Department of Mathematics

Maths 190 Assignment 2 Solutions

September 15, 2010 Due:

1. (4 marks).

   (a) Write $X = 0.27272727\ldots$. Then

   $$100X = 27.27272727\ldots$$

   so

   $$99X = 27$$

   and

   $$X = \frac{27}{99} = \frac{3}{11}.$$

   (b) Write $Y = 1.23012012012012\ldots$. Then

   $$100,000Y = 123012.012012012\ldots$$

   and

   $$100Y = 123.012012012\ldots$$

   so

   $$99,900Y = 123,012 - 123 = 122,889$$

   and

   $$Y = \frac{122,889}{99,900} = \frac{40,936}{33,300}.$$

2. (4 marks)

   (a) We have to solve:

   $$7\times 2 + 3\times 1 + 9\times 1 + 7\times 8 + 3\times 7 + 9\times 2 + 7\times 9 + 3\times 4 + 9\times x = 0 \mod 10$$

   $$196 + 9x = 0 \mod 10$$

   $$6 - x = 0 \mod 10$$

   $$x = 6$$

   (b) We have to solve:

   $$7\times 1 + 3\times x + 9\times 3 + 7\times 2 + 3\times 8 + 9\times 8 + 7\times 0 + 3\times 1 + 9\times 9 = 0 \mod 10$$

   $$228 + 3x = 0 \mod 10$$

   $$8 + 3x = 0 \mod 10$$

   $$3x = -8 = 2 = 12 \mod 10$$

   $$x = 4$$

3. (3 marks) Yes, it is possible, for example, the number with decimal expansion constructed from one 1 then one 2, then two 1's, two 2's, etc:

$$0.121122111222111122221111122222\ldots$$

4. (12 marks)

   (a) The cardinality of B is less that the cardinality of N.
   As there is a finite volume of water available to fill the Auckland Harbour and the bucket used has a volume greater than some minimal amount, then the division of these results in a finite maximum number of possible buckets of water that can be removed. A finite set has a smaller cardinality than N.

   (b) The cardinality of F is equal to the cardinality of N.
   The same argument used in class works here also. Build the set of rationals with only the supplied digits in the following way:

   Start with $\{1, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{22}, \frac{2}{23}, \frac{2}{24}, \frac{2}{25}, \ldots\}$

   Fill in the missing ones $\{1, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots\}$

   Add the reciprocals $\{1, \frac{2}{3}, \frac{3}{2}, \frac{2}{4}, \frac{4}{2}, \frac{3}{4}, \frac{4}{3}, \frac{2}{5}, \frac{5}{2}, \frac{3}{5}, \frac{5}{3}, \frac{4}{5}, \frac{5}{4}, \ldots\}$

   Add in the negatives $\{1, \frac{2}{3}, -\frac{2}{3}, \frac{3}{2}, -\frac{3}{2}, \frac{2}{4}, -\frac{2}{4}, \frac{4}{2}, -\frac{4}{2}, \frac{3}{4}, -\frac{3}{4}, \frac{4}{3}, -\frac{4}{3}, \frac{2}{5}, -\frac{2}{5}, \ldots\}$

   This gives an ordering of the fractions that can be logically paired off with the natural numbers, hence a one-to-one correspondence exists.

   (c) The cardinality of L is equal to the cardinality of N.
   Number the legs of the trousers and make the set $\{LT_1, RT_1, LT_2, RT_2, LT_3, \ldots\}$ and make a one-to-one pairing between odd numbers and the left legs and then another one-to-one pairing between the right legs and the even numbers. This generates a one-to-one paiting between all the legs of all the trousers and the natural numbers.

   (d) The cardinality of P is larger than the cardinality of N.

   I can construct a one-to-one correspondence between points on the circle and points on the (boundary of the) pentagon in one of two ways. Both rely on the fact that a circle has the same number of points as the interval $[0, 1]$.

   First, I could put the circle inside the pentagon. The circle will touch the pentagon at five points. Then any line segment drawn from the point at the centre of the circle out to a point on the pentagon will intersect the circle at precisely one point. This defines a pairing between a point on the circle and a point on the pentagon. The pairing is clearly one-to-one, as each point on the circle corresponds to one and only one point on the pentagon. Thus there is a one-to-one correspondence between points on the circle and points on the pentagon and so the set of points on the circle has the same cardinality as the set of points on the pentagon.

   Alternatively, I could straighten out the circle to make a line segment of length $2\pi r$ cm (r = radius of the circle) and straighten out the pentagon to make a line segment of length $5x$ cm (x = side length of pentagon). As shown in class, I can always construct a one-to-one correspondence between points on two line segments of different lengths. First draw a line $S1$ joining the left ends of both line segments, then draw a second line $S2$ joining the right ends of both line segments. The lines $S1$ and $S2$ will meet at precisely one point P. Then define a one-to-one correspondence between points on the two line segments by drawing lines through P and the line segments. Each line that crosses P and one line segment will also intersect the other line segment; there will be exactly one intersection of the line with each line segment. This defines a one-to-one correspondence between points on the two line segments.

Both these argument are easier to explain with the help of a decent picture, but a picture alone is not sufficient to gain more than 2 marks.

5. (5 marks)
1 mark for - Ivo needs only 1 infinite bus.
1 mark for - as the total number of passengers trying to get to the All Black training has the same cardinality as the natural numbers.
3 marks for an explanation that is satisfactory, one such explanations is:

Assign all the passengers from Room 1 on Ship 1 to a seat on the infinite bus. Now move all the seated passengers to even numbered seats, this allows all the passengers in Room 2 of Ship 1 to sit in the odd numbered seats. Now move all the seated passengers to even numbered seats, this allows all the passengers in Room 3 of Ship 1 to sit in the odd numbered seats. continue this until all passengers from Ship 1 are seated. Now start on Ship 2. Move all the seated passengers to even numbered seats, this allows all the passengers in Room 1 of Ship 2 to sit in the odd numbered seats. Now move all the seated passengers to even numbered seats, this allows all the passengers in Room 2 of Ship 2 to sit in the odd numbered seats. Continue this until all ships have all passengers seated on the infinite bus.