Maths 190 Lecture 2

- **Topic for today:** **Counting!**

- **Vitally important question**:

  Do there exist two nonbald people on the planet who have exactly the same number of hairs on their body?

M&M's

- I have a packet of M&M's, in which I know there are 55 sweets.
- The sweets come in 6 different colours - red, blue, green, yellow, orange and brown.
- Without opening the packet, which of the following statements are *definitely* true?

1. There must be at least 1 red M&M's.
2. There must be at least 0 blue M&M's.
3. There must be at least 10 M&M's of the same colour.
4. There must be at least 11 M&M's of the same colour.
5. There must be at least 11 yellow M&M's.

M&M's

- Now I eat 10 M&M's, without telling you which ones. What is the largest number you can replace the ? with and have these statements still be *definitely* true?

1. There must be at least ? red M&M's.
2. There must be at least ? blue M&M's.
3. There must be at least ? M&M's of the same colour.

Tennis Balls

- I have six tennis ball cans. Each can can hold at most four tennis balls.
- How do I know that at least two cans contain the same number of tennis balls?

The Pigeonhole Principle

- If we have a collection of things to put in categories, and there are more things than categories, then one category must contain more than one thing.

Some notes on language

Suppose we divide 13 sweets between 3 people, Claire, Vaughan and Ivo.

- Claire: 4, Vaughan: 2, Ivo: 7

What is the largest number you can write in the box?

- One of Claire, Vaughan and Ivo has at least □ sweets.
- Each of Claire, Vaughan and Ivo have at least □ sweets.

Some notes on language

Suppose we divide 13 sweets between 3 people, Claire, Vaughan and Ivo.

- Claire: 13, Vaughan: 0, Ivo: 0

What is the largest number you can write in the box?

- One of Claire, Vaughan and Ivo has at least □ sweets.
- Each of Claire, Vaughan and Ivo have at least □ sweets.

Some notes on language

Suppose we divide 13 sweets between 3 people, Claire, Vaughan and Ivo.

- Claire: ?, Vaughan: ?, Ivo: ?

What is the largest number you can write in the box, and be sure of being correct, however the sweets are divided?

- One of Claire, Vaughan and Ivo has at least □ sweets.
- Each of Claire, Vaughan and Ivo have at least □ sweets.

Tennis Balls again

- How many tennis balls would it take to fill this room?

- How would you estimate how many grains of sand are on the beach?

Back to our important question...

- How may hairs do you have on your body?
- Make a rough estimate on a small area.
- Find an upper limit.

Finding an upper limit

- Roughly 100 hairs in 5mm $\times$ 5mm square.
- So about 400 hairs in a square centimetre.
- *Surely* no-one can have more than 4000 per square centimetre.

- Estimate surface area as a cylinder: 2m high, 100cm around: area of $200 \times 100 = 20,000$ square centimetres.
- *Surely* no-one can have a surface area greater than 200,000 square centimetres.

Maximum number of hairs

- So a person 10 times as hairy (all over!), with 10 times the surface area, would have at most

$$200,000 \times 4000 = 800,000,000 \text{ hairs}$$

How many people?

- Current estimates put the world population at about 6.6 billion

$$6,600,000,000 \text{ people}$$

What do these two numbers tell us?

- At most 800,000,000 hairs on a person.
- At least 6,600,000,000 people in the world.

- Two must have exactly the same number of hairs....
- But we can't tell who these people are.

More mathematically

- If there are k times more things than categories, then one category must contain *at least k* or more things.
- So actually, since

$$\frac{\text{people}}{\text{hairs}} = \frac{6,600,000,000}{800,000,000} = 8.25$$

  there must be at least *nine* people in the world with the same number of hairs.
- Note that we don't know which category has k things - that is, we don't know how many hairs the nine people have.

A harder example

The numbers $1, 2, \ldots, 8$ are written in a circle, in any order. Show that there are 3 adjacent numbers whose sum is 14 or greater.

Important ideas from today:

- Its possible to estimate very large numbers of things by breaking one large problem down into a lot of smaller ones.
- The pigeonhole principle. This principle says that if you have to put N things into N-1 boxes, at least one box has to have two things in it.

For next time

- Read 2.2 in the text and think about other patterns you see in nature, or around you in your daily life.
- Bring **a pineapple** to the next class.