DEPARTMENT OF MATHEMATICS

**MATHS 190 Lecture 6 Summary**

---

In this lecture we illustrate three important observations:

- Some numbers are rational, others are not
- There are very few rational numbers compared to irrational numbers.
- But there are still infinitely many!

Lecture 6 was based around the following question:

**Question: Can we prove that all numbers are rational? If not, how many are?**

We began by discussing what a rational number is (it's a number that can be written as a fraction), and we showed how rational numbers are infinitely close together. We can always find a rational number between any two other rational numbers.

We then proved, using a lovely argument from the ancient Greeks, (another example of proof by contradiction) that $\sqrt{2}$ is not a rational number. We did this by assuming it is rational, and then deriving a contradiction. You then proved, using the same method, that $\sqrt{3}$ is also not rational.

We then showed how any number with a repeating decimal expansion must be a rational number, and vice versa. (This uses the pigeonhole principle in a really cool way!).

We discussed how, in the number line, there is always a rational number between any two irrationals, and an irrational number between any two rationals.

Finally, we discussed how very FEW rational numbers there really are. If you choose a number between 0 and 1 at random, the chance of its being rational is zero! The number line is made up almost completely of irrational numbers.

Before you come to the next lecture: You should spend an hour or two thinking and reading about the ideas presented in the lecture. You should also:

- Read Section 3.1 of the textbook. Beyond Numbers.

Other things to think about:

- Why can you not prove that $\sqrt{4}$ is not a rational number? Where does the proof break down?
- Try to find the positive number closest to 0 on the number line. Ask your friends if they can find it. What about the number closest (but not equal) to 1?
- Is 0.999999... the same as 1?