Maths 190 Lecture 17

Topic for today:

Change!

Dynamical Systems & Cellular Automata

Question of the day:

How many people will live on this planet in the year 2525?

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Three ingredients of change

1. From where do we **start**?—the *initial condition*
2. How does **change** happen?—a *rule*
3. **How long** does it take?—*time*

Note:

Parameters (like, e.g., a *growth rate*) can be considered as a part of the rule.

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Iterative Dynamical Systems

A **bank balance:**

- Let B_n be the amount of money in a bank account at time n.
  If no withdrawals are made and the interest rate is 3% per year, then

  $$B_{n+1} = 1.03B_n$$

- If the balance is initially B_0 =\$5000, then

  $B_1 =$

  Balance after 10 years is

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  $$B_{10} = 1.03^{10} \cdot \$5000 \approx 1.34 \cdot \$5000 \approx \$6719.58$$

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A really **simple population model:**

- Let P_n be the 'population density' at time n. A population increasing by a constant percentage each year could be modelled by:

$$P_{n+1} = kP_n$$

- If $k = 2$ and $P_0 = 0.1$, then

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$$P_{n+1} = k(P_n - P_n^2) = \underbrace{k(1 - P_n)}_{\text{corrected growth}} P_n$$

- $k = 2$, $P_0 = 0.1$:

$$P_1 = 2 \cdot (0.1 - 0.01) = 2 \cdot 0.09 = 0.18$$

- $k = 4$, $P_0 = 0.1$:

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- **Important:** Often simple rules can lead to complicated behaviour.

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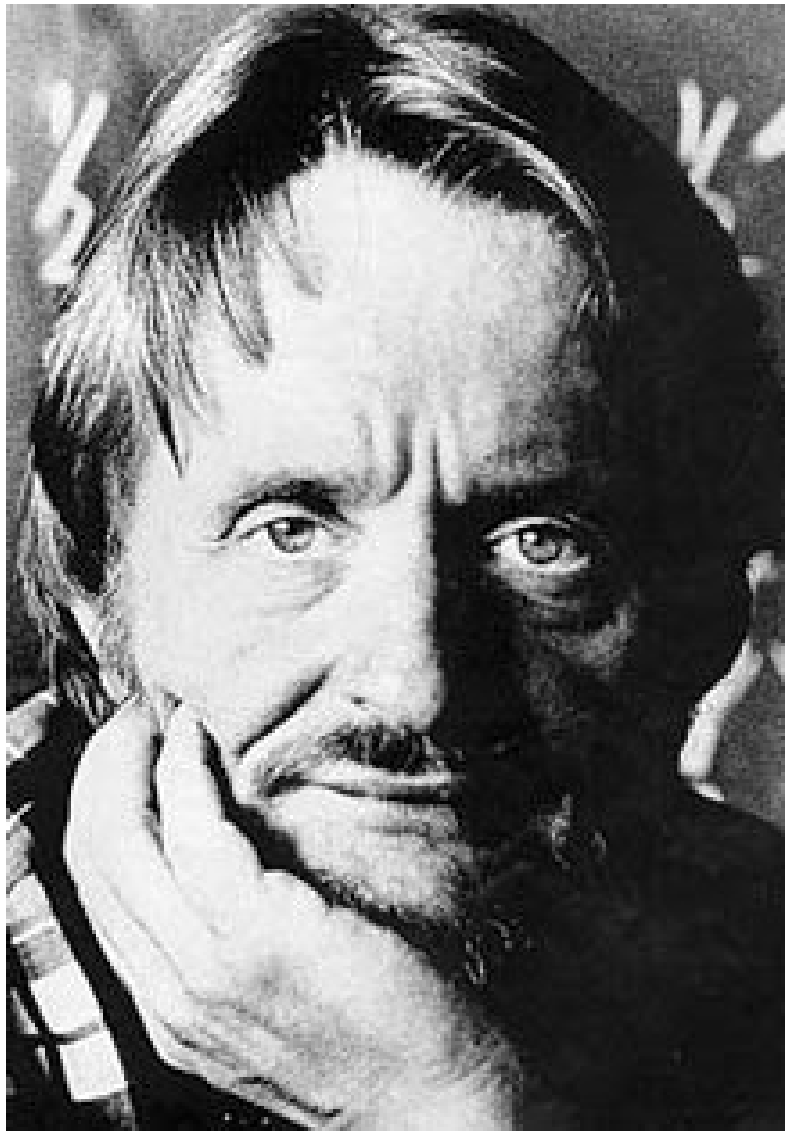
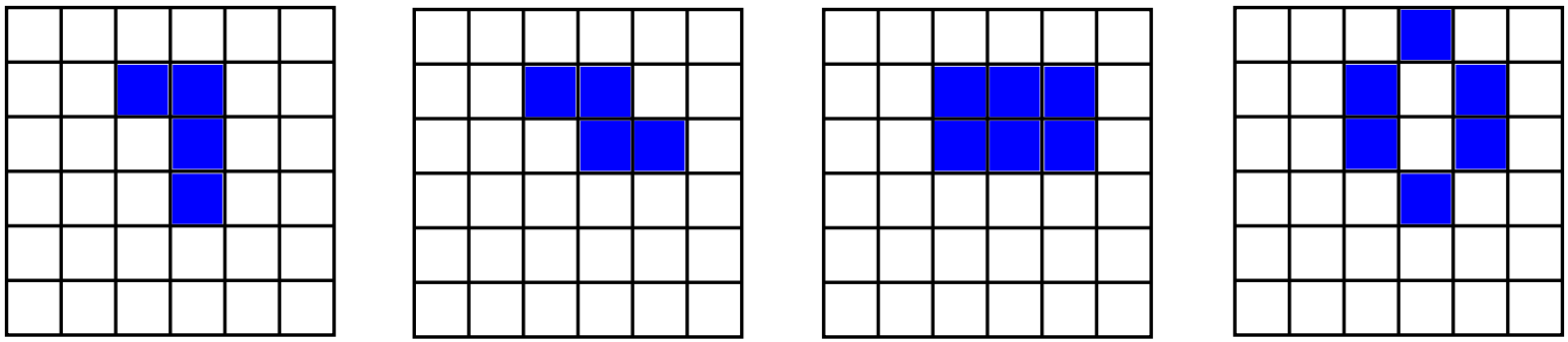
Conway's Game of Life



