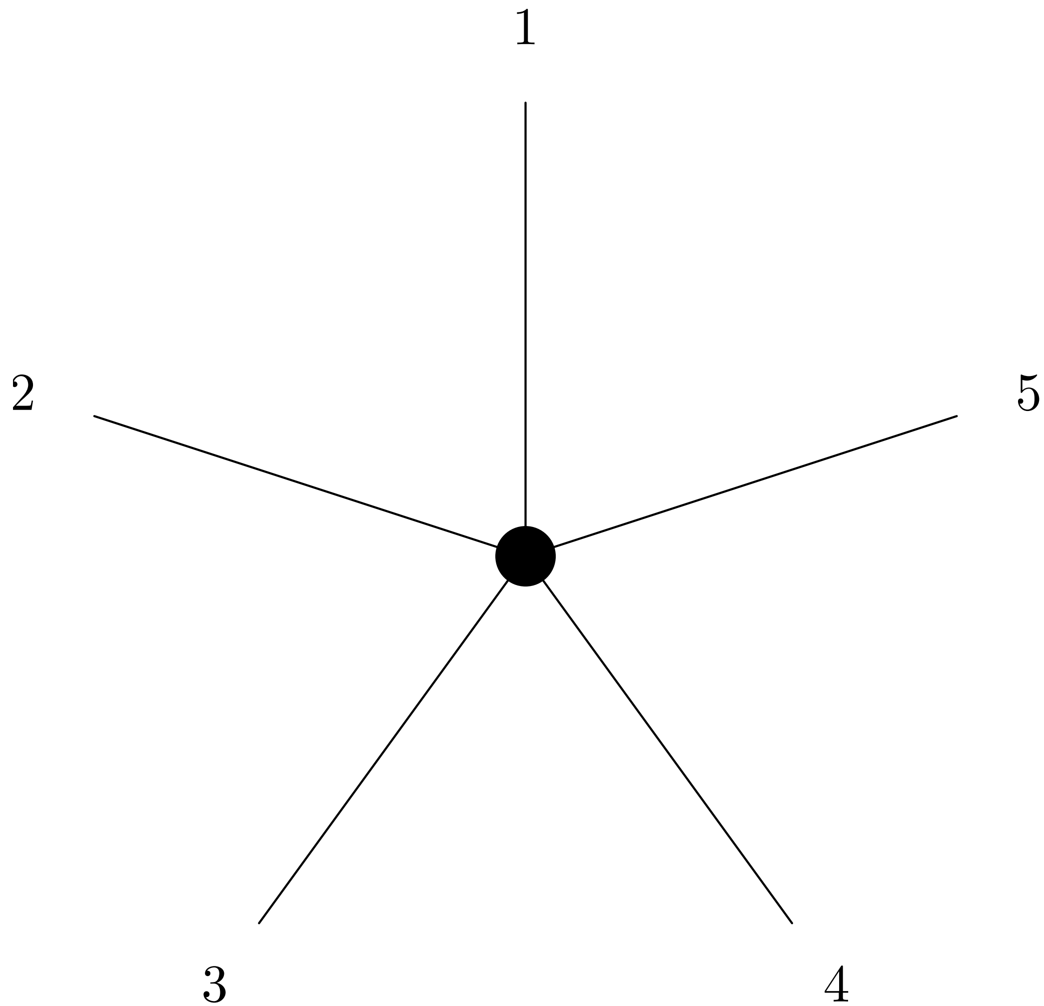
$\mathcal{M}^{H_2}$



$\mathcal{M}$

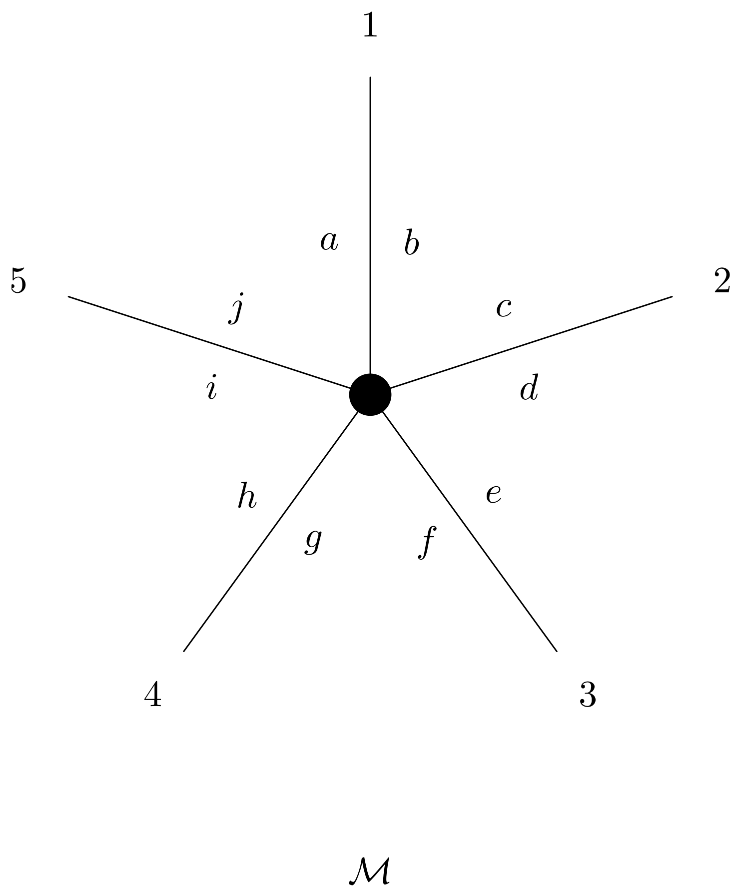


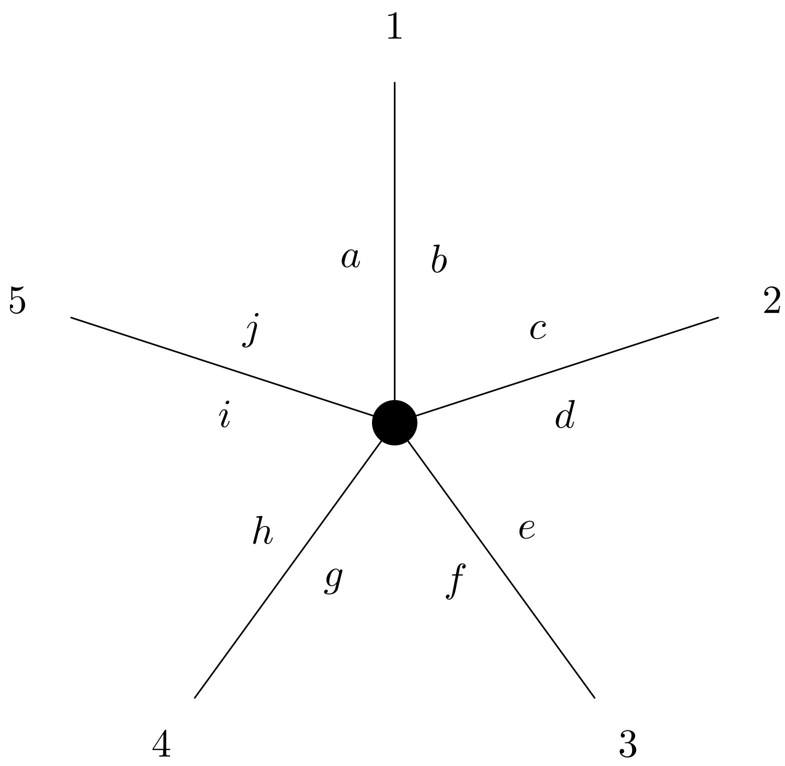
