Abstracts for ATCAGC 2010 (Auckland, 15-19 February)

Marston Conder (University of Auckland, New Zealand)

Graph and surface coverings via combinatorial group theory

Families of symmetric structures (such as arc-transitive graphs, and regular maps on surfaces) can often be constructed as families of covers of a given example. In this talk I will describe some combinatorial group-theoretic methods that can be helpful in this task.

Mike Fellows (University of Newcastle, Australia)

Algorithmic issues of operators on ideals

If J is a lower ideal in the minor order of graphs, and J' is the set of graphs that have a cover (or branched cover) in J, then J' is also an ideal. This is an example of an operator on ideals. Given the finite obstruction set that characterizes J, is the obstruction set for J' computable? Few such operators are known, and as for the algorithmic issues they pose, little is known, and the mathematics needs new ideas. The general theme is about how one graph is represented in another.

Jiří Fiala (Charles University, Prague, Czech Republic)

Computational Complexity of Local Injective Homomorphisms: Why Lists Help

We prove full dichotomy for the list version of locally injective homomorphisms (that is, List-H-LIHom is polynomial time solvable if H has at most one cycle in each compoment, and NP-complete otherwise). In the second part of the talk I will discuss the troubles and woes of trying to apply similar methods to the locally bijective case.

Gabino González-Diez (Universidad Autónoma de Madrid, Spain)

Dessins d'enfants, Riemann surfaces and covers

A *dessin d'enfant* is a finite graph $\mathcal{G}$ embedded in an oriented topological surface X such that $X \setminus \mathcal{G}$ is the disjoint union of a number of topological discs. It is well known that such graphs permit the definition of holomorphic atlases on X, thus giving rise to compact Riemann surfaces $X_{\mathcal{G}}$. Not all Riemann surfaces arise in this way, but in this talk we shall show that if $f : S \to S'$ is a morphism of compact Riemann surfaces and $S \simeq X_{\mathcal{G}}$, then it is also true that $S' \simeq X_{\mathcal{G}'}$ for certain dessin $\mathcal{G}'$.

Milagros Izquierdo (Linköping University, Sweden)

Coverings of Riemann and Klein surfaces

A morphism between compact Riemann and Klein surfaces is a silvered branched covering. Coverings of surfaces provide a powerful topological/combinatorial tool in studying geometrical properties of surfaces such as automorphisms, ovals and real representations. In this talk we present coverings of surfaces, and some of the many results about Riemann and Klein surfaces (and their moduli spaces) obtained by means of coverings.

Robert Jajcay (Indiana State University, USA)

On the use of generalized truncation and anti-truncation in cage constructions

One way to think of truncation is as of attaching cycles to dangling edges created by removing vertices of the original graph. The generalization we will talk about relies on the very same idea but instead of attaching cycles it involves attaching arbitrary graphs of suitable orders. As the usefulness of the truncation is widely acknowledged, it should not be surprising that its generalization has also a wide range of applications. In our talk, we choose to focus on applications to cage constructions – that is, on constructions of small regular graphs of given girth. We also address the possibility of contracting subgraphs of a given graph into single vertices – a technique that may be thought of as the opposite of truncation. The presented techniques are the results of cooperation with Gabriela Araujo Pardo, Jozef Siran, and Tomaz Pisanski.

Jan Kratochvíl (Charles University, Prague, Czech Republic)

Computational Complexity of Graph Covers: The Never Ending Story Part II

I will review the progress in computational complexity questions related to locally bijective homomorphisms (ergo graph covers) and locally injective ones (ergo partial covers) made since the Finse workshop in 2009. Pending open problems will be stressed.

Bernard Lidický (Charles University, Prague, Czech Republic)

Computational Complexity of LIHomomorphisms to Weight Graphs: A Full Classification of Simple Weights

In this talk we link to the previous work of Marek Tesař about LIHomomorphisms (locally injective homomorphism) to Θ graphs. The ultimate goal is to characterize the computational complexity of LIHomomorphisms to weight graphs. A weight graph is a connected multi-graph G with two special vertices u and v, which are of degree at least three and all other vertices are of degree two. Moreover, there is a cycle (loop allowed) in $G - u$ as well as in $G - v$. We resolve the computational complexity of LIHomomorphisms to weight graphs where u and v are of degree 3, and give some partial results for the other cases. This is joint work with Ondřej Bílka, Jiří Fiala, Jan Kratochvíl and Marek Tesař.

Martin Mačaj (Comenius University, Bratislava, Slovakia)

Strongly regular graphs from coverings?

This contribution is motivated by search for strongly regular graphs with 'moderately large' automorphism groups. A possible approach is to find all collapsed matrices of appropriate orbit decompositions and identify those which can be lifted. We present concrete examples related to putative Moore graphs of degree 57. (Talk based on joint work with Jozef Siran.)

Aleksander Malnič (University of Ljubljana, Slovenia)

Let's lift an automorphism, shall we?

Combinatorial theory of graph covers emerged approximately forty years ago in the context of maps on surfaces, with the final solution of the long standing Heawood's Map Colour Problem as the primary incentive. It soon became an indispensable tool in Algebraic graph theory as well, mostly due to problems such as classification of graphs and maps in terms of their symmetries. An important topic in this setting is the problem of lifting automorphisms; indeed, one has good control over symmetries of the covering graph provided that sufficiently many automorphisms of the base graph lift. In this talk I will review some basic aspects of the lifting problem, and discuss certain complexity problems one might encounter along the way.

