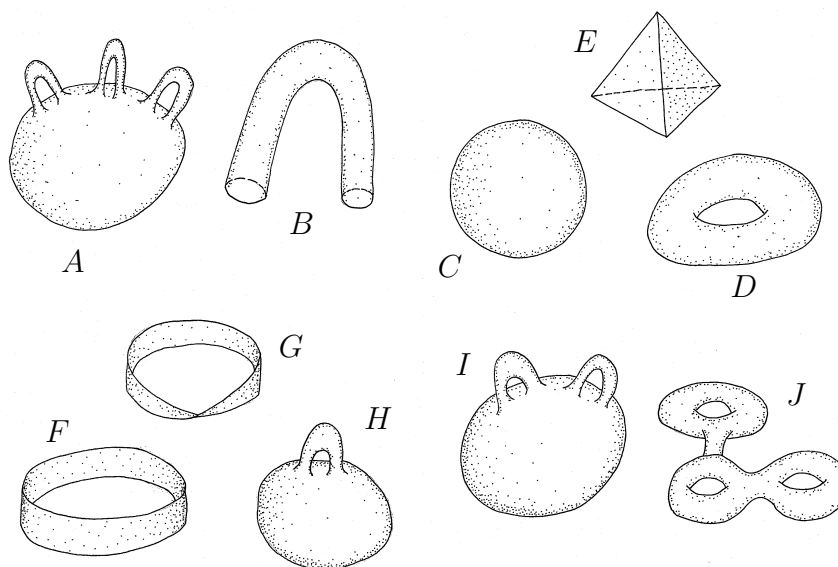


Maths 190 Assignment 4

October 7, 2010

Due: 4pm, Monday October 18th, 2010

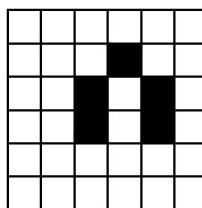
- Your completed assignment should be handed in to the appropriate box on the ground floor of the Mathematics/Physics Building **before** 4pm on the date due.
 - Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.
 - Late assignments or assignments placed in the wrong box **will not be accepted**.
 - You must show all your working, and write careful, clear explanations of your solutions, stating all your assumptions.
 - You may discuss your attempts to answer the questions with other members of the class. However, **you should write up your answers yourself (do not copy from another person or allow another person to copy from you)**. If you worked with someone on your answers, state who you worked with.
1. (10 marks) Referring to the following **surfaces** by their letter labels, A, B, C, \dots , group the surfaces together that are equivalent by distortion. *Note: the tube labelled B is open-ended.*



2. (6 marks)
- (a) Take two strips of paper and put them on top of each other. Twist them together as if you were making a Möbius strip, taping the tops together and the bottoms together. How many sides does the object you have just made have? What do you get if you cut it in half lengthwise? (keeping the two bands together whilst cutting).
 - (b) Take a strip of paper and put three half twists in it, and tape the ends together. How many sides and edges does the object have? Cut it lengthwise down the centre. What do you get?

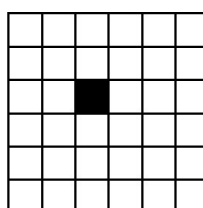
3. (6 marks)

(a) Consider the following initial population in the game of life.



Show the population for the next two steps.

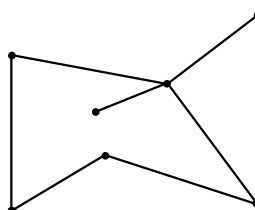
(b) Find a population in the game of life whose next step is the following picture.



You may use the additional sheet of blank grids to show your answers if you wish.

4. (10 marks)

(a) How many vertices and edges does the following graph have?



(b) Show that this graph cannot have an Euler circuit.

(c) What is the minimum number of edges that need to be added to this graph to obtain a graph which has an Euler circuit? Show on a copy of the figure the edge(s) you need to add.

(d) Write down Euler's formula relating the number of vertices, edges and faces of a graph.

How many faces does the graph in part (a) have? Verify Euler's formula in this case.

(e) You will now prove a special case of Euler's formula: Suppose one has a graph which satisfies Euler's formula. Now suppose one adds a single edge between two existing vertices to get a new graph. Explain why the new graph satisfies Euler's formula.

Tutorial write up: Remember to hand in with your assignment your written solutions to question 4 on tutorial 9 (5 marks), question 5 on tutorial 10 (5 marks) and question 6 on tutorial 11 (5 marks).