$\mathcal{M}^{H_4}$



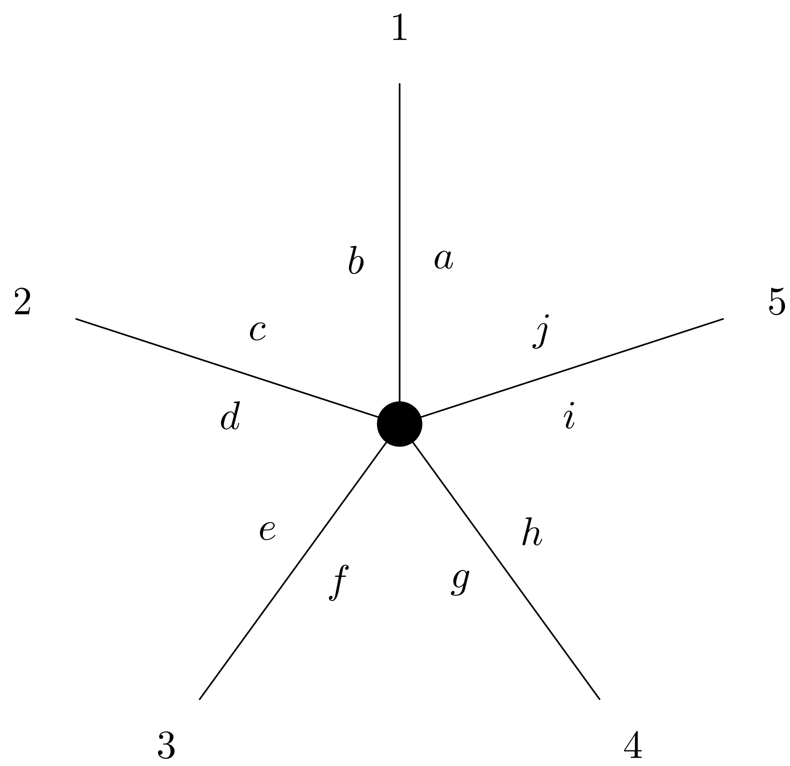
$$\mathcal{M}^{H_4} = \mathcal{M}^{H_{-1}} = \mathcal{M}^{M}$$

What about flags?