The Game of Life

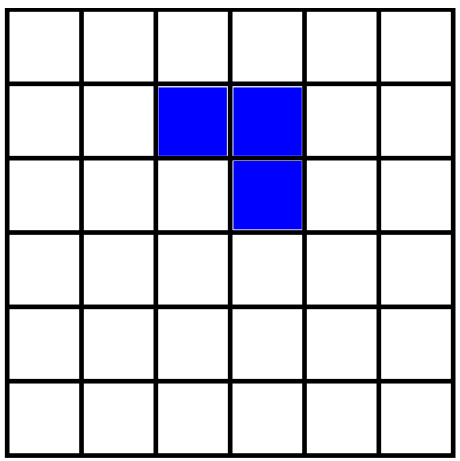


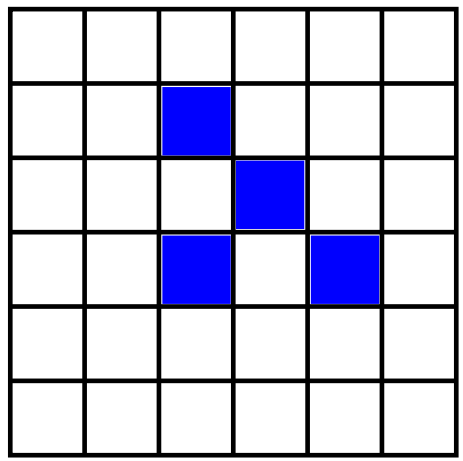
- The game is played on an infinite grid of ‘alive’ and ‘dead’ cells. Live cells are shaded, dead cells are blank.
- Each cell has eight “neighbours”. The current status of a cell’s neighbours determines the status (dead or alive) of the cell in the next generation.
- No random element in the game; just a few simple rules.

