# Closed Form Solutions for the Fundamental Solution of Laplace's Equation in the Hyperboloid model of Hyperbolic Geometry

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In this talk I will discuss the computation of closed form hyperbolic function expressions of the fundamental solution of the Laplacian, in $\mathbf{H}^n$ yielding the solution for inhomogeneous Poisson-type problems in this space. We work in a model of hyperbolic geometry called the hyperboloid model (or the pseudo-sphere) in which one chosen sheet of a two sheeted hyperboloid is embedded in a higher dimensional Minkowski space. This is accomplished by computing the appropriate Laplace-Beltrami operator and then solving Laplace's equation for a radially symmetric solution. The solutions can be seen to match up to the corresponding fundamental solutions, in $\mathbf{R}^n$ by comparing the local functional form of the singularity and incorporating a global additive constant so that the solutions vanish in the far-field limit. The fundamental solutions are given in terms of a minimum distance function for two arbitrary points along a geodesics on the corresponding manifold and the functional behaviour of the fundamental (radial) solutions and the corresponding (general angular) harmonics are seen to be given in terms of associated Legendre functions of the first and second kind with order and degree given by $n/2-1$.