DEPARTMENT OF MATHEMATICS

MATHS 761 Worksheet 2 - Stability and linear systems

1. For each of the equilibria in the figures in Worksheet 1, question 1, describe the stability of the equilibria (Liapounov stable, quasi-asymptotically stable, asymptotically stable or none of these)

2. (Strogatz 5.1.10) For each of the following systems, decide whether the origin is Liapounov stable, quasi-asymptotically stable, asymptotically stable or none of these. (Hint: draw the phase portrait first.)

   (a) $\dot{x} = y$, $\dot{y} = -4x$

   (b) $\dot{x} = 2y$, $\dot{y} = x$

   (c) $\dot{x} = 0$, $\dot{y} = x$

   (d) $\dot{x} = 0$, $\dot{y} = -y$

   (e) $\dot{x} = -x$, $\dot{y} = -5y$

   (f) $\dot{x} = x$, $\dot{y} = y$

3. Consider the system of equations:

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & 1 \\ -a & b \end{pmatrix} \mathbf{x}$$

   For each of the following cases sketch the phase portrait, identify the type of stationary solution (e.g., saddle, node sink, spiral sink, etc.) and describe the qualitative behaviour of solutions to the differential equation.

   (a) $a = 1, b = 0$

   (b) $a = 0.5, b = -1$

   (c) $a = 0.5, b = 1$

   (d) $a = -2, b = 1$

   (e) $a = 2, b = -3$

   (f) $a = 2, b = 3$

   (g) $a = 1, b = -2$

   (h) $a = 1, b = 2$

4. (Harder) Consider the system of equations:

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha & -1 & 1 \end{pmatrix} \mathbf{x}$$

   Find five values of α corresponding to five different types of stationary solution at the origin. (Hint: first show that for all α there is only one real eigenvalue of the matrix, that the product of the eigenvalues is α and that the sum of the eigenvalues is 1.)