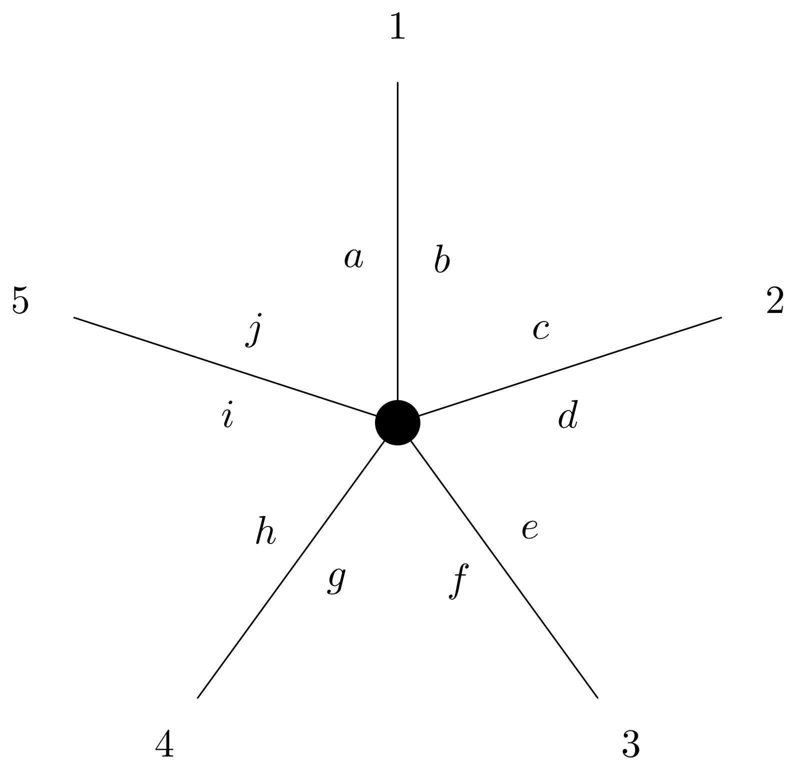




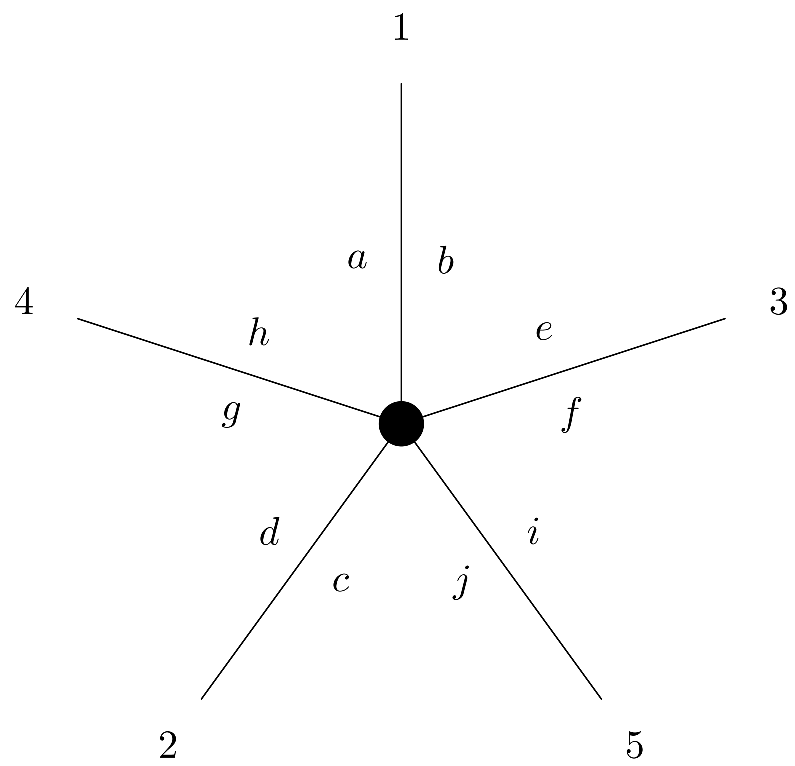
$\mathcal{M}$



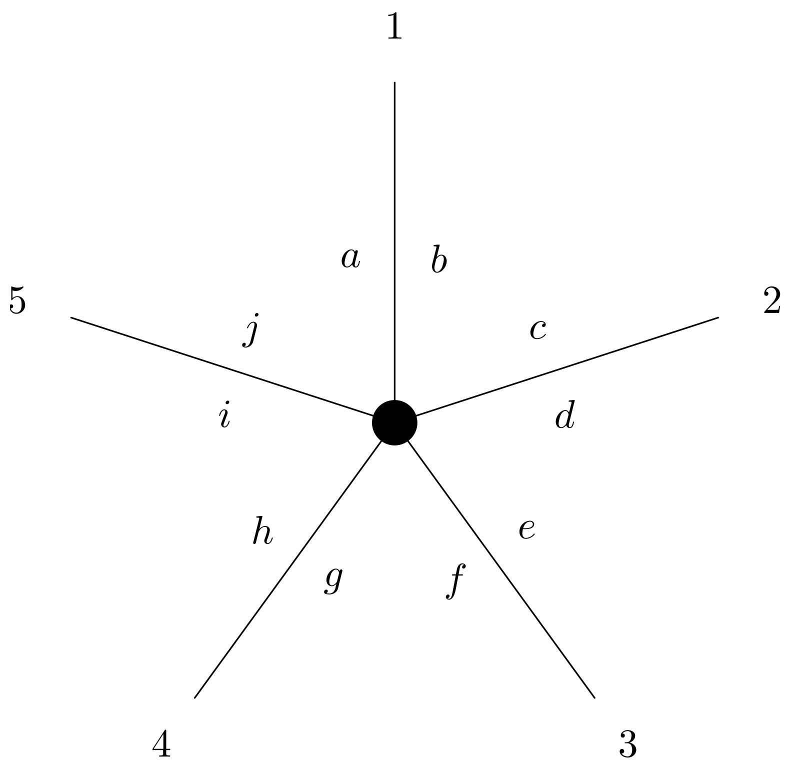
$\mathcal{M}^M$



