# Maths 190 Assignment 3

September 23, 2010 Due: 4pm, 29th September 2010

- Students should hand in their assignments at the boxes in the basement of the Mathematics/Physics Building.
- Your completed assignment should be handed in to the appropriate box **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be accepted.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.

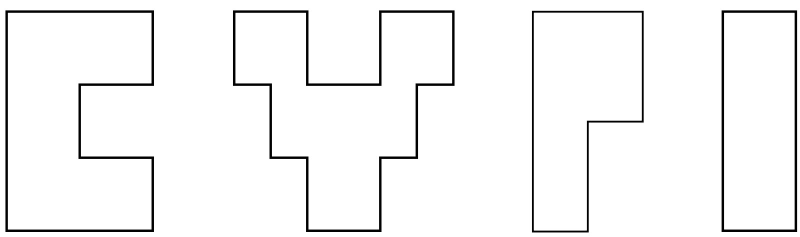
1. (4 marks) Your friend is crazy about fantasy role-playing games. He is especially fascinated about the many different dice which are so much more interesting than the ordinary ones with just six faces. One day he comes to you and asks the following questions:

   (a) "For my new adventure I want to throw a fair die to find out on which day of the week my player character will be struck by lightning. But it seems to be impossible to find dice with seven faces. Do you have an idea why no company bothers to make these?"

   (b) "Last night I had a dream of my player character, the mighty sorcerer Nezahualpilli of the Hag: He told me that if I count the corners and the edges of an arbitrary die he could tell me how many faces the die has. Is this some kind of black magic?"

   Can you help your friend with these questions?

2. (16 marks) Claire, Paul, Vaughan and Ivo would like to give their bathrooms a slightly more personal touch. Therefore, they all decide to tile the walls only using their favourite tiles: Claire especially likes the C-shaped tiles, Vaughan only wants to see V-shaped tiles in his bathroom and Paul and Ivo also prefer tiles which slightly resemble their initials.



(a) Claire (b) Vaughan (c) Paul (d) Ivo

Figure 1: Four favourite bathroom tiles

   (a) For each of the four types of tile show one possible tiling of their bathroom walls. In each case you are only allowed to use one tile (as well as rotations or reflections of this tile). Be sure that the whole plane can be covered, in particular, leave no holes between tiles!

(b) For each of your four tilings, list the rigid symmetries which you can find in the tiling patterns. Indicate translations by arrows and be sure to give lines of reflection and angles of rotation.

(c) Paul is proud that his tile can be used to make a pattern which has a nice symmetry of scale. Show how you can build a super-tile with 16 copies of his favourite P-shaped tile (there is more than one correct solution).

(d) Show that if Paul's bathroom is tiled so that it has the symmetry of scale which you discovered in (c), then it also has rigid symmetries. Give one example.

3. (8 marks) On the extra sheet you will find the first six iterations of the Hilbert curve (Figure 2), a fractal which lies inside the unit square. The Hilbert curve consists of line segments. At each iteration, four smaller copies of the preceding iteration are assembled and glued together by additional line segments.

   (a) On the additional page, for iterations 2 to 6 indicate the four copies of the preceding iteration in a different colour. Hand in this page with your solutions. How many additional line segments are needed for glueing the copies together?

   (b) From the first six iterations of the Hilbert curve, what do you expect it to look like if this process is continued forever?
   **Hint:** This type of curve is also called a *space-filling curve.*

   (c) The dimension of the unit square is 2, of course. *Without calculation*, which value do you expect for the fractal dimension of the Hilbert curve?

4. (22 marks) Now, we shall calculate the fractal dimension of the Hilbert curve.

   (a) Have a look at your pictures from 3(a) and explain why the additional line segments become less and less important for higher iterations by completing the following sentence:
   "The additional line segments become less and less important for higher iterations because there are always _____ additional line segments and their length becomes _____ and _____ in comparison with the copies of the preceding iterations."

   (b) Now calculate the length of the Hilbert curve at each iteration. To do this, complete columns 2-4 of Table 1, as follows:

      i. In column 2 of Table 1, add the *number of line segments* at iteration 3 by looking at Figure 2c. Try to spot a pattern to find out the numbers for iterations 4-6.
      **Hint:** How many copies and additional line segments do you need for the next iteration?

      ii. For iterations 3 and 4 determine the *length of one line segment* by looking at Figure 2. Add these values to column 3 of Table 1. Again, try to find a pattern for iterations 5 and 6.

      iii. Calculate the *total length* of each iteration by multiplying the number of line segments by the length of each line segment. Add this information to column 4 of Table 1.

