World Mathematical Year 2000 and the New Millennium

Hilbert marked the end of the nineteenth century, and the start of the twentieth century, by enunciating his 23 famous problems. We are now approaching the end, not just of a century, but of a millennium, and we should be prepared to think on an even more lavish scale. We could even try to reflect what has happened to mathematics during the nearly 2000 years of the Christian Era – or Common Era – that have so far elapsed. The very fact that there is some disagreement as to when the new millennium starts, reminds us of one of the greatest advances that mathematics has made: the introduction of the number zero, not only into standard mathematical usage, but also into popular consciousness. In my early days of computing much was made of the advantages of counting from zero, rather than one, but when the first compilers came along, counting from one suddenly became obligatory again. When telephone numbers had to be dialled, people from other countries seemed to think the arrangement of the digits on our dials, 0 to 9 in clockwise order, was eccentric but I still think we were the only ones in the right.

If you try to see what is going on in the plans for the World Mathematical Year 2000 by consulting the website `http://wmy2000.math.jussieu.fr/` you soon learn that countries in the Northern Hemisphere habitually use names of seasons as though they were dates. For example the newsletters issued in connection with WMY 2000 are identified as Newsletter 1 Summer 1993 to Newsletter 6 Autumn 1998 and Newsletter 7 Spring 1999. This reminds us that the earth is no longer considered to be flat and that there exists a Southern Hemisphere with differently-phased seasons. A wider significance, which has been crucial to scientific developments, with consequent effects on the way mathematics is used in science, is the modern insistence, at least in principle, on relating scientific theory to experimental verification.

What are the great mathematical achievements of the last 2000 years? I suppose the questions that were being thought about at the beginning of this epoch were "the duplication of the cube", "squaring the circle" and a proof from the axioms of congruence of the Euclidean parallel axiom. The first two questions describe geometrical constructions that are now known to be impossible and the reasons go well beyond geometry. Ruler and compass constructions produce only multiples of a basic length that lie in a field consisting of the rational numbers extended so as to be closed under the function $x \mapsto \sqrt{1+x^2}$. And neither $\sqrt[3]{2}$ nor π is in this field. These considerations are related to the discovery of irrational and ultimately transcendental numbers and to Galois theory. Attempts to prove the parallel axiom led to constructions that eventually became basic tools of hyperbolic geometry. The possibility that questions that can be stated in an unambiguous way but might not have definite answers was a revolutionary idea and led eventually to the work of Gödel. The continuum hypothesis and the axiom of choice now have to be thought of, not as conjectures to prove or disprove, but as options for us to choose from. Far from causing doubts about fundamental questions, mathematics has now been revealed as a much deeper and much richer subject than anyone might have supposed 2000 years ago.

Returning to the World Mathematical Year, a browse through its list of projects reveals one New Zealand item. This is the international TIME (Technology in Mathematics Education) conference to be held in Auckland on 11-14 December 2000. For details see `http://math.auckland.ac.nz/TIME2000`. I don't know if other New Zealand activities can be officially scheduled at this late stage but it would be wonderful if a WMY flavour could be incorporated into events that are happening anyway. I am aware of at least two suitable events, the ANZIAM conference in Waitangi 8-12 February and the NZ Mathematics Colloquium in Hamilton 26-29 November, and I am sure that some WMY spin is possible. The next of the series of ANODE numerical analysis workshops that we organise in Auckland will be at the end of the year, or possibly in the first few days of 2001. This will be a good time to review a century of scientific computation and I hope our programme can recognise this.

This (Southern Hemisphere) summer marks the retirement of Vamanamurthy and the seventieth birthday of Alex McNabb. Even though they work in completely different parts of mathematics, they both enjoy problems and each is a dab hand with special functions. Hence, I suspect one of them will be the first to send me a new proof of a little identity that I once needed and eventually managed to prove. Of course a more interesting problem, and one that I have no ideas about, is to discover a more general result, of which this is a special instance. Perhaps the more general result will involve an arbitrary orthogonal polynomial system, rather than just the Laguerre polynomials, $L_0 = 1$, $L_1(t) = 1-t$, $L_2(t) = 1-2t+\frac{1}{2}t^2$, ... And now the problem:

Let n be a positive integer and t a real number. Define $a_0, a_1, a_2, \ldots, a_n$, and $b_0, b_1, b_2, \ldots, b_n$, by

$$\begin{aligned} a_0 &= 1, & b_0 &= 1,\\ a_1 &= t-1, & b_1 &= 1-t,\\ a_k &= \tfrac{(-1)^{k-1}}{k+1} L'_{k+1}\big((k+1)t\big),\ k \ge 2, & b_k &= \tfrac{(-1)^{k-1}}{k} t L'_k\big((k-1)t\big),\ k \ge 2. \end{aligned}$$

Prove that

$$a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \cdots + a_n b_0 = 0.$$

John Butcher, `butcher@math.auckland.ac.nz`