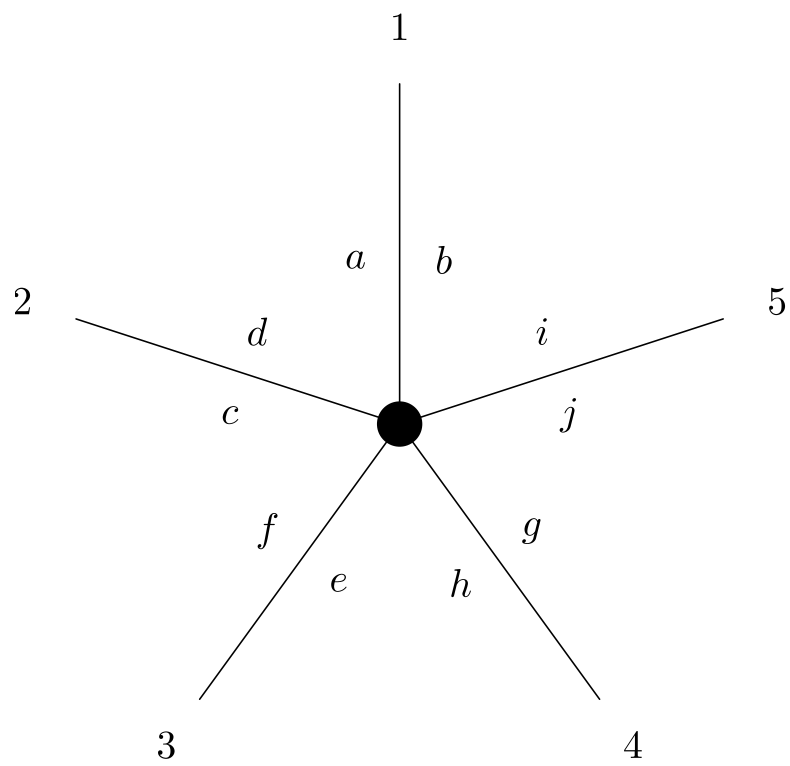
$\mathcal{M}$



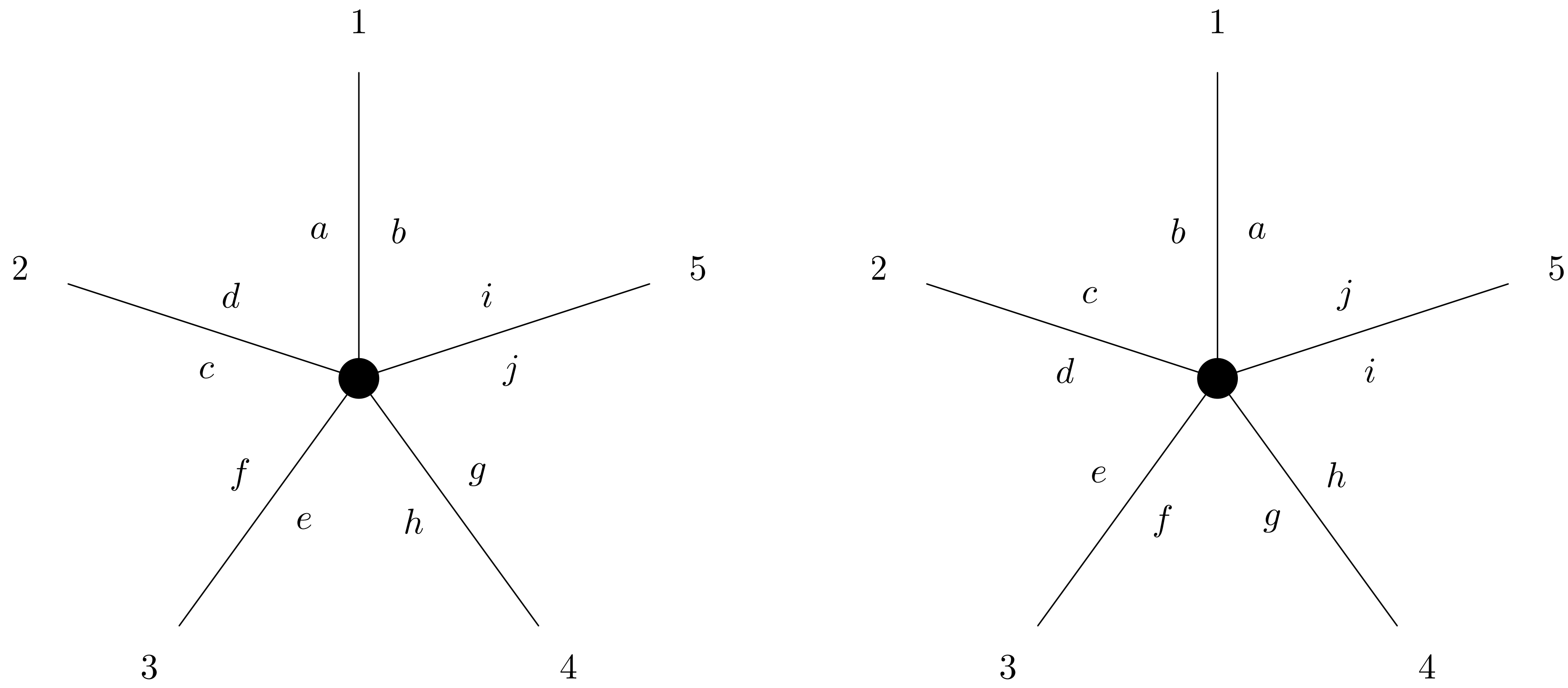
$\mathcal{M}^{H_2}$



$\mathcal{M}$

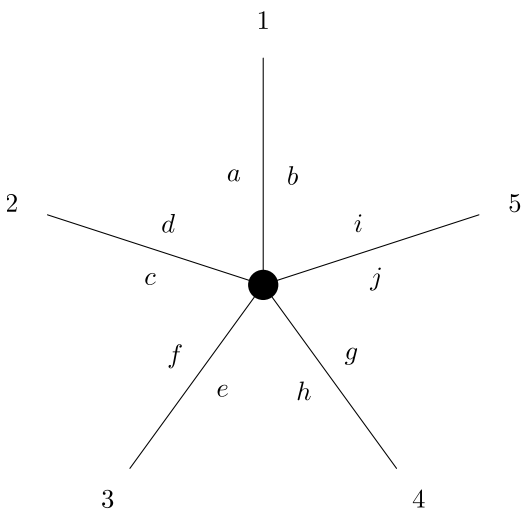