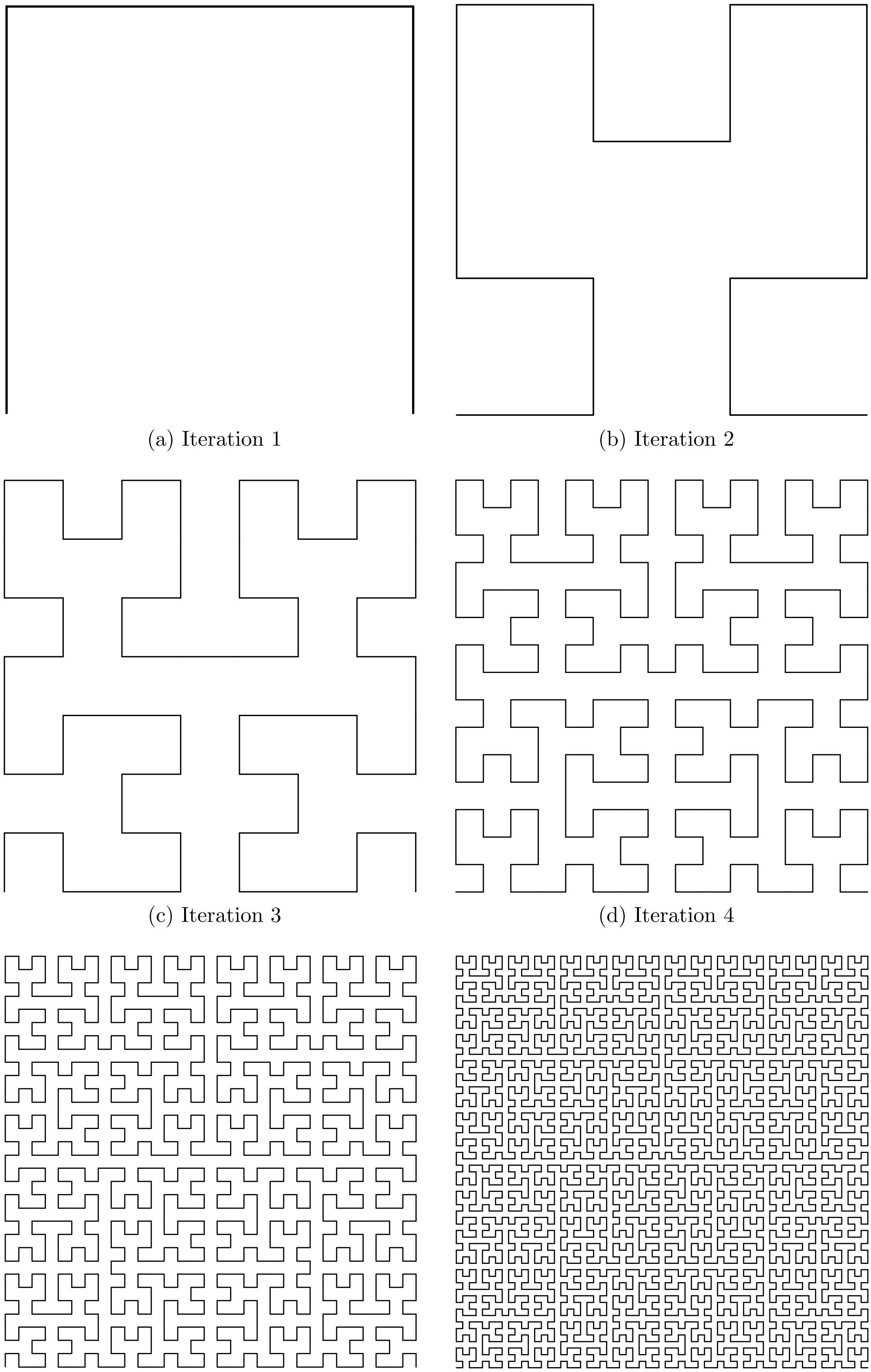
   (c) We have observed in 4(a), that the additional line segments which are needed to glue our copies together become less and less important. Therefore, assume that you only need four copies of an iteration for building the next one. For iterations 2-5, calculate the *scaling factor* by multiplying the total length of an iteration by the number of copies and divide by the length of the next iteration. Add your results to column 5 of table 1. To which value does the scaling factor tend to?

Table 1: Length and scaling ratios of the Hilbert curve at subsequent iterations

| Iterations | No. of line segments | Length of each line segment | total length | scaling factor |
|---:|---:|---:|---:|---:|
| 1 | 3 | 1 | $3 \cdot 1 = 3$ | $12/5 = 2.4$ |
| 2 | 15 | $\frac{1}{3}$ | $15 \cdot \frac{1}{3} = 5$ | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |

(d) Finally, take the freedom to be sloppy: Ignore the additional line segments, i.e., assume that you need only four copies to build the next iteration. Also, take the scaling factor which the values in column 5 of Table 1 seem to tend to. Using these values calculate the fractal dimension of the Hilbert curve. Looking back at exercise 3(c), are you surprised by your result?

**Tutorial write up:** Remember to hand in with your assignment your written solutions to Question 5 on Tutorial 6 (5 marks), Question 4 on Tutorial 7 (5 marks) and Question 5 on Tutorial 8 (5 marks).



(a) Iteration 1
(b) Iteration 2
(c) Iteration 3
(d) Iteration 4
(e) Iteration 5
(f) Iteration 6

Figure 2: The first six iterations of the Hilbert curve