Roman Nedela (Slovak Academy of Sciences, Slovakia)

Graphs with semiedges and their coverings

Following a paper by Malnič, Nedela and Skoviera (European J. Comb., 2000), we first define graphs that allow one-ended edges, called semi-edges. Such graphs naturally arise as quotients of standard graphs, by groups of automorphisms acting freely on vertices but not on edges. A consistent theory of coverings between such graphs was developed in the above-mentioned paper. In this talk we present some basic ideas about such coverings, emphasizing the main differences from the classical graph coverings. We will try to convince the audience that such a generalization is very natural and useful.

Jaroslav Nešetřil (Charles University, Prague, Czech Republic)

On the algebraic properties of graphs and homomorphisms between them

[**Special 50-minute colloquium-style lecture**]

An adjacency-preserving function from one graph to another is called a graph homomorphism. From the algebraic point of view, homomorphisms give perhaps the most natural way to compare graphs (and similar discrete structures). In this lecture we present a survey of recent results related to abstract representations of groups, monoids, posets and categories by various classes of graphs. This topic, although classical, is a source of many interesting open problems that lie on the interface between several disciplines - including model theory, set topology, and the theory of partially ordered sets. Of particular interest are representations that are simple (”finite presentations”), by means of simple structures (e.g. graphs of bounded vertex-degree), and/or ”small” (e.g. having linear order).

Primož Potočnik (University of Ljubljana, Slovenia)

Census of tetravalent arc-transitive graphs

When investigating a class of combinatorial objects it is very useful if a list of small representatives of that class is available. An example of such a list is the famous Foster census of cubic arc-transitive graphs. In 2002, Conder and Dobcsányi completed Foster's work by finding all cubic arc-transitive graphs on up to 768 vertices.

After cubic graphs are done, the next step would be to produce the analogue of the Foster census for tetravalent graph. Can this be achieved using the same approach as in the cubic case? The answer is “No”. The main ingredient of the approach of Conder and Dobcsányi was the fact that the order of the vertex-stabiliser in a cubic arc-transitive graph is bounded by 48. Unfortunately, this is not the case in the tetravalent case, where the order of the vertex-stabiliser can be as big as $2^{\frac{n}{2}}$ where n is the order of the graph.

This difficulty calls for other techniques. One of the possible approaches, that I want to discuss in my talk, uses the fact that every arc-transitive tetravalent graph can be obtained by a series of elementary abelian covers from certain *basic* graphs. One family of such basic graphs form the cycles with every edge doubled. It turns out that computation of elementary abelian arc-transitive covers of these graphs is closely related to the theory of cyclic and negacyclic error correcting codes.

Constructing the census of tetravalent arc-transitive graphs is a part of a large project, initiated by Steve Wilson and myself several years ago. A preliminary census can be found on Steve's webpage. Several results from my talk are parts of joint work with Gabriel Verret, Aleksander Malnič and Boštjan Kuzman.

Martin Škoviera (Comenius University, Slovakia)

Branched coverings of maps and voltage assignments

The first occurrence of coverings in graph theory is closely related to the solution of the Heawood map colouring problem by Ringel, Youngs and others in the 1960s. Since then, graph covers have provided a powerful tool for the construction of graph embeddings in surfaces, and the study of various properties of maps. In this talk, we will survey some of the techniques used to describe such coverings — and in particular, voltage assignments on graphs and on maps.

Marek Tesar (Charles University, Prague, Czech Republic)

Locally injective homomorphisms to Theta graphs

A locally injective homomorphism from graph G onto graph H is a 'local injective' mapping from the vertex set of G onto the vertex set of H, such that for every $v \in V(G)$, the neighborhood of v is mapped injectively onto the neighborhood (in H) of the image of v. For a fixed graph H we can define the H-*LIHom* problem as a problem of deciding if there exists a locally injective homomorphism from a given graph G to the graph H.

We consider the computational complexity of the Θ-*LIHom* problem, when Θ is graph with two special vertices A and B of degree d, which are connected with d undirected paths from A to B (so all vertices except A and B have degree 2). We also present a characterization of computational complexity of the Θ-*LIHom* problem.

Gabriel Verret (University of Ljubljana, Slovenia)

On tetravalent arc-transitive graphs with large vertex-stabilisers

A significant obstacle when trying to obtain a census of small tetravalent arc-transitive graphs is the fact that some of them have very large vertex-stabilisers. For example, the vertex-stabiliser of the wreath graph on $2n$ vertices has order 2^n. In this talk, we will describe some results which severely restrict the structure of tetravalent arc-transitive graphs with large vertex-stabilisers and hence make the construction of such a census manageable computation-wise.

Let Γ be a connected, tetravalent G-arc-transitive graphs with n vertices and let uv be an arc of Γ. Our first result is that, unless Γ is part of a well-understood family of exceptional examples related to wreath graphs, then $|G_{uv}| \leq n^3$.

The proof of this result uses very classical techniques, dating back to Tutte. Unfortunately, the bound that this result provides is not tight enough to allow us to get very far with a census of small graphs. To improve this bound, we have used some heavier machinery.

The group G is called *semisimple* if it has no non-trivial abelian normal subgroup. By using the classification of finite simple groups, we prove that if G is a finite semisimple group and S is a 2-subgroup of G, then $|S| < |G : S|^{1.14}$.

The case where G has a normal abelian subgroup lends itself to a natural reduction, by considering quotient graphs. This allows us to improve the bound in our earlier result to $|G_{uv}| \leq n^{1.14}$, at the cost of allowing solvable covers of the exceptional graphs mentioned earlier.

Part of this talk is joint work with Primož Potočnik.