$\mathcal{M}^{H_4}$

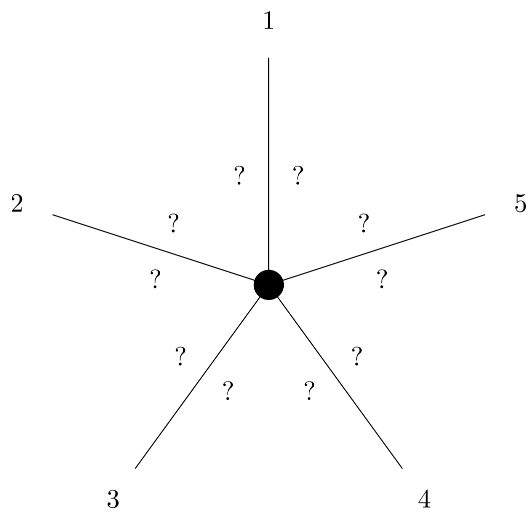


$$\mathcal{M}^{H_4} \neq \mathcal{M}^{M}$$

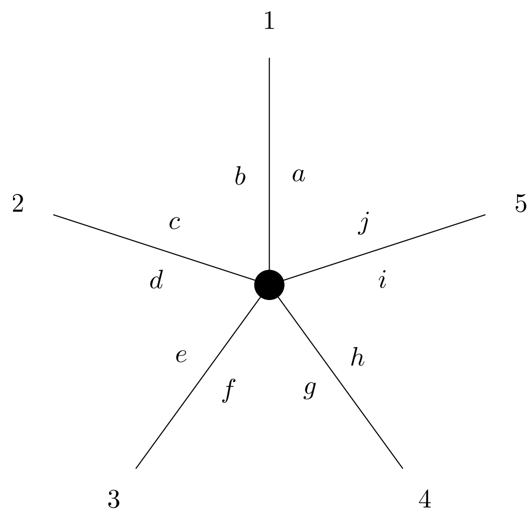
How to define $\mathcal{M}^{H_{-1}}$?



$\mathcal{M}^{H_4}$



$\mathcal{M}^{H_{-1}}$?



$\mathcal{M}^{M}$

Representing oriented maps using flags

Let $\mathcal{M}$ be an oriented map.