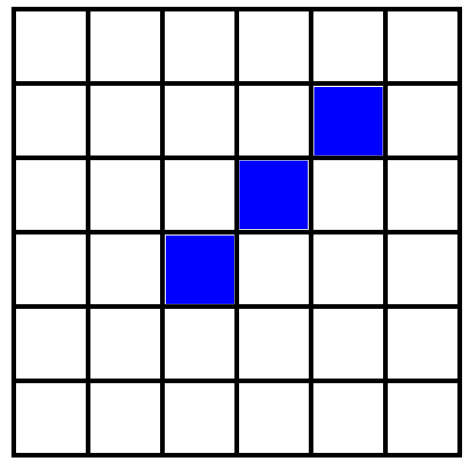
Rule 1: Staying alive

A living square will **remain alive** if it has **two or three neighbours**, otherwise it will die.

With the following initial configurations, which cells remain alive in the next generation?



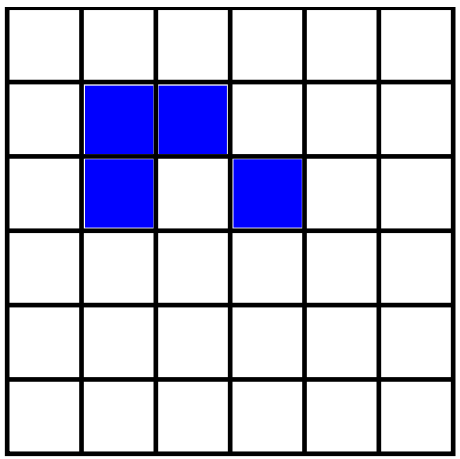


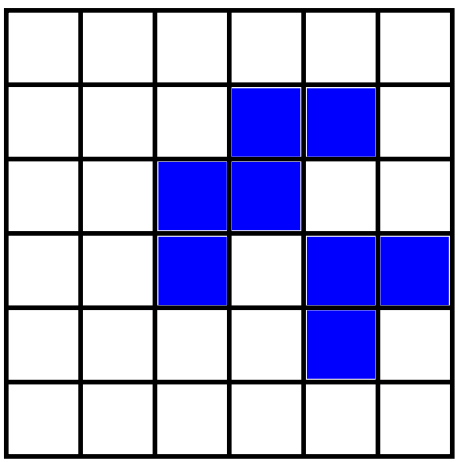


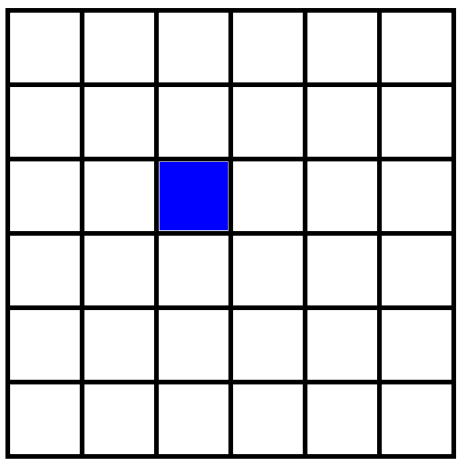
Rule 2: Dying

A living square will **die** if it has **fewer than two**, or **more than three neighbours**.

With the following initial configurations, which cells will die in the next generation?



