Maths 761 Lecture 10 Summary

Topics for today

Discrete maps derived from flows

Reading for today's lecture

Glendinning: §6.1-§6.4
Strogatz: §8.7

Recommended problems

Glendinning: §6: 8
Strogatz: 8.7.2, 8.7.9

Reading for next lecture

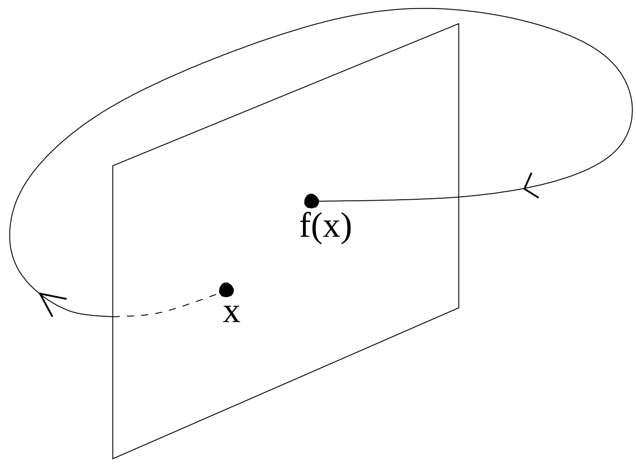
Glendinning: §4.3, §4.4

Discrete maps derived from flows

Discrete maps arise from the study of differential equations in two main ways:

1. **Return maps** (for autonomous differential equations)

   Pick a surface (or cross-section) in the phase space, with the dimension of the surface being one less than the dimension of the phase space. The flow in the phase space then induces a map on the surface: for each point, x_0, on the surface, define its image under the return map to be the place that the solution curve passing through x_0 next intersects the surface as it crosses the surface in the same direction (see figure on next page).

   For example, in $\mathbb{R}^3$ a suitable surface might be the plane $x = 0$. Orbits can cross this plane with x increasing or decreasing. We will be interested in either those intersections for which x is increasing or those for which x is decreasing, but not both.

   For a return map to be well defined and continuous, orbits must in fact return to the surface and the flow must be transverse to the surface.

2. **Poincaré maps** (for non-autonomous differential equations)
   In periodically forced systems there is a distinguished time interval, T, the period of the forcing. A Poincaré map is derived from the flow by defining the image under the Poincaré map of a point located at x_0 at time t_0 to be the position in phase space of the orbit through x_0 after time T has elapsed. Different choices of t_0 will lead to different Poincaré maps, but they will all be equivalent.

Construction of return maps from particular flows is usually difficult analytically, although often straightforward numerically. We often get around this difficulty by abstracting the property we are interested in and throwing away the equations. For an example of this approach, see the discussion of homoclinic bifurcations later in the course.

A return map will not always capture all the information contained in the original flow. For example, what orbit in a return map corresponds to a stationary solution in the original flow? What about a homoclinic orbit in the original flow?