DEPARTMENT OF MATHEMATICS

MATHS 190 Lecture 10 Summary

In this lecture we explored infinity in geometrical objects.

We used one-to-one correspondences defined geometrically (i.e., with pictures) to show that two line segments of different length have the same cardinality, and that a line segment (with end points missing) has the same cardinality as the whole real number line. We showed that there is the same number of points in a square as on a line segment.

We saw some pictures of geometric patterns illustrating different sorts of infinite collections.

We saw that our explorations of infinity have opened up a lot of questions about infinity. For instance:

- Is there an infinity greater than the cardinality of the set of natural numbers but less than the cardinality of the set of real numbers?
- Is there an infinity greater than the cardinality of the set of real numbers?
- Are there infinitely many different sizes of infinity?
- Is there a largest infinity - one that encompasses all the others?

Three of these questions have answers (yes or no) but the other cannot be answered (it is known that neither 'yes' nor 'no' is a correct answer to that question). Read the textbook to find out which is the weird question.

Understanding ideas about infinity is an intellectual triumph. The ideas we have explored are challenging, but they allow us to grasp concepts of infinity that for thousands of years seemed beyond human comprehension.

Before you come to the next lecture: You should spend an hour or two reviewing the material from todays lecture. You should also

- Read §3.5 in the textbook.
- Try some of the Mindscapes at the end of §3.5 in the textbook.

Other activities you could do if you have time are:

- Think further about the questions about the size of infinities listed above. Read section 3.4 in the textbook.