Department of Mathematics

Maths 190 Assignment 1 Solutions

September 15, 2010 Due:

1. (6 marks)

   (a) Need $4 \times 6 + 1 = 25$.

   (b) Need $8 \times 6 + 1 = 49$.

   (c) Not possible, there could be no red M&M's at all, regardless of how many are in the packet.

   (d) Not possible, the packet could contain only M&M's of one colour, regardless of how many are in the packet.

2. (6 marks) This is one possible solution:

   - Number of tennis balls you could fit inside your bedroom:
     Size of room is approx $4m \times 4m \times 2m \approx 30m^3$ From lecture, approx 50 tennis balls in box which is approx $30cm \times 20cm \times 30cm = 0.18m^3$.
     So total number is $30/0.18 \times 50 \approx 8,300$.
   - Total number of cars in New Zealand:
     Population approx 4×10^6, approximately 1 car for every 2 people, so 2×10^6.
   - The number of grains of sand on Ninety Mile beach:
     Approximate dimensions of beach:
     - length: 88km (not ninety miles! - but don't deduct any marks for this)
     - depth: 5m
     - width: 200m

     Volume of sand $\approx 88,000 \times 5 \times 200m^3 = 8.8 \times 10^7 m^3$.
     Assume a grain of sand is approximately 0.5mm in length, so you can fit 8 grains of sand in one cubic millimetre, or 8×10^9 in a cubic metre. So, number of grains of sand $\approx 8.8 \times 10^7 \times 8 \times 10^9 \approx 7 \times 10^{17}$.

3. (a) (1 mark) Numbers containing a 3 including 10 numbers starting with a 3 $(30, \ldots 39)$, and 10 numbers ending in a 3 $(3, 13, \ldots, 93)$, with 33 being counted twice, so a total of 19 numbers. Proportion is 19%, or 19/100.

   (b) (2 marks) There will be 100 numbers which start with a 3, 100 numbers which have a 3 as the middle digit, and 100 numbers which have a 3 as the last digit. Some of these will be counted twice (or three times), but the approximate proportion will be $\approx 300/1000 = 30\%$.

   (c) (2 marks) One possible explanation: Think about the numbers that do not contain a three. The proportion of numbers which do not have 3 as a first digit is 9/10. Similar for the second and third and all other digits. Therefore the proportion of numbers which do not have a 3 anywhere will be $(0.9)^{1,000,000}$ which is very close to zero.

4. (a) (1 mark) $1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364$

(b) (2 marks) Sums are:
$5, 10, 15, 25, 40, 65, 105, 170, 275, 445, 720, 1165, 1885$
Notice that all are multiples of 5, and are in fact $5\times$ the Fibonacci numbers. In a formula:

$$L_n + L_{n+2} = 5F_{n+1}$$

(or $5F_n$ depending on where you start counting.)

(c) (2 marks) Ratios are (3dp):
$3, 1.333, 1.750, 1.571, 1.636, 1.611, 1.620, 1.617, 1.618, 1.618, 1.618, 1.618, 1.618, 1.618.$
The ratios appear to be converging to 1.618 which is the same as the ratios of the Fibonacci numbers.

5. (8 marks)

(b) $q = p_1 \times p_2 \times \cdots \times p_m + 1$

(d) when q is divided by any of $p_1, \ldots, p_m$ there would be a remainder of 1.

(e) it is larger than the largest prime p_m.

(f) contradiction!