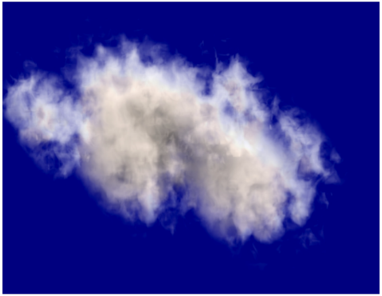
Maths 190 Lecture 16

Topic for today:

Degrees of brokenness or: Fractal Dimension

Question of the day:

What is the dimension of a cloud?



What do we mean by dimension?

- Why do we say a **line** is *one-dimensional* but a **square** is *two-dimensional?*
- Why is a **cube** *three-dimensional?*

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Here's another one...

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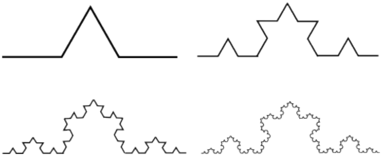
Here's another one...

- Why is **space-time** *four-dimensional?*

Fractal dimensions

- What is the dimension of the **Koch curve**?
  - It's not a straight line.
  - It's infinitely long and infinitely detailed



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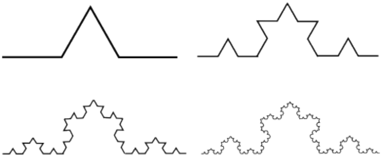
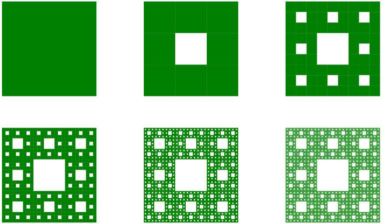


- What about the **Sierpinski Carpet**?
  - It's not a solid and it has zero area.



- How can we calculate the dimension of these fractals?

A scaling problem

Start with a line segment of a fixed length.

- **How many copies** of the line segment does it take to **make another line segment** that is **twice as long**?
- **How many copies** of the line segment does it take to **make another line segment** that is **three times as long**?

Original	Dimension	Scaling factor	Number of copies
Line	1	2	2
	1	3	
Square	2	2	
	2	3	
Cube	3	2	
	3	3	

A definition of dimension

Let d be the **dimension** of an object, let S be the **scaling factor** to make a larger version, and let N be the **number of copies** required to build the larger version.

We found the following pattern for line segments, squares and cubes..

$$S^d = N$$

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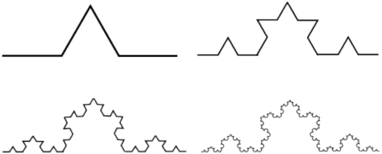
Use this to *define* **dimension:**

$$d = \frac{\ln N}{\ln S}$$

Here $\ln x$ means the "natural logarithm" of the number x.

Note: d does not need to be an integer for this definition to make sense.

Dimension of the Koch Curve



- How many copies are needed?

$$N =$$

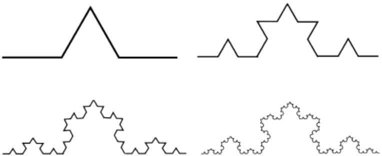
- What is the scaling factor?

$$S =$$

- What is the dimension?

$$d = \frac{\ln N}{\ln S} =$$

Dimension of the Koch Curve



- How many copies are needed?

$$N = 4$$

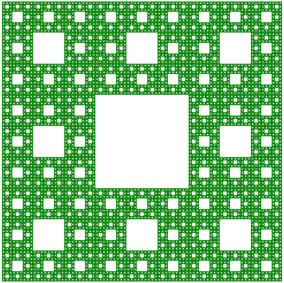
- What is the scaling factor?

$$S = 3$$

- What is the dimension?

$$d = \frac{\ln N}{\ln S} = \frac{\ln 4}{\ln 3} = 1.26186\ldots$$

Dimension of the Sierpinski Carpet



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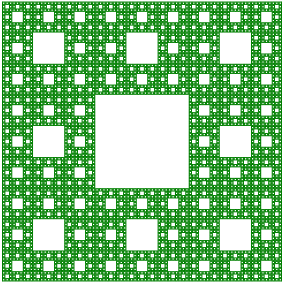

- How many copies are needed?

$$N = 8$$

- What is the scaling factor?

$$S = 3$$

- What is the dimension?

$$d = \frac{\ln N}{\ln S} = \frac{\ln 8}{\ln 3} = 1.89279\ldots$$

Building fractals

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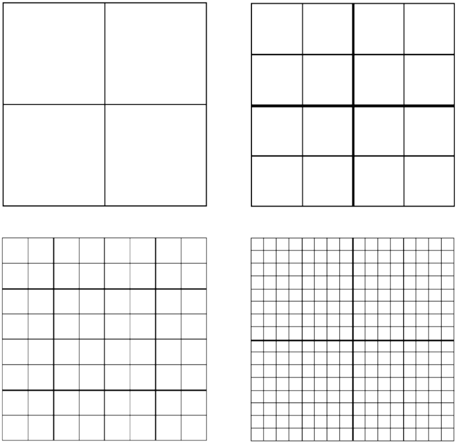

Important idea from today:

- The **dimension** of an object *need not be an integer!*

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- Read section 6.2

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- Can we create a fractal with dimension $\frac{\ln 3}{\ln 2}$ using line segments?

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- The **dimension** of an object *need not be an integer!*
- We defined the notation of a **"fractal dimension"**.

For next time:

- Read section 6.2
- Can we create a fractal with dimension $\frac{\ln 3}{\ln 2}$ using line segments?
- Can you construct a curve with dimension exactly 1.5?