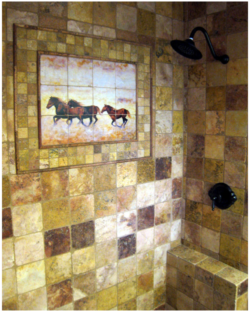
Maths 190 Lecture 13

REMINDER: Test September 20th at 6:00.

Topic for today:

Tiling the plane
or: New Zealand's most beautiful bathroom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Question(s) of the day:

- What is the most symmetric way of covering the plane with tiles?
- What is the least symmetric tiling?

Tilings of the plane

In pairs or threes: Use the handout and the transparencies provided to investigate the symmetries corresponding to each of the tiling patterns on the front page of the handout. Write a list of the symmetries for each case.

Symmetry in tilings of the plane

We distinguish between two types of symmetries:

- A **rigid symmetry** of a pattern in the plane is a motion of the plane that preserves the pattern and does not shrink, stretch, or otherwise distort the pattern.
- A pattern in the plane has a **symmetry of scale** if the tiles that make up the pattern can be grouped into super-tiles that still cover the plane and, if scaled down, can be rigidly moved to coincide with the original pattern.

Note that these definitions refer to symmetries of a pattern, not to symmetries of the individual tiles.

Rigid symmetries in the plane

All rigid symmetries are formed by combining the following three basic rigid symmetries:

- shifts (also called translations)
- rotations around a point
- flips (also called reflections) along a line.

Theorem

Any **rigid symmetry** *can be formed by first* **reflecting** *(if necessary), then* **rotating** *(if necessary), and finally* **translating** *(if necessary).*

Which symmetries?

- Some tilings have **rigid symmetries** *and* **symmetry of scale** (e.g., tilings using square tiles or equilateral triangle tiles).
- Other tilings have **rigid symmetries** but *no* **symmetry of scale** (e.g., tilings using hexagonal tiles).

Prize question:

Can tilings have **symmetry of scale** but *no* **rigid symmetries**?

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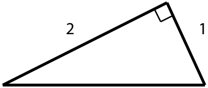
The Pinwheel Pattern

The pinwheel pattern is a tiling of the plane using a triangular tile: a right angle triangle with sides of length 1, 2 and $\sqrt{5}$ units.



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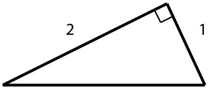
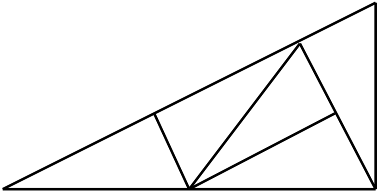


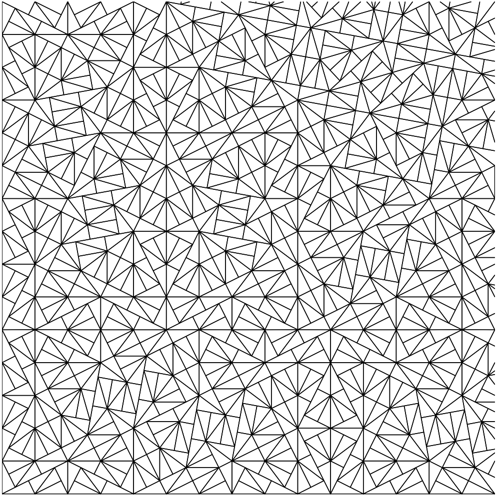
Five pinwheel triangles are combined to make one super-tile.



Five pinwheel super-tiles can be combined to make one super-super-tile, and so on ...

Symmetry of scale but no rigid symmetry

The pinwheel pattern has symmetry of scale but no rigid symmetries. We say the pattern is **aperiodic**.



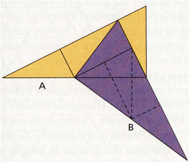
No rigid symmetry!

Figure: Can the interior tile "change its role"? Let's check!

- Rigid symmetry: A tile in a super-tile A is also part of another super-tile B
- But... to be part of different super-tiles, tiles must "change roles"!

No rigid symmetry!

Figure: Can the interior tile "change its role"? Let's check!



(a) No!

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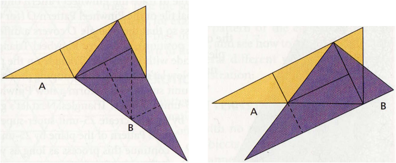

(a) No! (b) No!

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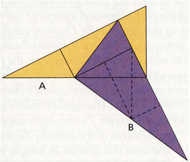
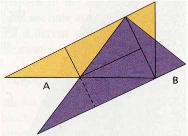
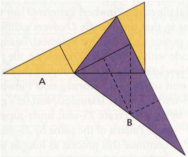


(a) No!



(b) No!



(c) No!

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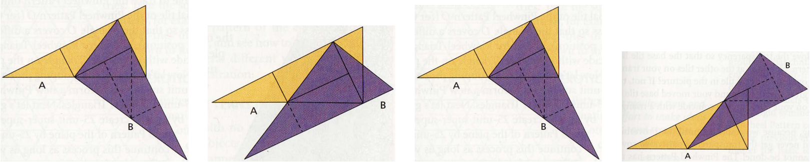

(a) No! (b) No! (c) No! (d) No!

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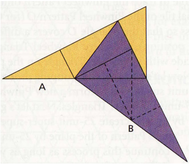

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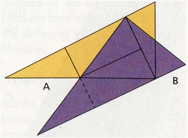

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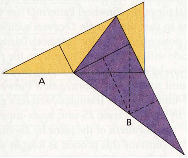
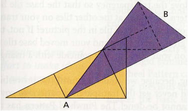


(c) No!



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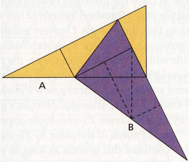

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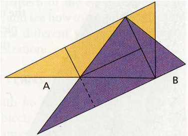

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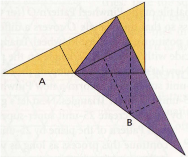
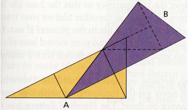


(c) No!



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- Rigid symmetry: A tile in a super-tile A is also part of another super-tile B
- But... to be part of different super-tiles, tiles must “change roles”!

Important ideas from today:

- Patterns in the plane can have **two types of symmetries**: rigid symmetries and symmetry of scale.
- Some patterns have **symmetry of scale** but **no rigid symmetries**. The pinwheel pattern is an example of this type of pattern.

For next time

- Read §4.4 in the textbook.
- Try some Mindscapes at the end of §4.4 of textbook.
- Look about you as you go about your life in the next few days and notice the tiling patterns you see. What symmetries can you see?

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Beauty = Symmetry?

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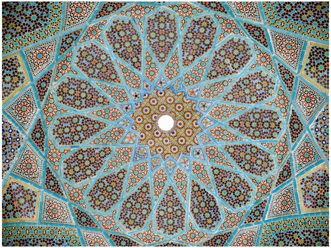

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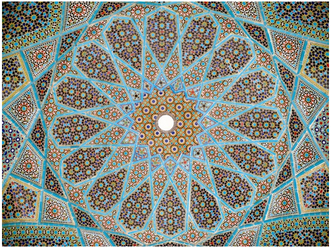

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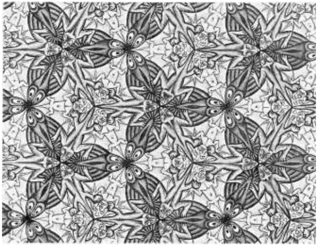

(b) Beautiful?