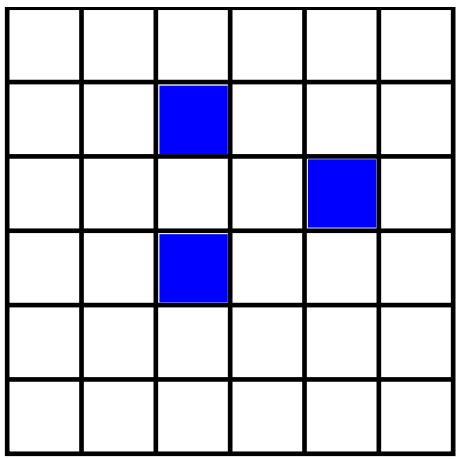


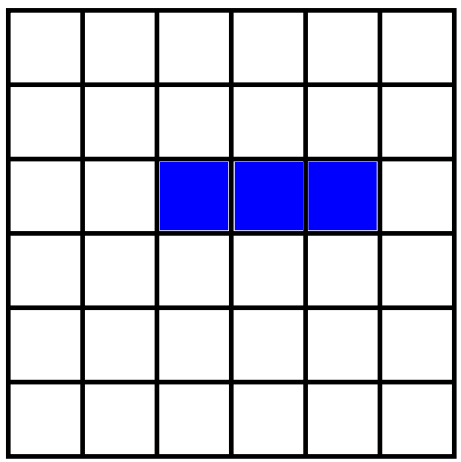


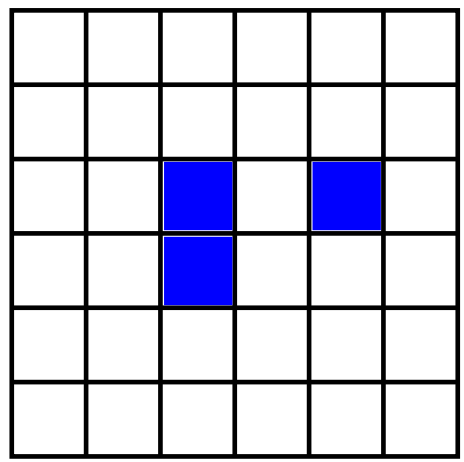
Rule 3: Coming alive

A dead square will **come alive** if it has *exactly* **three living neighbours**.

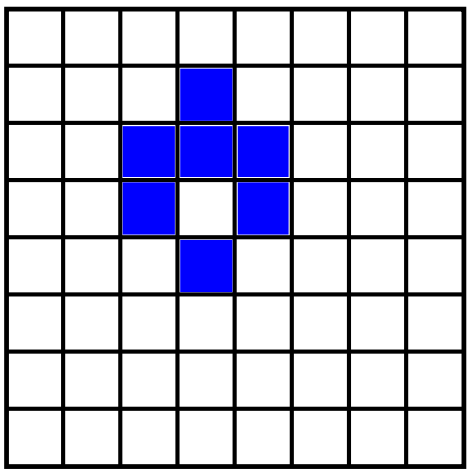
With the following initial configurations, which cells will come alive in the next generation?

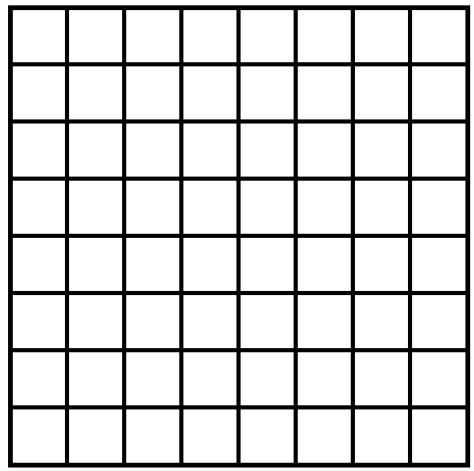


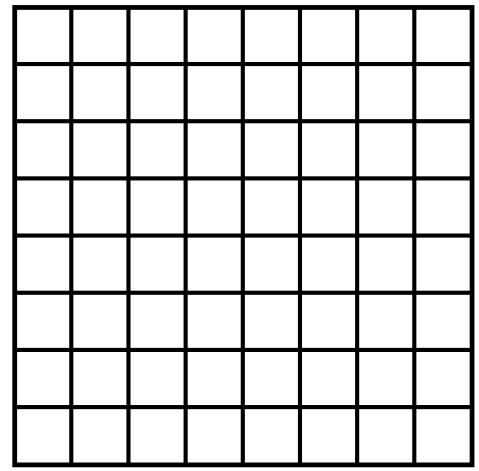


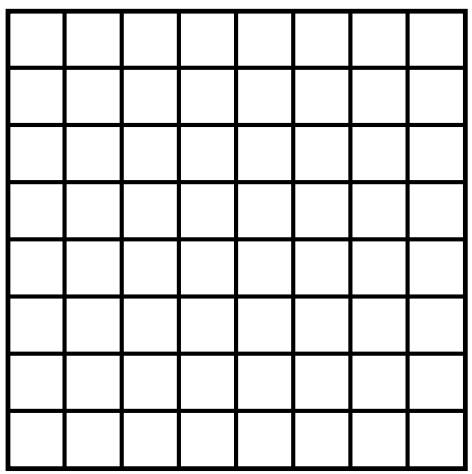


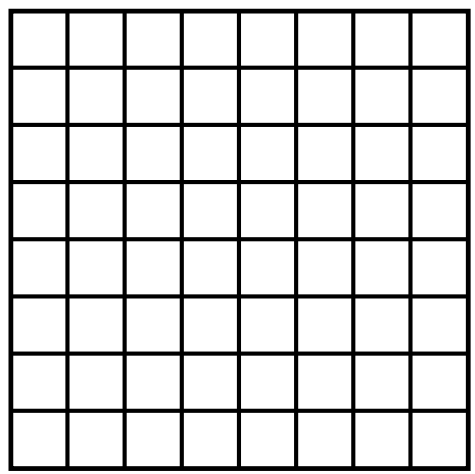
Have a go!

