DEPARTMENT OF MATHEMATICS

MATHS 190 Lecture 7 Summary

In the next two weeks of the course we study infinity.

The fundamental idea in our study of infinity is that two sets are of the same size if there is a one-to-one correspondence between members of one set and members of the other set. We illustrated this idea by comparing some finite sets. We found one-to-one correspondences between the following sets:

- Red and blue tennis balls in a box
- $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$
- $\{1, 2, 3, 4, 5, 6, 7\}$ and $\{-3, -2, -1, 0, 1, 2, 3\}$
- $\{1, 2, 3, 4, 5, \square\}$ and $\{\square, -2, 1, 0, 1, 2, \square\}$ where $\square$ indicates that an unknown (finite) number of elements in a set have been covered up (the same number in total from each set).

We also found one-to-one correspondences between some infinite sets:

- $\{1, 3, 5, 7, \dots\}$ and $\{2, 4, 6, 8, \dots\}$
- $\{1, 2, 3, \dots\}$ and $\{2, 3, 4, \dots\}$
- $\{1, 2, 3, \dots\}$ and $\{1, 4, 9, 16, 25, \dots\}$

In the case of infinite sets, we found that not all pairings give a one-to-one correspondence. Having a pairing that is not a one-to-one correspondence does not mean that there are no one-to-one pairings.

Before you come to the next lecture: You should spend an hour or two reviewing the material from todays lecture. You should also

- Read §3.1 and §3.2 in the textbook.
- Try some of the Mindscapes at the end of §3.1 in the textbook.

Other activities you could do if you have time are:

- Ask a friend or family member what ideas come to mind when they think of infinity. Does the person you talk to have similar ideas to you? Do either of you think there could be different sizes of infinity?