DEPARTMENT OF MATHEMATICS

MATHS 190 **Tutorial 5**

Tutorials in Maths 190 are **collaborative tutorials**. You should work in groups of 3 or 4 students, discussing the situations and puzzles listed below, or issues arising from lectures. Part of your final mark depends on your participation in tutorials.

A written answer to question 6 should be handed in for marking with your answers to Assignment 2 (due August 25th).

1. A Double Decker package contains two delicious chocolate bars. Suppose you had infinitely many Double Decker packages: one for each natural number. Does the collection of individual chocolate bars have the same cardinality as the natural numbers? If not, explain why? If so, provide a one-to-one correspondence.

2. Not-Finite City is made up of infinitely many roads running north and south (one road for each natural number) and infinitely many streets running east and west (one street for each natural number). A traffic light is placed at every intersection of a street with a road. How many traffic lights are there? Does the set of traffic lights have the same cardinality as the set of natural numbers? If not, explain why. If so, provide a one-to-one correspondence.

3. Your friend gives you a list of three, five-digit numbers, but she only reveals one digit in each: 3????, ?8???, ??2??. Can you write down a five-digit number you know for certain will not be on her list? If so, give one; if not, explain why not.

4. Consider the following infinite collection of real numbers. First, describe in your own words how these numbers are constructed (that is, describe the procedure for generating this list of numbers). Then, using Cantor's diagonalization argument, find a number not on the list. Justify your answer.

   0.12345678910111213141516171 8...
   0.24681012141618202224262830 32...
   0.36912151821242730333639424 5...
   0.48121620242832364044485256 60...
   and so on.

5. Prove that a small circle has the same number of points as a large triangle. Stated precisely, prove that the cardinality of points on a small circle is the same as the cardinality of points on a large triangle. Do this by describing a one-to-one correspondence between these two sets.

6. **Write up your answer to this question and hand it in with your answers to Assignment 2 (due August 25th).** Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

   Consider the infinite collection of boxes below:

   $\square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \ldots$

   Suppose you have two markers, one red and one blue, and you colour each box one of the two colours. How many different ways could you colour the collection of boxes? Show that the set of all possible box colourings has a greater cardinality than the set of all natural numbers.

7. Prove that the cardinality of points in a solid cube is the same as the cardinality of points on a line segment.

8. **Harder:** Recall the Hotel Cardinality from the last tutorial. It is a hotel with as many rooms as there are natural numbers. The room numbers are 1,2,3,4,5,.... Create a collection of people so that it would be impossible for the night manager to give each person a room. Explain why it is not possible to give each person from the collection a room.