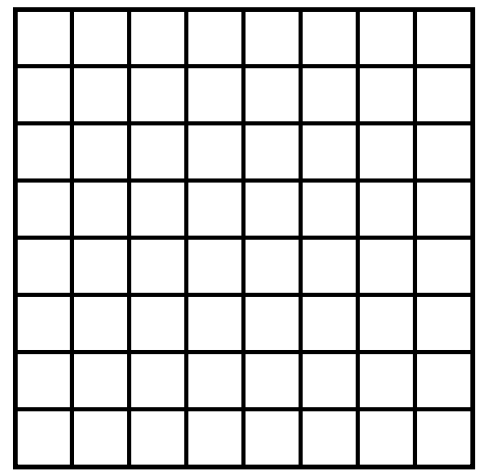












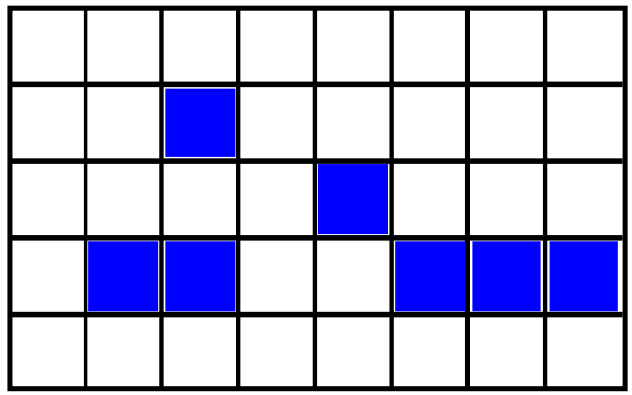
Types of behaviour

Types of behaviour we might see include:

- population **explosion**
- population **extinction**
- **steady pattern**
- **periodic pattern**
- **migratory pattern**

Types of behaviour

Sometimes the behaviour is very complicated. For instance, this initial configuration takes 5206 steps to settle down to a steady pattern.

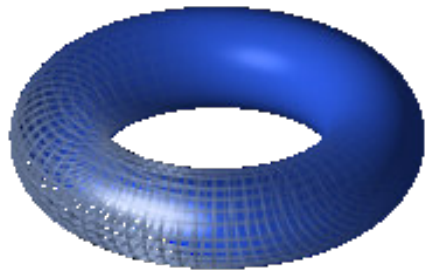


There's no way to tell from an initial configuration what will happen — just have to try it and see.

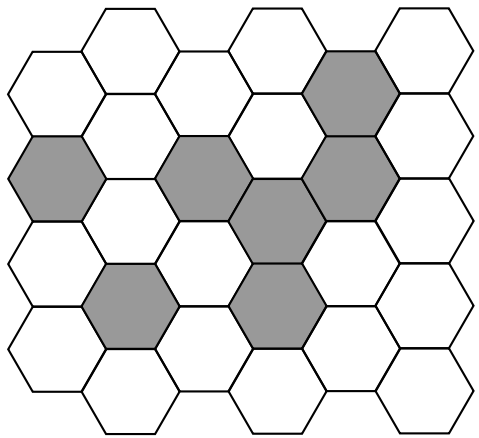
See `http://www.bitstorm.org/gameoflife/` for more information.

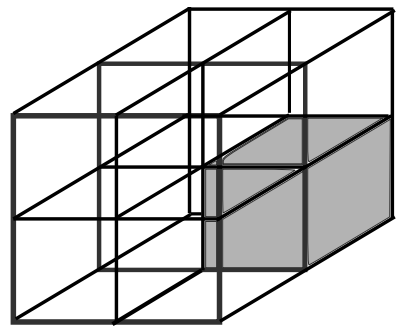
Other cellular automata

- Change the rules.
- Change the space.



- Change the grid.





Important ideas from today:

- The growth of populations can be modeled using fairly **simple repeating rules.**
- The **Game of Life** is one example of a **simple population model**.

For next time

- Read section 6.5