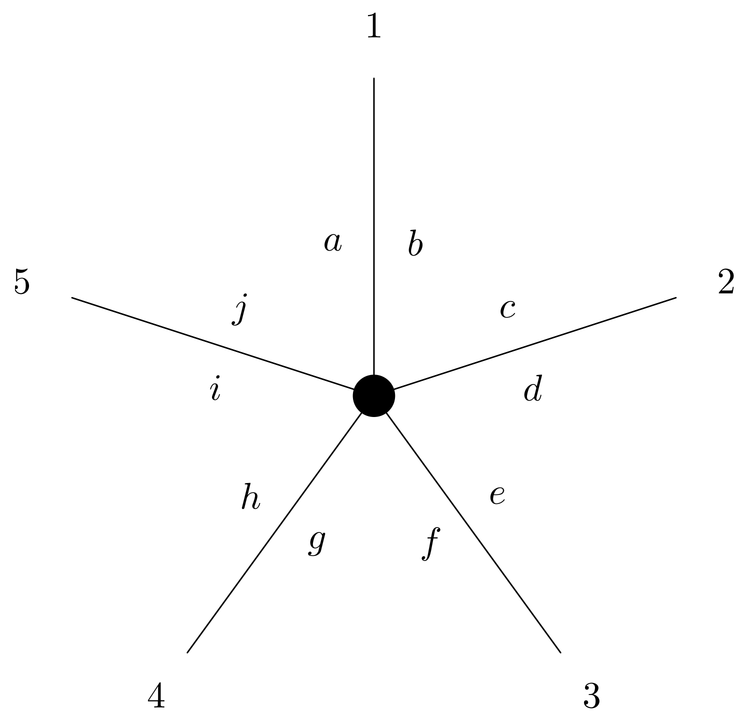
As an unoriented map it has a representation (F, λ, τ, ρ).

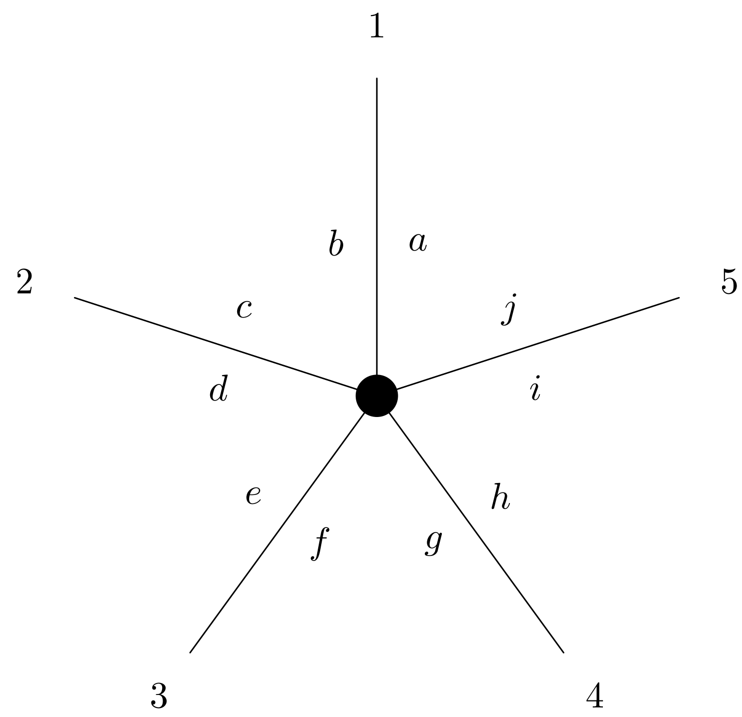
Let O_R and O_L denote the sets of flags which lie to the right and to the left of their darts, respectively.

O_R and O_L are orbits of $\langle \rho\tau,\ \tau\lambda \rangle$.

To get $\mathcal{M}$ it suffices to know F, τ, λ, ρ, and O_R.

Mirror image of an oriented map





$$(F, \lambda, \tau, \rho, O) \qquad \longmapsto \qquad (F, \lambda, \tau, \rho, F \setminus O)$$

What is the mirror image
of an unoriented map?

Thank You