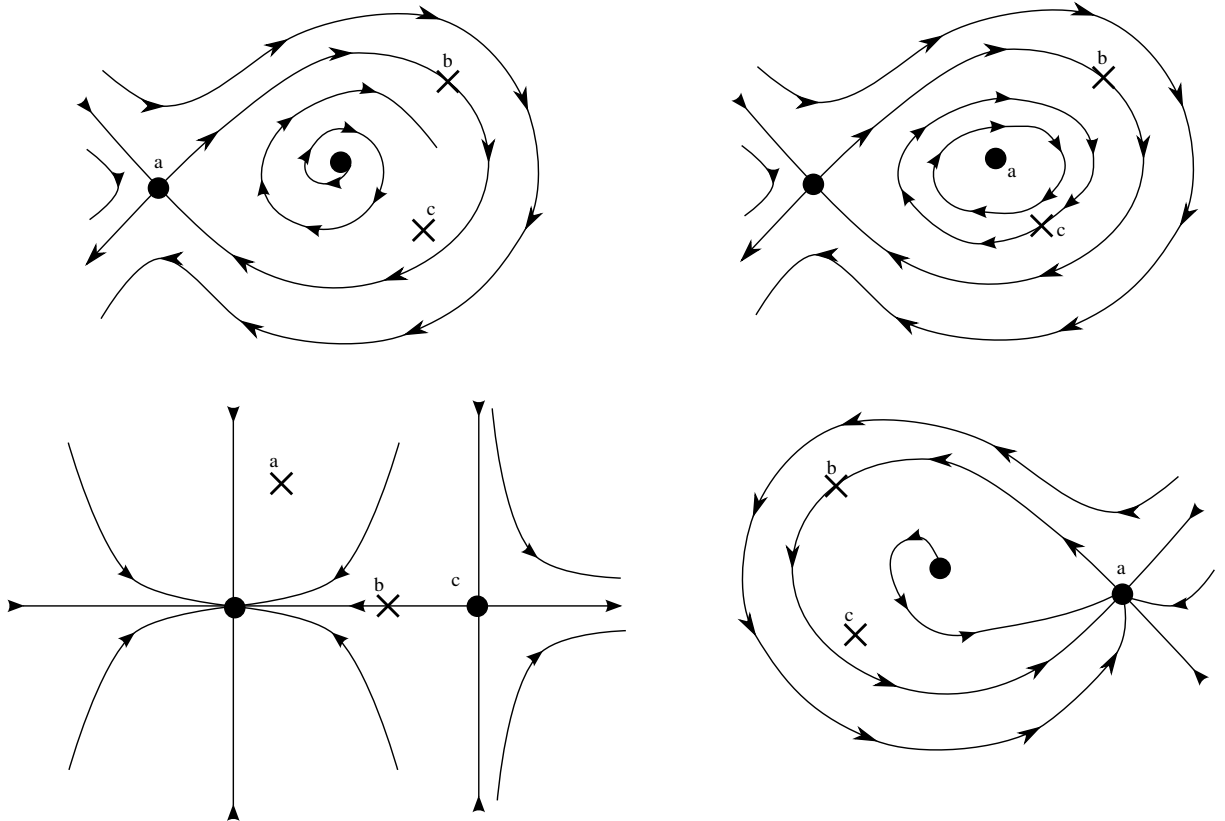


DEPARTMENT OF MATHEMATICS
MATHS 761 Worksheet 1 - Limit sets

1. The following pictures show the qualitative features of some planar flows. Equilibria are marked by black dots. In each case:

- (a) Find the ω -limit set and the α -limit set for each of the points a, b, c.
- (b) Find a subset of phase space that is forward invariant under the flow but not backward invariant under the flow.



2. (a) Draw a phase portrait in which the ω -limit set of a point x is a homoclinic orbit plus one equilibrium point. Label the point x clearly.
- (b) Draw a phase portrait in which the α -limit set of a point x contains a heteroclinic orbit. Label the point x clearly.
- (c) Draw a phase portrait in which the α -limit set of a point x is a periodic orbit and the ω -limit set of the same point x is a homoclinic orbit plus one fixed point. Label the point x clearly.

3. Write down a system of differential equations

$$\begin{aligned} \dot{x} &= f(x, y), \\ \dot{y} &= g(x, y), \end{aligned}$$

such that the y -axis is invariant under the flow, some orbits tend to the origin as $t \rightarrow \infty$ and all other orbits have a y -component tending to $-\infty$ as $t \rightarrow \infty$. *Hint: first draw a picture.*

4. Write down a system of differential equations

$$\begin{aligned}\dot{x} &= f(x, y), \\ \dot{y} &= g(x, y),\end{aligned}$$

which has only two equilibrium solutions, at $(x, y) = (1, 0)$ and $(-1, 0)$.

5. Consider the initial value problem

$$\dot{x} = \sqrt{|x|}, \quad x(0) = 0.$$

(a) Show that

$$x(t) = \begin{cases} 0 & t < T \\ \frac{(t - T)^2}{4} & t \geq T \end{cases}$$

is a continuous and differentiable solution to the IVP for any T .

(b) Explain how the existence and uniqueness theorems apply (or do not apply) to this IVP.