

- ▶ **Topic for today: Infinity**
  
- ▶ **Question of the day:** What comes to mind when you hear the word “infinity”? Write down your thoughts and questions.

## Without counting

- ▶ How can we find out if two groups (e.g., of people, tennis balls, sweets, etc) have the same number of members?
  
  
  
  
  
  
  
  
  
  
- ▶ What do we do if we can't count the members of the group?

# One-to-one correspondence

- ▶ Two collections of objects are equally numerous, precisely if there is a one-to-one correspondence between the elements of the two collections.

## Some notation:

- ▶ When we write  $\{1, 2, 3, \dots\}$  we mean the set consisting of the numbers 1, 2, 3 and all the other numbers following in the sequence.
- ▶ The curly brackets indicate that we are talking about a set. The dots indicate that we continue the sequence. Dots with no number after them means the sequence goes on forever.
- ▶ **Examples:**
  - ▶  $\{1, 3, 5, \dots, 13\}$  means the set of odd numbers from 1 up to 13.
  - ▶  $\{1, 3, 5, \dots\}$  means the set of all positive odd numbers.
  - ▶  $\{1, 3, 5, 7, 9, \dots\}$  also means the set of all positive odd numbers.

## Examples of one-to-one correspondence:

Find a one-to-one correspondence between the elements in the following pairs of sets.

1.  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

2.  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

3.  $\{1, 2, 3\}$

$$\{5, 6, 7, 8\}$$

4.  $\{1, 2, 3, 4, 5, 6, 7\}$

$$\{-3, -2, -1, 0, 1, 2, 3\}$$

5.  $\{1, 2, 3, 4, 5, \square \dots \square\}$

$$\{\square, \dots \square, -2, -1, 0, 1, 2, \square, \dots, \square\}$$

In this example we can assume that there are the same number of elements in both sets but we do not know how many. The boxes are covering up some numbers.

## Important Fact:

- ▶ The only way two sets can be the same size is if their elements can be put in one-to-one correspondence.

## Some definitions

- ▶ A non-empty set  $D$  is said to be **finite** if there is a natural number  $n$  such that there is a one-to-one pairing between the elements in the set  $D$  and the elements of the set  $\{1, 2, 3, \dots, n\}$ .
  
- ▶ An **infinite set** is a set that is not finite.

## Another definition

- ▶ Two sets have the same **cardinality** if there is a one-to-one correspondence between the elements of one set and the elements of the other.

## Try this exercise in pairs:

Compare the cardinality of the following sets. Which set of each pair is larger? If the sets are the same size as each other, find a one-to-one correspondence.

▶  $\{1, 3, 5, 7, \dots\}$

$$\{2, 4, 6, 8, \dots\}$$

▶  $\{1, 2, 3, \dots\}$

$$\{2, 3, 4, \dots\}$$

▶  $\{1, 2, 3, \dots\}$

$$\{1, 4, 9, 16, 25, \dots\}$$

## Comparing infinite sets

- ▶ When comparing two infinite sets, some pairings between the two sets might be one-to-one correspondences while other pairings are not.

- ▶ For example, compare the sets

$\{2, 3, 4, \dots\}$

$\{1, 2, 3, \dots\}$

- ▶ Failure to find a one-to-one correspondence between infinite sets does not necessarily mean that there is no one-to-one correspondence.

## Important ideas from today:

- ▶ The only way two sets can be the same size is if their elements can be put into a one-to-one correspondence.
- ▶ A non-empty set is finite if there is a natural number  $n$  such that there is a one-to-one pairing between the elements in the set and the elements in  $\{1, 2, 3, \dots, n\}$
- ▶ An infinite set is a set that is not finite.
- ▶ For infinite sets, some pairings may give one-to-one correspondences even though other pairings do not. Having a pairing that is not a one-to-one correspondence does not mean that there are no one-to-one correspondences.

## For next time

- ▶ Read §3.1 and §3.2 in the textbook.
- ▶ Try some Mindscapes at the end of §3.1 of textbook.
- ▶ Ask a friend what ideas come to mind when they think of infinity. Do they have similar ideas to you? Do either of you think there could be different sizes of infinity?