Roughly Weighted Games

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Abstract

One of the most fundamental questions of the theory of simple games is what makes a simple game a weighted majority game. The necessary and sufficient conditions that guarantee weightedness are known: Taylor and Zwicker (1992) showed that a simple game is weighted majority game if no sequence of winning coalitions (up to a certain length) can be converted into a sequence of losing coalitions by exchanging players. If a simple game does not have weights, then rough weights may serve as a reasonable substitute (see von Neumann and Morgenstern (1944)). A simple game G on the set $[n] = \{1, 2, ..., n\}$ of players is called roughly weighted if there exists a weight function $w : [n] \to R^+$, not identically equal to zero, and a positive real number q, called quota, such that for $X \in 2^P$ the condition $\sum_{i \in X} w_i < q$ implies X is loosing, and $\sum_{i \in X} w_i > q$ implies X is winning. My work currently concerns the existence of rough weights.

The game G is called weak k-trade robust if no sequence of winning coalitions of length k can be converted into a sequence of losing coalitions by exchanging players, where grand coalition is among winning coalitions and empty set is among loosing coalitions. We show that for the game to have rough weights it is necessary and sufficient to be weak k-trade robust for all k. If we don't know the size of the game, then all conditions of weak k-trade robustness are needed (or may be needed) to establish the existence of rough weights, but if the size of the game is fixed, say n, only finite number of them is necessary. Let $g(n)$ be the smallest positive integer such that every simple game with n players which is weak $g(n)$-trade robust is roughly weighted. We describe $g(n)$ for games with small number of players and show that all games with n greater than or equal to four players are roughly weighted. We demonstrate that $g(n)$ depends on the type of a game, for example all six players constant-sum games are roughly weighted, but we can find strong or proper game with six players without rough weights. The smallest constant sum game that is not roughly weighted is a game with seven players. We prove that for any game with n players holds

$$2n + 3 \leq g(n) < (n + 1)2^{\frac{1}{2} n \log_2 n}.$$

The gap between this two bounds is large, but it is actually smaller than Taylor and Zwicker got for weighted games.

Also we study projective games - one of the most interesting examples of games without rough weights.